COULD TORSIONAL OSCILLATIONS EXCITE ROSSBY WAVES IN THE PHOTOSPHERE OVER A SUNSPOT? A (SIMPLISTIC?) TOY MODEL...

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INITIAL ASSUMPTIONS OF THE MODEL

Under the following assumptions, we calculate the possible forced response in terms of Rossby waves to the torsional-oscillation zonal flow crossing the photosphere over a sunspot.

- We only consider motions in the photosphere. Following Lou (2000) and Kaladze & Wu (2006) who studied photospheric Rossby waves, we use the shallow-water approximation because the photosphere is thin (height $H \approx 500$ km) compared to the length scale of horizontal motions studied (typically $\approx$ hundreds of Mm).
- The torsional oscillations (Howard and LaBonte 1981) are a large-scale zonal flow with a time scale of ≈ 1 year and a ≈ 5 m s^{-1} velocity at the photospheric level. Following Spruit (2003), we assume it is in quasi-geostrophic balance.
- The sunspot impacts the vertical stratification in the photosphere, but dynamical and magnetic-field effects are ignored. This is simplistic: with actual sunspots, the magnetic field hampers crossing by a flow, as do coherent horizontal flows in the photosphere like the Evershed flow.

EFFECTIVE DEPTH OF A FLUID PARCEL IN THE PHOTOSPHERE OVER A SUNSPOT

- Potential temperature θ: temperature that a fluid parcel would have if it were expanded/compressed adiabatically from its existing (p,T) to a reference p (e.g. Vallis 2006) (lines of constant θ=isentropes). For an ideal gas, θ is (Holton 2004):

$$\theta(T,p) = T\left(\frac{p_r}{p}\right)^{\frac{R}{cp}} \tag{1}$$

  with the usual notations. p_r is a reference pressure, c_p a specific heat capacity at constant pressure, and R is the ideal gas constant.

- Ertel potential vorticity of a fluid parcel in an horizontal flow and in isentropic coordinates (Holton 2004):

$$Q = (\zeta + f)\left(\frac{1}{\rho}\frac{\partial\theta}{\partial z}\right) \tag{2}$$

  ρ is the fluid density, z is the vertical coordinate, ζ is the vertical component of the relative vorticity ω ($\omega = \nabla \times \mathbf{v}$) given at a specific θ, and f is the Coriolis parameter ($f = 2\,\Omega\sin(\vartheta)$) for a latitude ϑ and rotation rate Ω.

Only the vertical component of vorticity is of interest because the motion studied is horizontal. **Potential vorticity is better described as the ratio of the absolute vorticity $\zeta + f$ of a parcel to its effective depth** $(\frac{1}{\rho}\frac{\partial\theta}{\partial z})^{-1}$, and is conserved during an adiabatic motion.

Basic idea: the magnetic field of a sunspot locally impacts the stratification. This produces a change in effective depth of a fluid parcel crossing the photosphere over the sunspot, resulting in a change in absolute vorticity of this parcel (for Q to be conserved): changes in Q drive Rossby waves.

Indeed, Fig 1 shows that in the photosphere over the umbra of Fontenla et al. (2006), the effective depth is ≈ 0.47 times the effective depth in the quiet Sun. Therefore the absolute vorticiy is also expected to decrease over the sunspot.

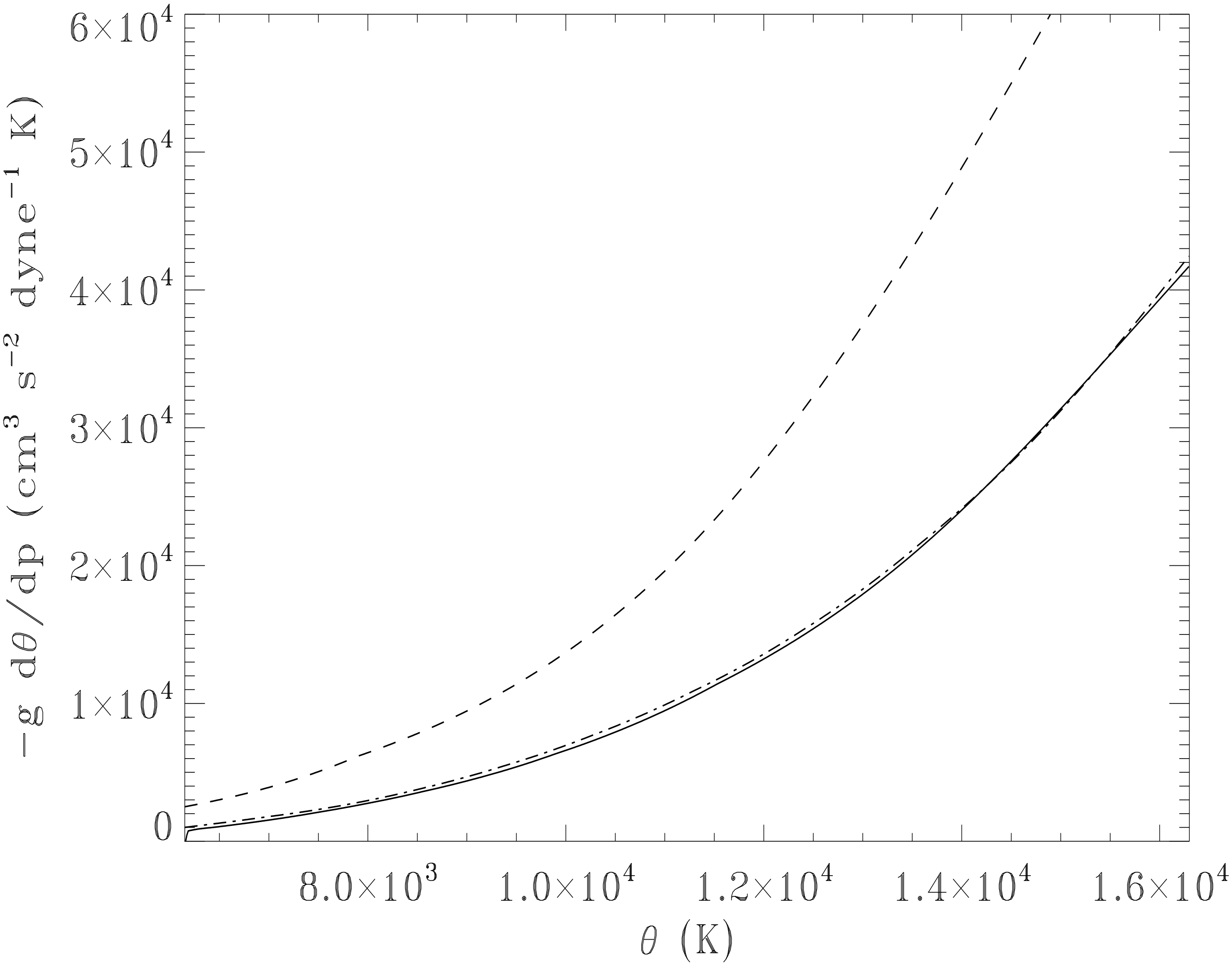

Figure 1: Inverse of the effective depth of a fluid parcel in the photosphere as a function of potential temperature θ in the quiet Sun (solid line), in a sunspot penumbra (dash-dotted line), and in a sunspot umbra (dashed line), based on the models C, R, and S of Fontenla et al. (2006). The reference pressure $p_r = 10^5$ dyne cm^{-2}.

FORCED RESPONSE TO A CHANGE IN EFFECTIVE DEPTH

To study the excitation of Rossby waves resulting from a change in absolute vorticity produced by a change in effective depth, we use the quasi-geostrophic potential vorticity equation (QGPVE) in a one-layer shallow-water system. In a shallow-water system, the effective depth of a fluid parcel is replaced by an actual depth: the thickness of the fluid layer.

- requirements for quasi-geostrophy (Pedlosky 1987): first, Rossby number $Ro = \bar{u}/(2\Omega L) << 1$ ($\bar{u}$ = average zonal flow velocity, L = typical spatial scale of the perturbed flow). For torsional-oscillation velocities $\bar{u} = 5$ m s^{-1} and $L \approx R_\odot$, $Ro < 0.002$; second, $1/(fT) << 1$ (T = typical timescale). For $T = 1$ year, the timescale that Spruit (2003) associates to the torsional oscillations, $1/(fT) < 0.02$ (for f calculated at latitude $\vartheta = 20°$).
- requirements for shallow-water approximation: Lou (2000) demonstrated how for large-scale and slow motions the photosphere can successfully be treated as nearly incompressible and the shallow-fluid approximation can be applied.
- β-plane approximation: the shallow-water equations are solved in Cartesian geometry, instead of spherical one. The Coriolis parameter $f = f_0 + \beta y$, where y is the local coordinate in the northward direction, f_0 is the Coriolis parameter at the reference latitude ϑ_0 (for which $y = 0$), and $\beta = df/dy$. This approximation is strictly valid only at ϑ_0. This approximation is used for photospheric Rossby waves by Lou (2000) and Kaladze & Wu (2006).

We start from a geostrophic flow $\mathbf{v}$ and the QGPVE for inviscid shallow-water systems: $Dq/Dt = 0$.

q is the shallow-water quasi-geostrophic potential vorticity, D/Dt the material derivative, and t the time (Vallis 2006). q is closely related to the potential vorticity Q (the same name "potential vorticity" is used to describe different quantities in fluid mechanics). Local Cartesian coordinates are (x, y, z): $x > 0$ westward (with the solar convention), $y > 0$ northward, and $z > 0$ above the bottom of the photosphere. $\mathbf{v} = (u, v, w)$ and $\mathbf{u} = u\mathbf{i} + v\mathbf{j}$. The vertical component of the relative vorticity is $\zeta = \partial v/\partial x - \partial u/\partial y$.

We linearize the potential vorticity equation:

$$\frac{\partial q'}{\partial t} + \bar{\mathbf{u}} \cdot \nabla_{\mathbf{h}} q' + \mathbf{u}' \cdot \nabla_{\mathbf{h}} \bar{q} = 0 \tag{3}$$

where $\nabla_{\mathbf{h}}$ is the horizontal gradient, $q(x, y, t) = \bar{q}(x, y) + q'(x, y, t)$, and $\mathbf{u}(x, y, t) = \bar{\mathbf{u}}(x, y) + \mathbf{u}'(x, y, t)$ (prime=perturbed quantity; bar=steady one).

In the quasi-geostrophic approximation, q can be expressed as (e.g., Pedlosky 1987; Vallis 2006):

$$q = \zeta + \beta y - \frac{f_0}{H}(\eta - h_b) \tag{4}$$

where $\eta(x, y, t) = H + \eta'(x, y, t)$ is the height of the free surface of the active layer and h_b is the height of the resting-fluid layer (the bottom of the photosphere). Any change in the effective depth of a fluid parcel in the stratified photosphere will be conveniently simulated in the shallow-water system by a change in $h_b = 0 + h'_b$: this directly impacts the thickness $\eta - h_b$ of the active fluid layer.

Because the flow $\mathbf{u}$ is quasi-geostrophic its horizontal divergence is zero to the first order and we introduce a streamfunction $\Psi(x, y, t) = \bar{\Psi}(x, y) + \Psi'(x, y, t)$ to describe it, such as $\zeta = \nabla^2 \Psi$.

Turbulence in the photosphere creates a lot of friction resulting in a large kinematic viscosity ν. Following Charney & Eliassen (1949), we include a linear damping term $r\zeta'$ (Rayleigh friction) in Eq (3), which can be written:

$$\frac{\partial}{\partial t}(\zeta' - \frac{\Psi'}{L_d^2}) + \frac{f_0}{H}\frac{\partial h'_b}{\partial t} + \bar{u}\frac{\partial \zeta'}{\partial x} + \bar{v}\frac{\partial \zeta'}{\partial y} + \beta v' + u'\frac{\partial \bar{\zeta}}{\partial x} + v'\frac{\partial \bar{\zeta}}{\partial y} + r\zeta' = -\frac{f_0}{H}(\bar{u}\frac{\partial h'_b}{\partial x} + \bar{v}\frac{\partial h'_b}{\partial y}) \tag{5}$$

where $L_d = \sqrt{gH}/f$, g is the gravitational acceleration, and $r = \nu/H^2$ when ignoring any Ekman layer. r is the inverse of the spin-down time, i.e. the e-folding time of vorticity perturbations.

Now, $\bar{\mathbf{u}}$ is assumed to be purely zonal ($\bar{v} = 0$), and meridionally and zonally uniform ($\bar{u}$ is independent of x and y). Meridional invariance is a crude approximation for torsional oscillations.

As is usual (e.g., Lou 2000), we assume that the solutions to Eq (5) are linear combinations of plane waves:

$$\Psi'(x, y, t) = \mathrm{Re}\{\hat{\Psi}\exp(i(kx + ly - \omega t))\} \tag{6}$$

and, because h_b' depends on time:

$$h_b'(x, y, t) = \mathrm{Re}\{\hat{h_b}\exp(i(kx + ly - \omega t))\} \tag{7}$$

where k and l are the horizontal wavenumbers in the, respectively, x and y directions, and ω is the angular frequency.

After injecting Eqs (6) and (7) into Eq (5), and discarding the terms in $\bar{v}$:

$$\hat{\Psi} = \frac{\frac{f_0}{H}(1 - \frac{\omega}{\bar{u}k})\,\hat{h_b}}{-\frac{\omega K^2}{\bar{u}k} - \frac{\omega K_d^2}{\bar{u}k} + K^2 - K_s^2 - i\frac{K^2 r}{\bar{u}k}} \tag{8}$$

where $K^2 = k^2 + l^2$, $K_d = 1/L_d$, and $K_s = \sqrt{\beta/\bar{u}}$. Eq (8) shows that a stationary response ($\omega = 0$) can exist if $\hat{h_b}(k, l, 0) \neq 0$ and it reaches a maximum amplitude for $K = K_s$. The presence of r prevents the amplitude of this stationary solution from growing to infinity when $K = K_s$.

The solutions to the homogeneous part of Eq (5) are traveling free Rossby waves satisfying:

$$\omega = \frac{-\beta k + \bar{u}K^2 k - iK^2 r}{K^2 + K_d^2} \tag{9}$$

This dispersion relation is the usual one (e.g., Dickinson 1978) except for the viscous damping term. For a negligible K_d^2, the phase and group speeds in the east-west direction are:

$$\frac{\omega}{k} = \bar{u} - \frac{\beta}{K^2} \;\; ; \;\; \frac{\partial \omega}{\partial k} = \bar{u} + \frac{\beta(k^2 - l^2)}{K^4} \tag{10}$$

NUMERICAL SIMULATION

We solve Eq (8) numerically to calculate the amplitude of the forced response $\hat{\Psi}(k, l, \omega)$ for a range of k, l and ω values. We then apply the inverse Fourier transform to obtain $\Psi(x, y, t)$.

- Computational Domain: $(x, y) = 4373 \times 1093$ Mm with 512×256 data points. The time span $T = 11$ years with 2048 temporal points is such that the waves are significantly damped long before T. When analyzing the results, we only consider the first year (timescale of torsional oscillations).
- β parameter: is calculated at a reference latitude $\vartheta_0 = 20^\circ$, roughly the middle of the active-latitude range.
- Sunspot "Model": a simple Gaussian transient perturbation in h'_b:

$$h'_b(x, y, t) = d \exp\left(-\frac{x^2 + y^2}{\sigma^2}\right) \exp(-\tau t) \qquad (11)$$

  d defines the change in depth of the active fluid layer (> 0 means that the depth decreases), τ is related to the life span of the sunspot, and σ is related to its horizontal width. At each timestep t, we subtract the mean value of h'_b so that the average depth of the active layer remains H.
- Kinematic Viscosity: it varies steeply in the photosphere, from $\nu \approx 10^{12}$ cm^2 s^{-1} according to Nesis et al. (1990) at the bottom, to a few 10^4 cm^2 s^{-1} at the top if we assume no turbulence (strong hypothesis) and use the prescription of Goodman (2000). $\nu = 10^8$ cm^2 s^{-1} is an intermediate value.

SOME RESULTS

Fig 2 shows 2D snapshots of the free surface height $\eta'(x, y, t)$ at different times. Result obtained for: $\nu = 10^8$ cm^2 s^{-1}, $d = 150$ km, $\sigma = 30$ Mm, $\bar{u} = 5$ m s^{-1} (westward flow), and $1/\tau = 20$ days. The response to the flow crossing the sunspot at $(x, y) = (0, 0)$ is constituted of traveling Rossby waves radiated away into the far field. There is a wave pattern downstream of the sunspot. The east-west wavelength λ_x of these waves is smaller than their north-south wavelength λ_y, as expected from Rossby waves. A movie of $\eta'(x, y, t)$ highlights very large-scale Rossby waves traveling in the opposite direction to the solar rotation at a fast pace, while a slowly-changing pattern unfolds downstream of the sunspot and propagates westward. As time passes, this downstream pattern contains waves of shorter and shorter wavelengths. Another feature is the presence of a trough immediately west of the sunspot. Fig 3 shows $\eta'(x, y, t)$ at $y = 0$ as a function of x and t: the trough, the downstream wave pattern, and the fast eastward-traveling Rossby waves are all visible.

Had the perturbation been permanent (like on Earth with topographic forcing), the downstream wave pattern would be stationary with a dominant wavelength $\lambda_x \approx 162$ Mm, corresponding to $k = K_s$ (assuming $l = 0$) (Charney & Eliassen 1949). Even though the phase speed of a stationary Rossby wave is zero, its group velocity is westward and equal to 10 m s^{-1} $(= 2\bar{u})$. Therefore, energy would be transported away from the sunspot and the wave pattern would extend dowsntream of the perturbation.

The extent of the downstream wave pattern depends on kinematic viscosity: the amplitude of the wavefronts away from the sunspot decreases with distance, as a function of the time it takes for the wavepackets to reach these locations.

Because $\hat{h_b}(k, l, 0) \neq 0$ for some (k, l) values, there exists a stationary response. However, it does not dominate the downstream wave pattern because the total power at $\omega = 0$ is much smaller than at higher ω. The lifetime of the perturbation being finite, waves with higher ω (which have longer λ_x and propagate faster) are excited unlike in the case of a permanent perturbation. Therefore, the response downstream of a transient perturbation is constituted of waves with longer wavelengths. Due to their higher group velocities, we see the longer-wavelengths wavepackets away from the sunspot before observing their shorter-wavelength counterparts. Due to the dispersive nature of Rossby waves, the wavepackets spread with time. The λ_x values of the waves constituting the downstream response is such that their group velocities are pointing downstream and their phase velocities are pointing upstream. Very long-wavelength waves have eastward phase speeds and circumnavigate in a relatively short time: for example, at $\lambda_x = 2\pi R_\odot$, and $l = 0$, the phase speed is such that circumnavigation takes only 14 days. Fig 4 shows a east-west cut through the middle of the sunspot: each line shows η' at a given time. It emphasizes how during the first month or so, a trough in Ψ' and η' is established downstream of the sunspot.

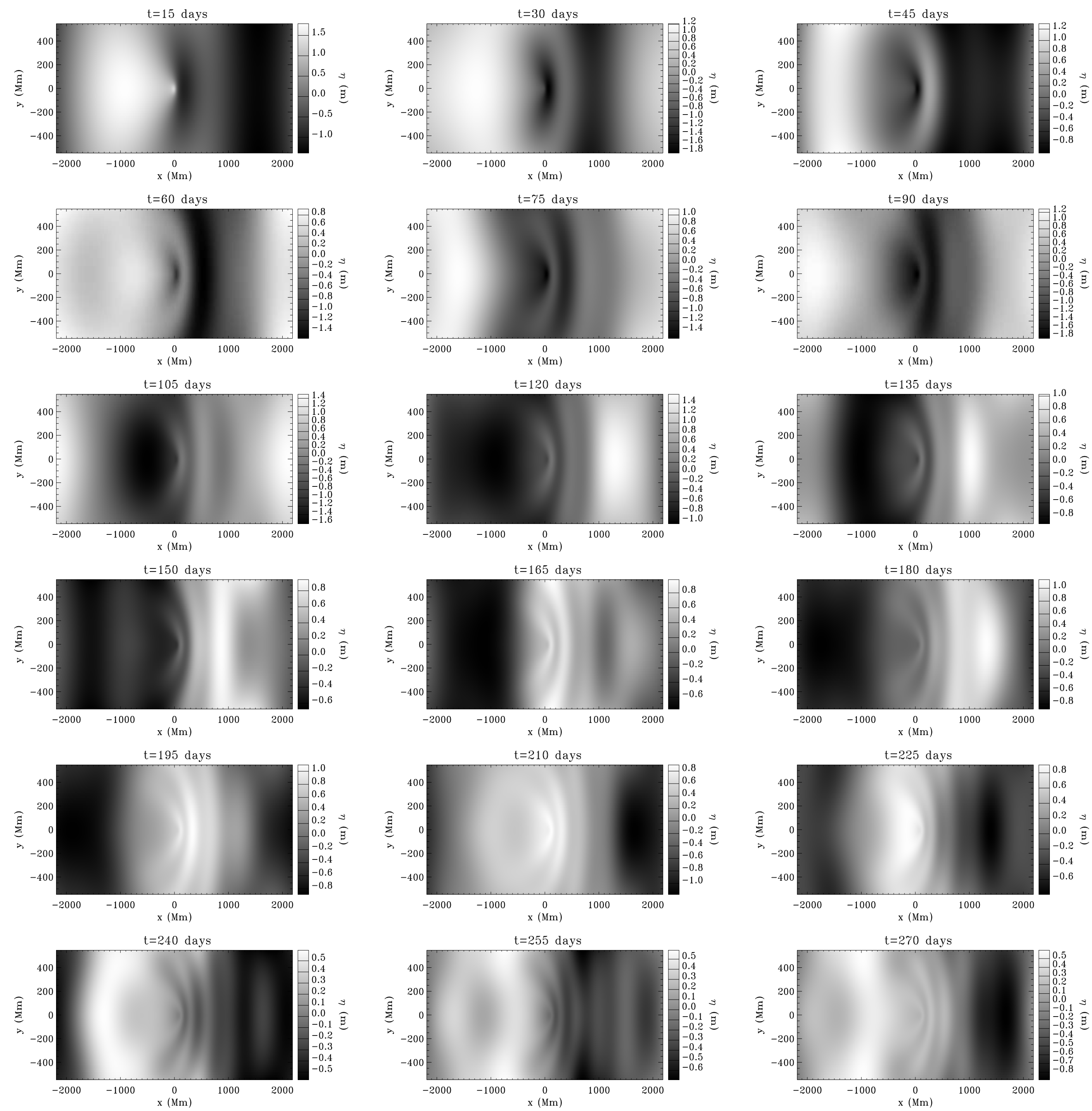

Figure 2: Free surface height $\eta'(x, y, t)$ in meters as a function of time. $\bar{u} = 5$ m s^{-1}, $\nu = 10^8$ cm^2 s^{-1}, $\sigma = 30$ Mm, $d = 150$ km, and $1/\tau = 20$ days. The sunspot is centered at $(x, y) = (0, 0)$.

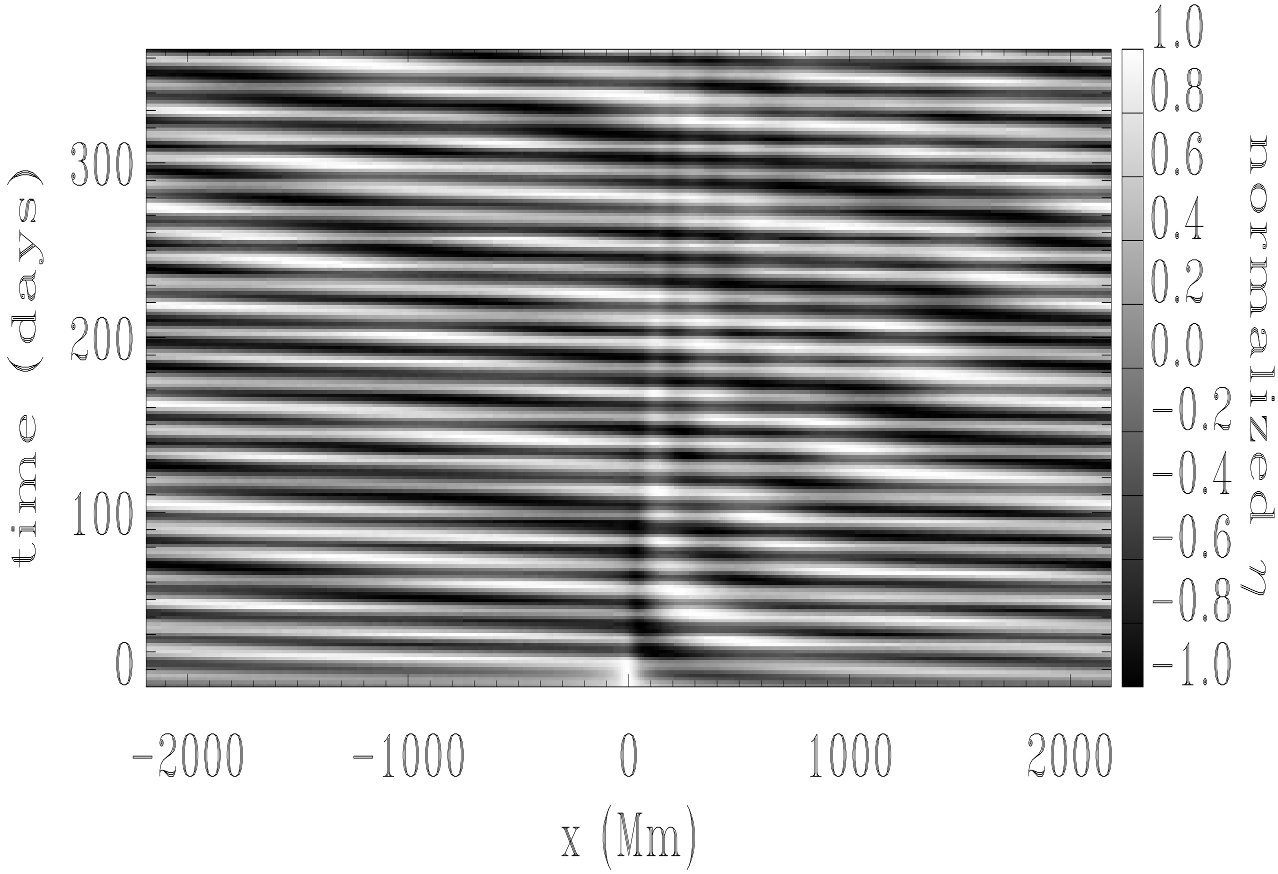

Figure 3: Free surface height $\eta'(x, 0, t)$ normalized by the maximum amplitude at each time t, as a function of both x and t. $\bar{u} = 5$ m s^{-1}, $\nu = 10^8$ cm^2 s^{-1}, $\sigma = 30$ Mm, $d = 150$ km, and $1/\tau = 20$ days.

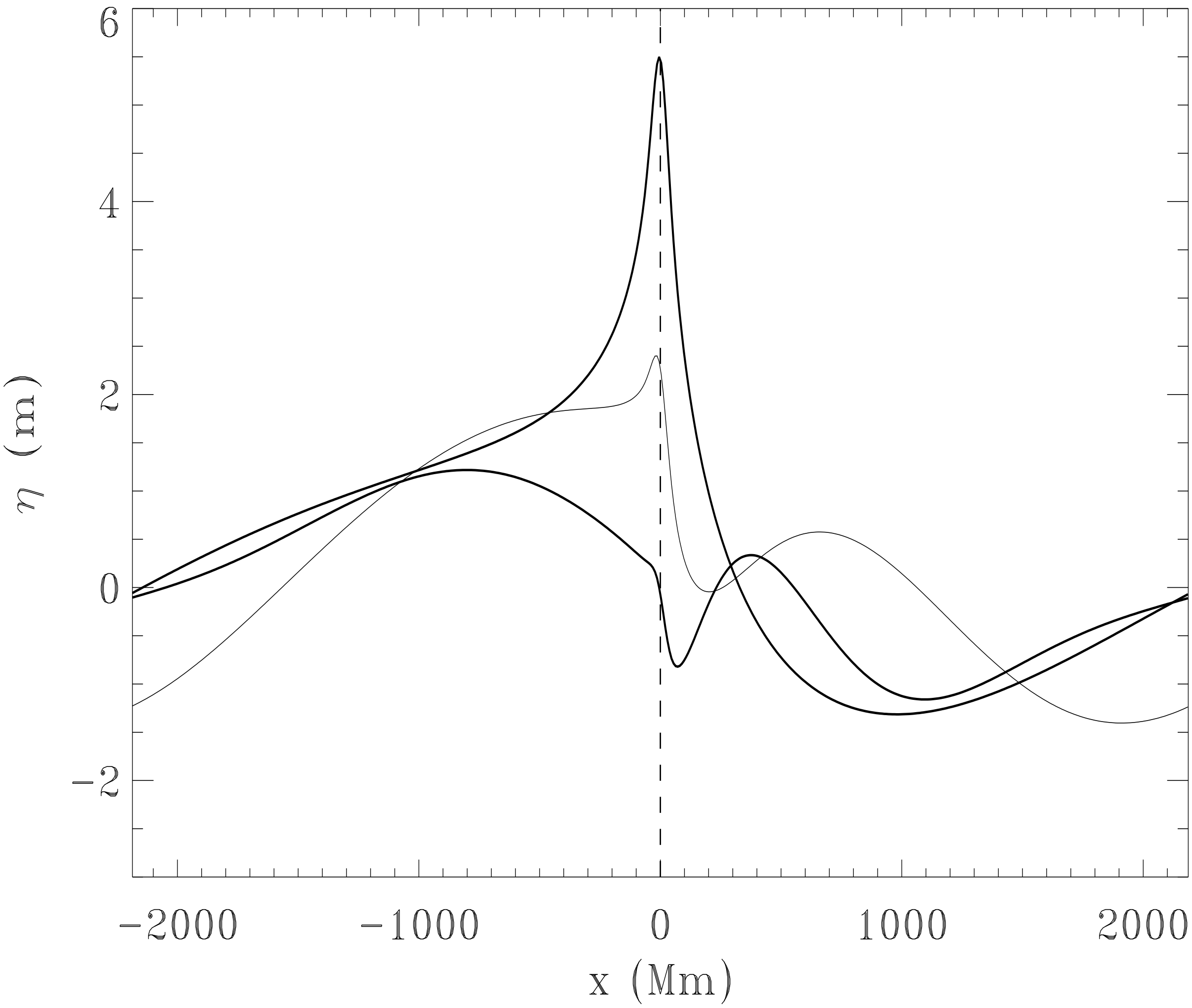

Figure 4: Free surface height $\eta'(x, 0, t)$ in meters as a function of time. Three time steps are shown: $t = 1$, $t = 14$ days, and $t = 29$ days. The dashed line shows the location of the sunspot center. The thick lines show the response at $t = 1$ and $t = 29$ days. $\bar{u} = 5$ m s^{-1}, $\nu = 10^8$ cm^2 s^{-1}, $\sigma = 30$ Mm, $d = 150$ km, and $1/\tau = 20$ days.

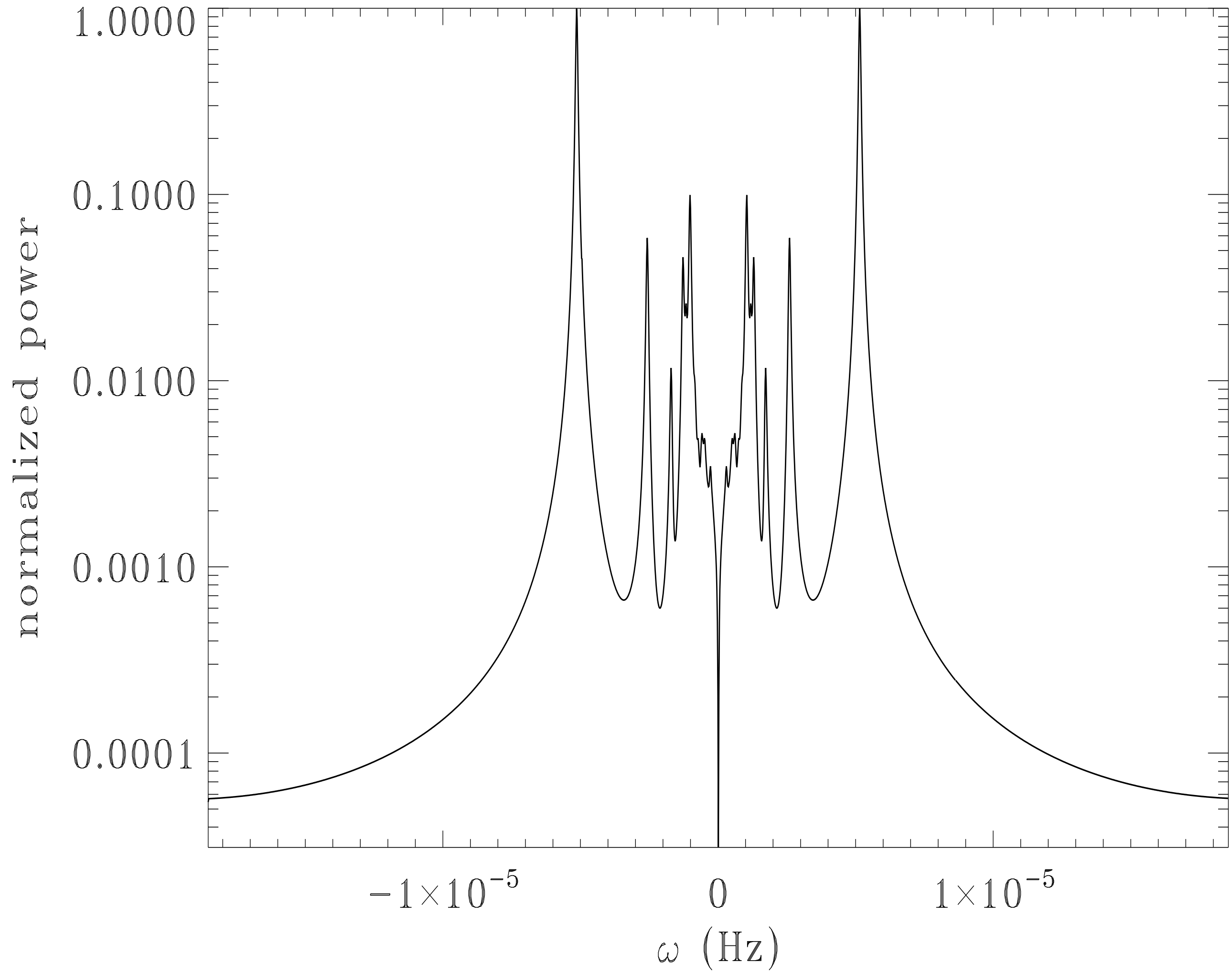

Figure 5: k-averaged power spectrum of the free surface height $\eta'(x, y, t)$. η' was averaged in 3 latitudinal bands centered on $y = 0$ Mm. $\bar{u} = 5$ m s^{-1}, $\nu = 10^8$ cm^2 s^{-1}, $\sigma = 30$ Mm, $d = 150$ km, and $1/\tau = 20$ days.

Fig 5 shows a power spectrum (averaged over all k) of a central latitudinal band of η'. It highlights the dominant temporal frequencies of the forced waves. These frequencies are very low, $\omega < 6.3\,\mu$Hz, and of the same order of magnitude as the solar rotation rate

($\omega = 2.97\,\mu$Hz at the equator), as expected from Rossby waves.

The surface elevation/depression η' of the waves seems to be at least an order of magnitude smaller than the 100-m high solar "hills" detected by Kuhn et al. (2000) and that were tentatively interpreted as a manifestation of r-modes (global Rossby waves that extend deeply into the convective zone). The horizontal phase velocity of the forced traveling photospheric Rossby waves studied seems to vary from of a few m s^{-1}, to a few km s^{-1} at most.

- Impact of Kinematic Viscosity: the main effect of an increased damping is to make the wave amplitudes drop faster. The trough is also closer to the center of the sunspot for a larger ν. For instance, with $\nu = 10^8$ cm^2 s^{-1} and at $t = 29$ days, the minimum of the trough is at 68 Mm from sunspot's center; with $\nu = 5 \times 10^8$ cm^2 s^{-1}, the minimum of the trough is at 43 Mm; and with $\nu = 10^9$ cm^2 s^{-1}, it is at 21 Mm. Finally, with a higher damping rate, the linewidths of the dominant power-spectrum peaks are increased.
- Impact of Sunspot Radius: except for its amplitude, the response appears largely unaffected by a reasonable change in radius. Indeed, η' drops for a smaller radius: σ was halved and the peak wave amplitude decreased by a factor 4. This was expected from the analytical expression of the Fourier transform of a 2D Gaussian: $\hat{h}_b$ varies as σ^2.
- Impact of Sunspot Lifetime: the Fourier transform of the decaying exponential exp$(-\tau t)$ is a Lorentzian function ($\propto \tau/(\tau^2 + \omega^2)$); when the sunspot lifetime increases the waves with lower temporal frequencies become more and more excited (the amplitude of $\hat{h}_b$ increases) compared to the waves with higher temporal

frequencies. These low temporal-frequency waves constitute the wave pattern unfolding downstream of the sunspot. Therefore, when $1/\tau$ increases, we expect this wave pattern to become more and more prominent. Instead of an exponential decay, we also tested rectangular functions for the sunspot life span: the results are very similar, but the maximum amplitude of the forced response is larger (for $1/\tau = 20$ days, the maximum amplitude as a function of time is ≈ 1.75 times larger with the rectangular function than with the exponential decay).

- Impact of Zonal Flow Velocity: the flow velocity does not affect the peak amplitudes. A change in $\bar{u}$ translates into a change in the phase and group speeds of the Rossby waves. This change affects all speeds the same way (for wavelengths much shorter than the deformation radius L_d). In a sense, the wave pattern is uniformly advected at a different velocity when $\bar{u}$ is changed.

CONCLUSION

We suggest the existence of photospheric Rossby waves forced by torsional oscillations traveling in the photosphere over a sunspot. Strong assumptions were applied: torsional oscillations are a geostrophic zonal flow steady over a timespan of 1 year, the photosphere can be isolated and the shallow-water theory applied, and the impact of a sunspot is reduced to a change in effective depth of a fluid parcel advected by the flow. We ignored dynamical and magnetic-field effects, except for kinematic viscosity resulting from turbulent motions. Coherent flows like the Evershed or moat flows might significantly alter the conclusions of this poster. Moreover, the strong magnetic field of sunspot, nearly vertical in the umbra, realistically hampers the crossing of the umbral photosphere by a flow. In other words, the model applied here is probably too simplistic.

With this caveat, this model showed that a sunspot could force the excitation of Rossby waves in presence of a large-scale zonal flow. A wave pattern is established downstream of the sunspot. For a westward flow, this downstream response propagates slowly westward and includes lower- and lower-wavelength waves as time passes. It vanishes after a few months, depending mainly on the kinematic-viscosity value. A trough is established immediately west of the sunspot. The amplitude of these Rossby waves depends on the sunspot: diameter, lifetime, and change in effective depth relative to the quiet Sun. The drop in amplitude with time is strongly dependent on kinematic viscosity (difficult to assess in the photosphere). With the tentative parameters applied here, the wave horizontal velocities vary from a few m s^{-1} to a few km s^{-1}, and the peak surface elevation/depression seems to be only a few meters.

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