Calculation of Observables from HMI Data (REVISED ON JULY 15, 2011)

Sébastien Couvidat and the HMI team

W.W. Hansen Experimental Physics Laboratory, Stanford University, Stanford, CA 94305-4085, USA

1. INTRODUCTION

This document provides a brief description of the algorithm used in the DRMS module HMI_observables to produce the line-of-sight observables (in series hmi.V_45s, hmi.M_45s, hmi.Ic_45s, hmi.Lw_45s, hmi.Ld_45s, and their _nrt counterparts). Only the MDI-like algorithm is discussed, even though long-term plans include the use of more robust algorithms (like the least-squares fit of an appropriate Fe I line profile). This algorithm is based on what was done for the MDI instrument onboard SOHO (the main difference being the use of 6 sampling points across the Fe I line instead of 5).

2. MDI-Like Algorithm

Ideally, we want to calculate the first and second Fourier coefficients a_n and b_n of the Fe I line profile $I(\lambda)$ where λ is the wavelength:

$$a_1 = \frac{2}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} I(\lambda)\cos\left(2\pi\frac{\lambda}{T}\right)d\lambda \quad (1)$$

$$b_1 = \frac{2}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} I(\lambda)\sin\left(2\pi\frac{\lambda}{T}\right)d\lambda \quad (2)$$

$$a_2 = \frac{2}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} I(\lambda)\cos\left(4\pi\frac{\lambda}{T}\right)d\lambda \quad (3)$$

$$b_2 = \frac{2}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} I(\lambda)\sin\left(4\pi\frac{\lambda}{T}\right)d\lambda \quad (4)$$

where T is the "period" of the line profile, taken to be 6 times the wavelength separation between 2 filters (nominally $T = 6 \times 68.8 = 412.8$ mÅ). We assume that the solar line has a

Gaussian profile:

$$I(\lambda) = I_c - I_d \exp\Big(-\frac{(\lambda-\lambda_0)^2}{\sigma^2}\Big) \tag{5}$$

where I_c is the continuum intensity, I_d is the linedepth, λ_0 is the Doppler shift, and σ is related to the linewidth. The Doppler velocity can be expressed as:

$$v = \frac{dv}{d\lambda}\frac{T}{2\pi}\,\mathrm{atan}\Big(\frac{b_1}{a_1}\Big) \tag{6}$$

where $dv/d\lambda = 299792458.0/6173.3433 = 48562.4$ m/s/Å. The second Fourier coefficients could also be used:

$$v = \frac{dv}{d\lambda}\frac{T}{4\pi}\,\mathrm{atan}\Big(\frac{b_2}{a_2}\Big) \tag{7}$$

The linedepth I_d is equal to:

$$I_d = \frac{T}{2\sigma\sqrt{\pi}}\sqrt{a_1^2+b_1^2}\,\exp\Big(\frac{\pi^2\sigma^2}{T^2}\Big) \tag{8}$$

while σ is equal to:

$$\sigma = \frac{T}{\pi\sqrt{6}}\sqrt{\mathrm{alog}\Big(\frac{a_1^2+b_1^2}{a_2^2+b_2^2}\Big)} \tag{9}$$

If the linewidth L_w of the Fe I line is defined as its FWHM, then $L_w = 2\sqrt{\mathrm{alog}(2)}\sigma$.

Since HMI samples the Fe line at only 6 points, and assuming that the HMI filter transmission profiles are delta functions, the Fourier coefficients can be approximated as:

$$a_1 \approx \frac{2}{6}\sum_{j=0}^{5} I_j \cos\Big(2\pi\frac{2.5-j}{6}\Big) \tag{10}$$

$$b_1 \approx \frac{2}{6}\sum_{j=0}^{5} I_j \sin\Big(2\pi\frac{2.5-j}{6}\Big) \tag{11}$$

$$a_2 \approx \frac{2}{6}\sum_{j=0}^{5} I_j \cos\Big(4\pi\frac{2.5-j}{6}\Big) \tag{12}$$

$$b_2 \approx \frac{2}{6}\sum_{j=0}^{5} I_j \sin\Big(4\pi\frac{2.5-j}{6}\Big) \tag{13}$$

The MDI-like algorithm consists in calculating a discrete approximation of the first and second Fourier coefficients of the Fe I line, using the 6 intensities (labeled I_0 to I_5 in order of decreasing wavelength) measured on the HMI CCDs and Eqs (10) to (13), separately for the LCP and RCP polarizations. Applying Equation (6) to the estimates of the first Fourier coefficients, we obtain two estimates of the Doppler velocity (one for the LCP polarization, and one for the RCP polarization): v_{LCP} and v_{RCP}.

2.1. Look-Up Tables

However the HMI filter transmission profiles are not delta functions, the discrete approximations to the Fourier coefficients are far from perfect because of the reduced number of sampling points, and the Fe I line does not have a Gaussian profile. Therefore, the Doppler velocities previously derived need to be corrected. To this end we use look-up tables, computed separately in another DRMS module (lookup) and put in the hmi.lookup series. These look-up tables are obtained the following way: a model of the Fe I line profile at rest is selected; this profile is shifted in wavelength to simulate the impact of a given Doppler velocity. At each Doppler shift, the shifted line profile is multiplied by the 6 HMI filter transmission profiles (derived from phase maps), and the corresponding integral values I_0 to I_5 are calculated. Equations (10) to (13) and Equation (6) are applied. Therefore, the Doppler velocity returned by the MDI-like algorithm is obtained as a function of the actual (input) Doppler velocity. The inverse function is called look-up table (a misnomer). In practice several tables are calculated: two per HMI CCD pixels, or per spatial average of pixels (one table for the velocity based on the first Fourier coefficients, and one for the velocity based on the second Fourier coefficients). Typically, these look-up tables are calculated for 821 input Doppler velocities, with a step of 24 m/s, covering the range [-9852,+9852] m/s.

The tables are linearly interpolated at v_{LCP} and v_{RCP}, producing the corrected Doppler velocities V_{LCP} and V_{RCP}.

2.2. Polynomial Correction

The look-up tables are derived from the HMI-filter transmission profiles. These profiles are obtained from calibration sequences taken on a regular basis (the profiles of the Michelson interferometers slowly drift in wavelength). Unfortunately, the calibration of the filter transmission profiles is not perfect and there are some residual errors (at the percent level) on their transmittances. Therefore, the corresponding look-up tables are also imperfect. The SDO orbital velocity is known very accurately, and it is possible to use it to correct the look-up tables. In practice, we produce the function $V = f(OBS\_VR)$ where OBS_VR is the radial velocity Sun-SDO. V is the median value of the Doppler velocities derived as previously described, over 99% of the solar radius. This function is fit by a polynomial of order three, and these polynomial coefficients (typically one or two sets of coefficients are calculated per day and are linearly interpolated in time) are used to correct the Doppler velocities V_{LCP} and V_{RCP} returned by the look-up tables. The resulting velocities V'_{LCP} and

$V'_{\rm RCP}$ are then combined to produce the final Doppler-velocity estimate:

$$V = \frac{V'_{\rm LCP} + V'_{\rm RCP}}{2} \tag{14}$$

while the line-of-sight magnetic field strength B is:

$$B = (V'_{\rm LCP} - V'_{\rm RCP})K_m \tag{15}$$

Where $K_m = 1.0/(2.0 \times 4.67 \cdot 10^{-5} \times 0.000061733433 \times 2.5 \times 299792458.0)$ for a Lande g-factor of 2.5.

2.3. Actual Implementation for the Linedepth, Linewidth, and Continuum Intensity in the HMI Pipeline

The actual implementation of the MDI-like algorithm in the HMI pipeline at Stanford University differs a bit from the previous equations. The linewidth σ returned by Equation (9) is multiplied by a factor $K_1 = 5/6$. Although the presence of this factor stems from an error in the derivation of the MDI-like algorithm equations, tests on simulated HMI data (using a Gaussian profile for the Fe I line) have shown that Equation (9) overestimates the true linewidth by $\approx 20\%$. Therefore, multiplying it by K_1 provides a much more accurate estimate, and it was decided to keep this "corrective" factor in the pipeline. Conversely, the same tests have shown that the linedepth I_d returned by Equation (8) is underestimated compared to the actual I_d. Currently, I_d returned by Equation (8) is multiplied by a factor $K_2 = 6/5$: this produces a more accurate linedepth, even though it is still underestimated. An advantage of having $K_2 = 1/K_1$ is that the continuum intensity I_c remains the same as what is returned by Equation (16) (see below): indeed, the integral of a Gaussian is proportional to $\sigma \times I_d$ and therefore it does not change when the factors K_1 and K_2 are introduced.

Moreover, in the pipeline, I_d is calculated based on a standard value of σ: the value returned by Equation (9) is not used for the calculation of I_d and I_c because it has various issues described in the next section. The standard σ value as a function of center-to-limb distance is obtained by azimuthally averaging σ returned by Equation (9) (and multiplied by K_1) about the solar disk center, using a linewidth image obtained during a period of low solar activity.

The estimate (proxy) of the continuum intensity I_c is obtained by "reconstructing" the solar line from the estimates of the Doppler shift λ_0, the linewidth σ, and the linedepth I_d:

$$I_c = \frac{1}{6}\sum_{j=0}^{5}\left[I_j + I_d \exp\left(-\frac{(\lambda-\lambda_0)^2}{\sigma^2}\right)\right] \tag{16}$$

2.4. Caveats

Currently, we only apply look-up tables and polynomial correction to the Doppler velocities. Therefore, only the Dopplergrams and magnetograms are corrected. We might also be able to calculate look-up tables for the linewidth, linedepth, and continuum intensity, but this has not been done so far. The factors K_1 and K_2 may need to be revised (in particular, K_2 should probably be revised upwards). Initially, we were planning to use the first and second Fourier coefficients to derive the Doppler velocities. However, several issues arose with the second Fourier coefficients: they are strongly dependent on the exact shape of the Fe I line. Any departure from a Gaussian profile creates discrepancies between the velocities returned by the first and second Fourier coefficients. The actual Fe I line profile, as seen by HMI, is not well known. This profile depends on the MTF/PSF of the instrument (due to, among others, convective blueshift), which is currently poorly known. Using various line profiles (from the Kitt Peak and Mount Wilson observatories, and from Voigt and Gaussian models), it was shown that the velocity returned by the first Fourier coefficient is relatively robust (it is relatively independent of the actual line profile), which is not the case for the velocity returned by the second Fourier coefficient. Moreover, the second Fourier coefficients seem to saturate faster (for smaller Doppler shifts) than the first Fourier coefficients, leading to sudden and unphysical variations in the quantitities using them (like the linewidth), especially in regions of strong and/or inclined magnetic field. Also, we decided to drop, for the time being, the second Fourier coefficient from the calculation of the Doppler velocities, and of the line-of-sight magnetic-field strength. Therefore the noise level due to photon noise is increased by $\sqrt{2}$ compared to its optimal value. Another issue is that the look-up tables are based on an Fe I line profile that is less realistic in strong and inclined magnetic-field regions. The polynomial correction is also not optimal in such active regions and near the solar limb, because the SDO OBS_VR velocity is limited to the range [-3.5,+3.5] km/s: outside of this range the polynomial correction is a basic extrapolation.