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Prediction of solar activity cycles by assimilating sunspot data into a dynamo model

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Abstract. Solar activity is a determining factor for space climate of the Solar system. Thus, predicting the magnetic activity of the Sun is very important. However, our incomplete knowledge about the dynamo processes of generation and transport of magnetic fields inside Sun does not allow us to make an accurate forecast. For predicting the solar cycle properties use the Ensemble Kalman Filter (EnKF) to assimilate the sunspot data into a simple dynamo model. This method takes into account uncertainties of both the dynamo model and the observed sunspot number series. The method has been tested by calculating predictions of the past cycles using the observed annual sunspot numbers only until the start of these cycles, and showed a reasonable agreement between the predicted and actual data. After this, we have calculated a prediction for the upcoming solar cycle 24, and found that it will be approximately 30% weaker than the previous one, confirming some previous expectations. In addition, we have investigated the properties of the dynamo model during the solar minima, and their relationship to the strength of the following solar cycles. The results show that prior the weak cycles, 20 and 23, and the upcoming cycle, 24, the vector-potential of the poloidal component of magnetic field and the magnetic helicity substantial decrease. The decrease of the poloidal field corresponds to the well-known correlation between the polar magnetic field strength at the minimum and the sunspot number at the maximum. However, the correlation between the magnetic helicity and the future cycle strength is new, and should be further investigated.

Keywords. Sun: activity, sunspots; methods: data analysis

1. Introduction

The interest to the problem of the solar cycle prediction has recently increased because of the unusually long minimum of solar activity. This minimum illustrates deficiencies in our understanding of the global processes in the Sun. The 400-year sunspot number record shows a chaotic behavior but also regular properties of the solar cycles, like the Waldmeier effect, which tells us that the stronger cycles have shorter razing times than the weaker cycles. Also, statistical analyses show the existence of long quasi-periodic variations of the solar activity amplitude in addition to the 11-years period (e.g. Miyahara *et al.* 2010). A number of methods for predicting solar cycles has been developed (e.g. Pesnell 2008), but the forecasts for the next solar cycle are widely different.

It is well-accepted that the variations of solar activity are a result of a complicate dynamo process in the convection zone. However, because of the imperfect dynamo models and deficiency of the necessary observational data, despite the known general properties of the solar cycles, a reliable forecast of the 11-year sunspot number is still a problem.

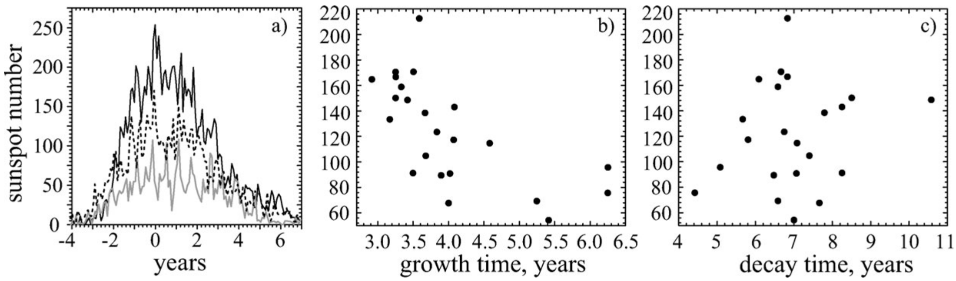

Figure 1. Basic properties of the solar cycles. Asymmetry of mean profile of sunspot number variations (a) shown for three solar cycles 14 (grey curve), 19 (black curve) and 23 (dotted curve), which are aligned according to their maxima ($t = 0$). Relationships between the amplitude of the sunspot number and the growth (b) and decay time (c) for the real solar cycles in 1755–2007.

The great variety of the predictions of the upcoming cycle 24 is caused by uncertainties in models and model parameters, and errors in both models and observations (Pesnell 2008). For this reason, we propose to use the data assimilation approach, which allows us to successively correct the model state according to observational data, and also take into account uncertainties of the model and data in the forecast solution.

Our idea is to combine observational data (in a first approximation, the sunspot data) and a dynamo model by a data assimilation method, and analyze the model and observational uncertainties. For application of the data assimilation approach we use the Ensemble Kalman Filter (EnKF) method, which works well for dynamical systems (Evensen 2007). In the first approximation, we consider the phenomenon of solar activity in the context of sunspot number variations, which have observational data during the past 23 solar cycles. In order to relate these data to dynamo models we propose to use the dependence of the sunspot number, W, on the toroidal component of magnetic field, B, in the form: $W \sim \mid B \mid^{3/2}$ (Bracewell 1953, 1988), for representation of the solar cycles.

For implementation of the EnKF method it is very important to reproduce in the dynamo model the basic properties of the solar cycle: in particular, the mean profile of the sunspot number variations (Fig. 1a), which is characterized by fast growth and slow decay, and the relationship between the cycle amplitude and the growth and decay times, (Fig. 1b and c). In the next section we consider a dynamo model, which includes the Parker's migratory dynamo model (Parker 1955) and an equation for magnetic helicity (Kleeorin & Ruzmaikin 1982; Kleeorin *et al.* 1995), which provides dynamical quenching of the dynamo process.

2. Parker's model with magnetic helicity and Waldmeier effect

Currently, there is no generally accepted model of the solar dynamo. However, most of the models are based on the Parker's oscillatory $\alpha\Omega$-dynamo mechanism (Parker 1955), which includes a turbulent helicity and magnetic field stretching by the differential rotation. Recent observational and theoretical investigations (e.g. Brandenburg & Subramanian 2005; Sokoloff 2007) revealed an important role of magnetic helicity (Pouquet *et al.* 1976). Thus, for this investigation we added to the original Parker's model an equation describing the evolution of the magnetic helicity, α_m. This equation was derived by Kleeorin and Ruzmaikin (1982) from a balance of the total magnetic helicity. Then, the dynamo model in local Cartesian coordinates can be written as

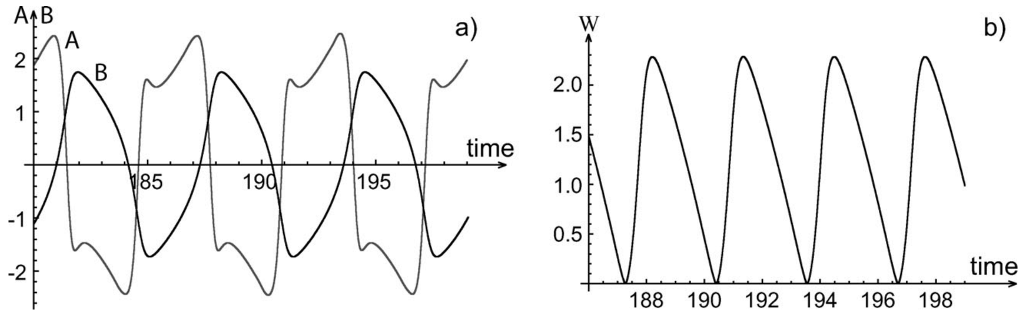

Figure 2. Periodical solutions of the dynamo model with included magnetic helicity for the middle convective zone parameters (panel a). Black and grey curves show variations for the toroidal component of magnetic field, B, and the vector-potential, A, of the poloidal magnetic field. Simulated variations of the sunspot number (panel b) obtained by applying the Bracewell's law: $W \sim\mid B \mid^{3/2}$ to the dynamo solution shown in panel a.

(Kitiashvili & Kosovichev 2009)

$$\begin{aligned}\frac{\partial A}{\partial t} &= \alpha B + \eta\nabla^2 A,\ \frac{\partial B}{\partial t} = G\frac{\partial A}{\partial x} + \eta\nabla^2 B, \\ \frac{\partial \alpha_m}{\partial t} &= \frac{Q}{2\pi\rho}\left[\langle\vec{B}\rangle(\nabla\times\langle\vec{B}\rangle) - \frac{\alpha}{\eta}\langle\vec{B}\rangle^2\right] - \frac{\alpha_m}{T},\end{aligned} \tag{2.1}$$

where B is the toroidal component of magnetic field, A is the vector potential of the poloidal component of the mean magnetic field, $\langle\vec{B}\rangle = \vec{B}_P + \vec{B}_T$ ($\vec{B}_P = \mathrm{curl}(0,0,A)$, $\vec{B}_T = (0,0,B)$ in the spherical coordinates); η describes the total magnetic diffusivity, which is the sum of the turbulent and molecular magnetic diffusivity; $\eta = \eta_t + \eta_m$ (usually $\eta_m << \eta_t$); $G = \partial\langle v_x\rangle/\partial y$ is the rotational shear; coordinates x and y are in the azimuthal and latitudinal directions respectively, parameter α is the total helicity represented in the form $\alpha = \alpha_h/(1+\xi B^2) + \alpha_m$, α_h and α_m are the kinetic and magnetic parts; ξ is a quenching parameter, ρ is density, T is a characteristic time of dissipation magnetic helicity (which includes dissipation though helicity transport) and, $Q \sim 0.1$.

Following the approach of Weiss *et al.* (1984) we average the system of equations (2) in a vertical layer to eliminate z-dependence of A and B and consider a single Fourier mode propagating in the x-direction assuming $A = A(t)e^{ikx}$, $B = B(t)e^{ikx}$. This dynamo model has been investigated in detail by Kitiashvili & Kosovichev (2009).

Figure 2 shows a typical nonlinear periodic solution (left panel) for the toroidal component of magnetic field, B (black curve), and vector-potential A (grey curve). We found that the dynamo model can reproduce the typical observed solar cycle profiles characterized by fast growth and slow decay (right panel). Figure 3 shows the relationships between of the amplitude of the sunspot number parameter, the growth and decay times and the cycle duration, for different values of α_h and D_0. These characteristic times were determined from the points of minima and maxima of the model sunspot number, W. The crosses represent the amplitude of the periodic field variations for $D_0 = -0.82$ and different values of kinetic helicity, α_h. The circles correspond to the case of constant $\alpha_h = 2.44$ and different values of dynamo number D_0. In the first case, the relationship between the cycle amplitude and the growth time is well-defined and monotonic. However, in the case of a fixed α_h and varying D_0 (circles), the amplitude initially, at small D_0, follows the same sequence as in the variable α_h case, but then at higher values of $|D_0|$ (shown by bigger circles) the amplitude decreases. The decay time is longer for the higher amplitude cycles (Fig. 3b).

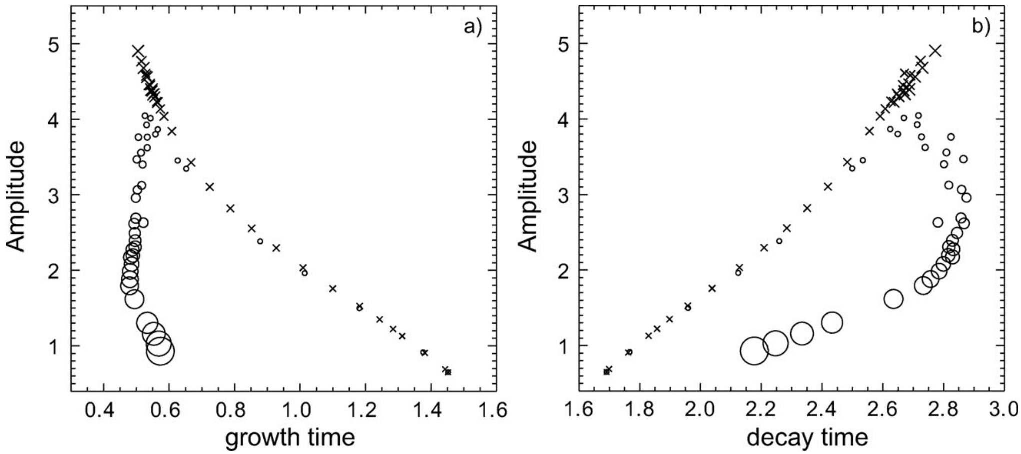

Figure 3. Relationships between the model sunspot number amplitude, W, and a) the cycle growth time and b) the decay time. The circles show a sequence for a fixed value of the kinetic helicity, $\alpha_h = 2.44$ and the dynamo number varying from -7 to -0.82. The crosses show the case of fixed $D_0 = -0.82$ and varying α_h, from 2.44 to 3. The size of the crosses and circles is proportional to the corresponding values of D_0 and α_h. Model parameters correspond to the middle convective zone: $\nu = 1.28, \lambda = 1.23 \times 10^{-6}$.

Thus, we have constructed the dynamo model, which reproduces qualitative properties of the sunspot cycles; and then using the data assimilation approach we can estimate the system state according to the observed sunspot number variations. In the next section, we briefly discuss the basic principles of the data assimilation approach.

3. Data assimilation methods

A primary target of any research is prediction of a physical system behavior. Often this is an extremely difficult problem, because the models are constructed with some approximations and assumptions, contain uncertainties, and deviate from the reality. Therefore, a theoretical model cannot describe the true condition of a system. On the other hand, observational data also include errors, which are often difficult to estimate.

The idea of data assimilation is in making continuous estimates of the system state and corrections to the initial conditions at subsequent moments of time according to the available observational data, and in creating a prediction of a future model state. Thus, the advantage of data assimilation methods is in their ability to combine the observational data and the models for possible efficient and accurate estimations of the physical properties, which cannot be observed directly. The data assimilation methods such as the Kalman Filter (Kalman 1960) allow us, with the help of an already constructed model and observational data, to determine the state of the model that is in agreement with a set of observations, and using this state as the initial conditions to obtain a forecast of future observations and error estimates (Evensen 2007; Kitiashvili & Kosovichev 2008). In our case, we know from observations the sunspot number (with some errors) and want to estimate the parameters of solar magnetic fields and helicity, described by the dynamo model given by equations (2.1).

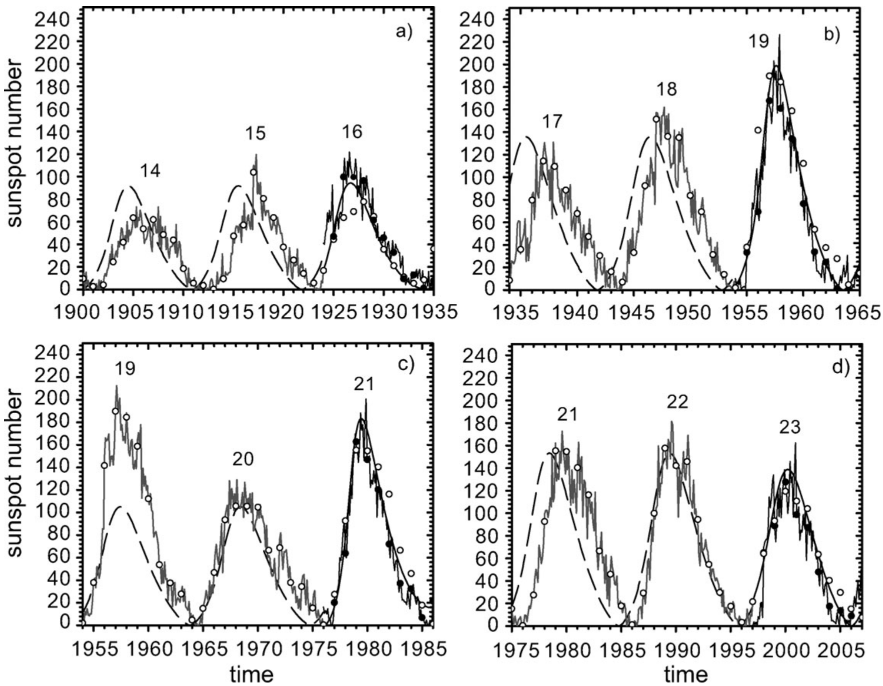

Figure 4. Predictions for solar cycles 16, 19, 20 and 23. Black dashed curves show the model reference solution. Gray curves show the best estimate of the sunspot number using the observational data (empty circles) and the model, for the previous cycles. Filled circles are simulated observational data. Black thick and thin curves show the reference solutions according new initial conditions and prediction results correspondingly.

4. Ensemble Kalman Filter in the solar cycle prediction problem

For predicting the solar cycle properties we use the Ensemble Kalman Filter (EnKF), a data assimilation method, which is effective for non-linear models formulated as dynamical systems and provides a statistical analysis of an ensemble of possible fluctuations (Evensen 2007). This method is tested by calculating predictions of the past cycles using only the observational data (annual sunspot numbers) until the start of these cycles.

Figure 4 shows examples of the EnKF implementation of the forecasting for solar cycles 16, 19, 20 and 23. In this approach, the exact model solution is corrected according to the previous observational data. This allows us to redefine the initial conditions of the model for the magnetic field components and helicity, and construct a model solution for the next time interval.

The experiments with the previous sunspot data show that this approach can provide reasonable forecasts of the strength of the following solar cycles. However, there are significant discrepancies between the forecasts and the actual data. For instance, the strength of cycle 16 is over-estimated, and the strength of cycle 19 is under-estimated. The main uncertainties are caused by inaccuracies in determining the time of the end of the previous cycle from the sunspot number data, and, of course, by the incompleteness of the model and insufficiency of the sunspot number data. In particular, we found the forecast can be inaccurate when the sunspot number changes significantly from the value of the previous cycle (Kitiashvili & Kosovichev 2008).

In our forecast experiments, we found a strong dependence on the phase relationship between the reference model solution and the observations. The phase difference appears

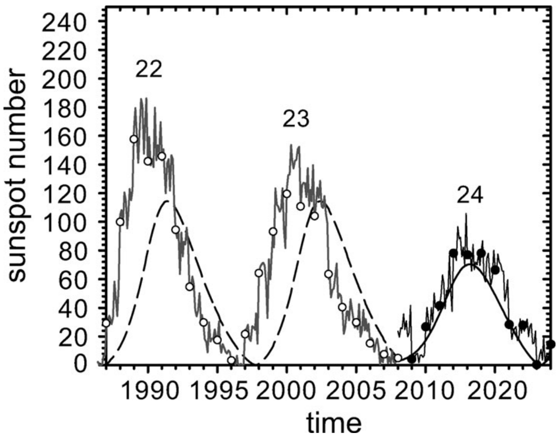

Figure 5. Prediction of the solar cycle 24. Notations are the same as in Fig. 4

to be due to the constant period of the model solution. Curiously, when the model phase is ahead of the solar cycle phase, adding a data point at the start of the cycle substantially improves the forecast. However, when the model phase lags, this improvement does not happen. This effect is taken into account by correcting the phase of a reference solution that it is slightly ahead of the solar cycle phase.

We used the same analysis scheme for predicting of the upcoming solar cycle 24 (Fig. 5). According to our analysis, the solar cycle 24 which starts in 2009 will be weaker than the current cycle by approximately 30% (Kitiashvili & Kosovichev 2008). The estimated formal error of our prediction is $\sim 10\%$.

5. Estimation of the state of the solar dynamo

The basic data assimilation procedure in our forecast scheme is in estimating the properties (or state) of the solar dynamo model at the solar minima by using the sunspot data available up to this minimum. In our simple model these properties include the toroidal component of magnetic field, B, the vector-potential of the poloidal component, A, and the magnetic helicity, α_m. These properties are estimated by using the Ensemble Kalman Filter method. Then, they are used as the initial conditions for calculating the properties of the next solar cycle by solving Eqs (2.1). Thus, it is interesting to see how these initial conditions correlate with the predicted maxima of the sunspot number.

In Figure 6 (top panel) we shows the "predicted" maximal annual sunspot number (squares) for solar cycles from 16 to 24, with an estimated the forecast uncertainty and the actual observed values for cycles 16–23 (circles). In the bottom panel, we show the corresponding initial conditions for the preceding solar minima. The variations of the toroidal field, B, do not show a particular pattern, and are close to zero as expected for during the solar minima. The vector-potential of poloidal field, A, shows changes the sign changes corresponding to the polar field reversal. The amplitude at the start of cycles 20 and 24 is substantially lower than during the other minima. This may correspond to the well-known correlation between the strength of the polar magnetic field and the following sunspot number (Schatten 2005; Svalgaard *et al.*, 2005). However, there is no such correlation for weak cycles 16 and 17. The variations of the magnetic helicity, α_m, (solid curve with triangles) shows significantly better correlation with the future sunspot numbers, indicating that the magnetic helicity substantially decreases prior the weak sunspot cycles. Perhaps, the combination of two factors, the strength of the poloidal

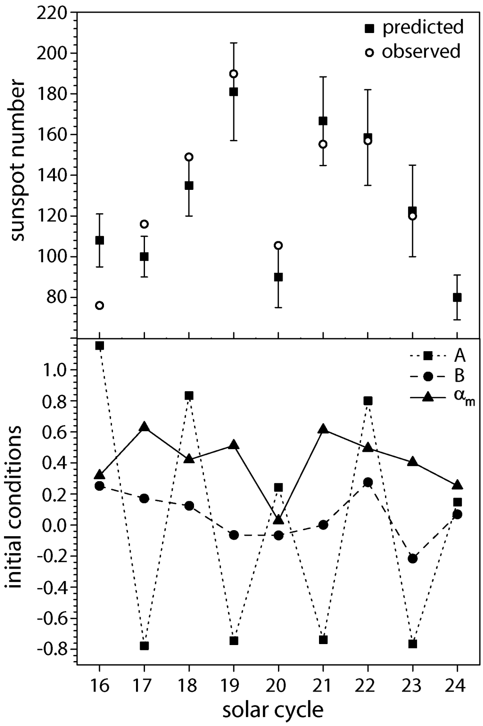

Figure 6. Correlation between the predictions of the sunspot maxima (squares with errorbars), the actual observed maximum sunspot numbers (open circles), shown in the top panel, and the initial conditions of the dynamo model (vector-potential of the poloidal magnetic field component, A, toroidal magnetic field component, B, and the magnetic helicity, α_m) at the preceding minima, obtained by the EnKF method for the solar cycle forecasts (bottom panel).

magnetic field and the level of the magnetic helicity determines the strength of the future cycles. Of course, this requires further investigation.

6. Conclusions

We have presented a numerical analysis of a simple non-linear dynamical model, which includes the classical Parker's dynamo equations (Parker 1955) with the standard α-quenching and an equation for the magnetic helicity evolution (Kleeorin & Ruzmaikin 1982). The formulation for the magnetic helicity is based on the balance between the large-scale and turbulent magnetic helicities. Using a low-order dynamical system approach we have examined the influence of the kinetic and magnetic helicities on the non-linear fluctuations of the dynamo-generated magnetic field in the conditions of the solar plasma, and compared the model solutions with the sunspot number variations observed during the solar 11-year cycles. The analysis of the model showed the existence of non-linear periodic and chaotic solutions (Kitiashvili & Kosovichev 2009). For this model we obtained profiles of the sunspot number variations, which qualitatively reproduce the typical profile of the solar cycles.

The results of assimilation of the annual sunspot number data into the solar dynamo model and the prediction of the previous solar cycles (Fig. 4 and 5) demonstrate a new method of forecasting the solar activity cycles. This method predicts a weak solar cycle 24 with a maximum of the smoothed annual sunspot number of approximately 80 (Fig. 5).

It is interesting to note that the simulations show a delay of the upcoming cycle that is observed at the present time. According to the prediction, the maximum of the next cycle will be reached approximately in 2013.

Using the data assimilation results we have estimated the parameters of the dynamo model during the solar minima and found that the vector-potential of the toroidal component substantially decreased prior the weak sunspot cycles 20 and 23, and also at the present time, prior cycle 24. This may corresponds to the observed weak polar magnetic fields. The level of the magnetic helicity seems to show a reasonably good correlation with the future sunspot maxima. This may have important implications for solar dynamo models, but, of course, requires further investigation.

Future investigations of the data assimilation approach will include more complete dynamo models and assimilation of the solar synoptic magnetic field data, which are available for the past 3 cycles.

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