PHYS 312. Basic Plasma Physics

- Time: 3:15-4:30, Tuesday, Thursday
- Place: SEQ Teaching Center room 101
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- Grades: biweekly assessments + presentations
- Course materials: http://quake.stanford.edu/~ sasha/PHYS312

Lecture Plan

- 1. January 9, Tuesday, Basic concepts. Debye shielding.
- 2. January 11, Thursday, Plasma ionization. Saha equation.
- 3. January 16, Tuesday, Coulomb collisions. Plasma resistivity.
- 4. January 18, Thursday, **Particle motion in magnetic** field.
- 5. January 23, Tuesday, Adiabatic invariants.
- January 25, Thursday, Kinetic theory of plasma.
 Vlasov equation.
- 7. January 30, Tuesday, Collisions. Fokker-Planck Equation.
- 8. February 1, Thursday, Plasma Resistivity. MHD approximation. Ohm's law.
- 9. February 6, Tuesday, Energy and momentum transport. Chapman-Enskog theory.
- 10. February 8, Thursday, Plasma transport in magnetic field. Ambipolar diffusion.
- 11. February 13, Tuesday, Propagation of electromagnetic waves in plasma.
- 12. February 15, Thursday, Plasma waves. Landau damping.
- 13. February 20, Tuesday, Plasma radiation: bremsstrahlung, recombination, synchrotron.

- 14. February 22, Thursday, MHD waves.
- February 27, Tuesday, Non-linear effects in plasma.
 Collisionless shocks. Quasi-linear theory of Landau damping.
- 16. March 1, Thursday, **Resistive instabilities. Magnetic** reconnection.
- 17. March 6, Tuesday, Plasma Applications. Thermonuclear fusion. Tokamak.
- 18. March 8, Thursday, Dynamo theory. Helicity.
- 19. March 13, Tuesday, Stochastic processes. Particle acceleration.
- 20. March 15, Thursday, Plasma experiments.

Books

- 1. T.J.M. Boyd and J.J. Sanderson, The Physics of Plasmas, Cambridge Univ.Press, 2003
- 2. J.A. Bittencourt, Fundamentals of Plasma Physics, 3rd edition, Springer, 2004
- 3. P.A. Sturrock, Plasma Physics, Cambridge Univ. Press, 1994
- 4. R.J. Goldston, P.H. Rutherford, Introduction to Plasma Physics, IOP Publ., 1995
- 5. F. L. Waelbroeck, R. D. Hazeltine, The Framework of Plasma Physics (Frontiers in Physics, V. 100), 1998
- R.Dendy (Ed.) Plasma Physics: an Introductory Course, Cambridge Univ. Press, 1993
- 7. R.O. Dandy, Plasma Dynamics, Oxford Sci. Publ., 1990
- 8. R. Fitzpatrick, Introduction to Plasma Physics, 1998, http://farside.ph.utexas.edu/teaching/plasma/plasma.html
- 9. R.D. Hazeltine, F.L. Waelbroeck, The Framework of Plasma Physics, Perseus Books, 1998.
- 10. L. Spitzer, Jr. Physics of Fully Ionized Gazes, Interscience Publishers, 1962
- Ya.B. Zel'dovich and Yu.P. Raiser, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena, Dover, 2002
- 12. NRL Plasma Formulary (handbook) http://wwwppd.nrl.navy.mil/nrlformulary

Topics for presentations

- Computer simulation of cold plasma oscillations (Birdsall and Langdon, Plasma Physics via Computer Simulations, p.90)
- 2. Two-stream instability (ibid, p.104)
- 3. Landau damping (ibid. p.124)
- 4. Particle motion in electric and magnetic fields (tokamak) http://www.elmagn.chalmers.se/courses /plasma1/matlab_spm.html
- 5. Jets from magnetized accretion disks
- 6. Plasma accelerators
- 7. Liquid ionization detectors
- 8. Magnetic reconnection in solar flares
- 9. Magnetic helicity
- 10. Tokamak experiments (Dendy, R. Plasma Physics, chap.8)
- 11. Magnetospheres of planets (ibid. chap.9-I)
- 12. Collisionless shocks (ibid. chap.9-II)
- 13. Laser-produced plasmas (ibid. chap. 12)
- 14. Industrial plasmas (ibid. chap. 13)
- 15. Dynamo experiments (Dynamo and Dynamics, ed P.Chossat et al, 2001)
- 16. Origin of cosmic rays (Longair, M., 1997, High-Energy astrophysics, Vol.2)

Basic Plasma Properties

([1] p.1-11; [2] p. 1-28; [8] p.2-13)

Definition of plasma

Plasma is *quasi-neutral* ionized gas. It consists of electrons, ions and neutral atoms.

Brief history of plasma

- A transparent liquid that remains when blood is cleared of various corpuscules was named *plasma* (after the Greek word πλασμα, which means "moldable substance" or "jelly") by Johannes Purkinje, the great Czech medical scientist.
- Irving Landmuir, Nobel prize-winner, first used "plasma" to describe an ionized gas. He studied tungsten-filament light bulbs to extend their lifetime, and developed a theory of *plasma sheaths*, the boundary layers

between the plasma and solid surface. He also discovered periodic variations of electron density - *Langmuir waves*.

Major areas of plasma research

- Radio broadcasting and Earth's ionosphere, a layer of partially ionized gas in the upper atmosphere which reflects radiowaves. To understand and correct deficiencies in radio communication E.V. Appleton (Nobel price 1947) developed the theory of electromagnetic waves propagation through a non-uniform, magnetized plasma.
- 2. Astrophysics, 95-99% of the visible Universe consists of plasma. Hannes Alfvén around 1940 developed the theory of *magnetohydrodynamics*, or MHD, in which plasma is treated as a conducting fluid (Nobel price in 1970). Two topics of MHD are of particular interest: *magnetic reconnection* (change of magnetic topology accompanied by rapid conversion of magnetic energy into heat

and accelerated particles) and *dynamo theory* (generation of magnetic field by fluid motions).

3. Controlled nuclear fusion, creation of hydrogen bomb in 1952. This research was mostly about hot plasma confinement and instabilities in magnetic field. Reaction:

 $d+t \rightarrow \alpha + n + 17.6 {\rm MeV}$

required $T = 10^8$ K and density $n = 10^{20}$ m⁻³.

- 4. **Space plasma physics.** James Van Allen discovered in 1958 the radiation belts surrounding the Earth, using data transmitted by the Explorer satellite. This led to discovery and exploration of the Earth's *magnetosphere*, plasma trapping in magnetic field, wave propagation, and magnetic reconnection, the origin of geomagnetic storms.
- 5. Laser plasma physics. Laser plasma is created when a high powered laser beam

strikes a solid target. Major application of laser plasma physics are *inertial confined fusion* (focus beams on a small solid target) and *plasma acceleration* (particle acceleration by strong electric field in laser beam.

Plasma can consist not only of electron and ions. Electron-positron plasma exists in pulsar atmospheres. In semiconductors, plasma consists of electron and holes. The unit systems in plasma physics

SI units

$$\begin{split} &[m] = \text{kg}; \ [x] = \text{m}; \ [t] = \text{s}; \ [F] = \text{kg} \text{ m/s}^2 = \text{N}; \\ &[E] = \text{Nm} = \text{J}; \ [T] = \text{K}; \\ &[I] = \text{A}; \ [q] = \text{As} = \text{C}; \ [U] = \text{J/C} = \text{V}; \ [\vec{E}] = \text{V/m}; \\ &[\vec{B}] = \text{Vs/m}^2 = \text{T}. \end{split}$$

Energy and temperature are often given in units of eV: 1 eV=1.60 \cdot 10⁻¹⁹ CV=1.60 \cdot 10⁻¹⁹ J. For temperature: $T[eV] \simeq k_B T[K]$

$$1 \text{eV} = \frac{1.60 \cdot 10^{-19} \text{J}}{1.38 \cdot 10^{-23} \text{J/K}} = 11600 \text{K}$$

CGS units

$$\begin{split} &[m] = \text{g}; \ [x] = \text{cm}; \ [t] = \text{s}; \ [F] = \text{g cm s}^{-2} = \text{dyn}; \\ &[E] = \text{g cm}^2 \text{ s}^{-2} = \text{erg}; \\ &[q] = \text{esu} = (10/c) \text{ C} \rightarrow e = 4.8 \cdot 10^{-10} \text{ esu} \\ &(\text{where } c = 3 \cdot 10^{10} cm/s) \\ &[\vec{B}] = \text{G} = 10^{-4} \text{ T}; \end{split}$$

In CGS, magnetic and electric fields have the same dimension.

Fundamental Constants

	SI units	CGS units
Speed of light	$c = 2.998 \ 10^8 \ m \ s^{-1}$	$2.998 \ 10^{10} \ \mathrm{cm \ s}^{-1}$
Gravity const.	$G = 6.6739 \ 10^{-11} N \ m^2 \ kg^{-2}$	$6.6739 \ 10^{-8} \ \mathrm{cm}^3 \ \mathrm{g}^{-1} \ \mathrm{s}^{-2}$
Planck's const.	$h = 6.626 \ 10^{-34} J s$	$6.626 \ 10^{-27} \mathrm{erg \ s}$
Boltzmann const.	$k = 1.381 \ 10^{-23} J \ K^{-1}$	$1.381 \ 10^{-16} \mathrm{erg} \ \mathrm{K}^{-1}$
Proton mass	$m_p = 1.673 \ 10^{-27} kg$	$1.673 \ 10^{-24} \mathrm{g}$
Electron mass	$m_e = 9.109 \ 10^{-31} kg$	$9.109 \ 10^{-28} \mathrm{g}$
Electron charge	$e = 1.602 \ 10^{-19} \ C$	$4.80 \ 10^{-10} \ \mathrm{esu}$

Astrophysical Constants

	SI units	CGS units
Mass of Earth	$M_E = 5.98 \ 10^{24} kg$	$M_E = 5.98 \ 10^{27} g$
Radius of Earth	$R_E = 6.37 \ 10^6 m$	$R_E = 6.37 \ 10^8 cm$
Mass of Sun	$M_S = 1.99 \ 10^{30} kg$	$M_S = 1.99 \ 10^{33} g$
Radius of Sun	$R_S = 6.96 \ 10^8 m$	$R_S = 6.96 \ 10^{10} cm$
Luminosity of Sun	$L_S = 3.9 \ 10^{26} W$	$L_S = 3.9 \ 10^{33} \mathrm{erg s}^{-1}$
Hubble's constant	$H_0 = 3.24 \text{ h} 10^{-18} \text{ s}^{-1}$	$H_0 = 3.24 \text{ h} 10^{-18} \text{ s}^{-1}$
CMB Temperature	$T_0 = 2.73 \text{ K}$	$T_0 = 2.73 \text{ K}$

Conversions

	SI units	CGS units
$1 \mathrm{eV}$	$1 \text{ eV} = 1.60 \ 10^{-19} \text{J}$	$1 \text{ eV} = 1.60 \ 10^{-12} \text{erg}$
1 Jy	$1 \text{ Jy} = 10^{-26} \text{W m}^{-2} \text{Hz}^{-1}$	
Light year	$lt-yr = 9.46 \ 10^{15} m$	$lt-yr = 9.46 \ 10^{17} cm$
Parsec	$1 \text{pc} = 3.09 \ 10^{16} \text{m}$	$1pc = 3.09 \ 10^{18} cm$

If a formula is known in SI, the corresponding formula can be found in CGS by the use of the following rules:

SI	\rightarrow	CGS
ϵ_0	\rightarrow	$1/4\pi$
μ_0	\rightarrow	$4\pi/c^2$
q	\rightarrow	q
\vec{B}	\rightarrow	$ec{B}/c$
\vec{E}	\rightarrow	$ec{E}$

Examples

• the speed of light

$$1/\sqrt{\mu_0\epsilon_0} \to 1/\sqrt{4\pi/c^2 1/4\pi} = c$$

• Coulomb force

$$\frac{qq'}{4\pi\epsilon_0 r^2} \to \frac{qq'}{r^2}$$

• Larmour force

$$q(\vec{E} + \vec{v} \times \vec{B}) \to q(\vec{E} + \vec{v} \times \vec{B}/c)$$

Note that these rules do not give the conversion of units.

Conditions affecting basic properties of plasma

- Quantum degeneracy
- Electrostatic coupling
- Quasi-neutrality
- Debye shielding
- Collisionality

Quantum degeneracy

Quantum effects become significant when plasma density is high. Plasma with relatively low density can be described by classical physics and is called *classical plasma*.

Consider conditions for quantum and classical plasmas. If the distance between particles d is much less than the quantum (De Broglie) wavelength λ_D then the quantum effects are important.

 $d\ll\lambda_D$ - quantum plasma

 $d \gg \lambda_D$ - classical plasma

Distance between particles: $d \simeq n^{-1/3}$

De Broglie wavelength: $\lambda_B \simeq \frac{\hbar}{p}$, where p is particle momentum $p = \sqrt{2mE} = \sqrt{2mT}$, temperature T is measured in energy units, eV.

Then
$$\lambda_D \simeq \frac{\hbar}{\sqrt{2mT}}$$
.
If $n^{-1/3} \gg \frac{\hbar}{\sqrt{2mT}}$
or $T \gg \frac{\hbar^2 n^{2/3}}{2m}$
then the plasma is *classical*.
The quantum effects first become significant for

electrons.

Temperature is often measured in eV. $1eV=1.6 \cdot 10^{-12}erg=11600$ K.

Electron temperature $T_e = \frac{1}{2}m_e \langle v_e^2 \rangle$, Ion temperature $T_i = \frac{1}{2}m_i \langle v_i^2 \rangle$.

Electron and ion velocities: $v_e = \sqrt{\frac{2T_e}{m_e}}$ and $v_i = \sqrt{\frac{2T_i}{m_i}}$. If a one-temperature plasma $T_e = T_i = T$ and $v_i = \sqrt{\frac{m_e}{m_i}} v_e$. Ions move slower than electrons. Example

Estimate significance of quantum effects in the Sun's core $(T = 1.5 \times 10^7 \text{K}, \rho = 150 \text{ g/cm}^3).$

Practical formula for the condition of classical plasma: $T_{\rm eV} \gg 3.5 \cdot 10^{-16} n_{cm^{-3}}^{2/3}$

Electrostatic coupling

Compare kinetic and electrostatic energy of plasma particles.

Kinetic energy: $E_K \simeq T$ Electrostatic energy:

$$E_E \simeq \frac{e^2}{2n^{-1/3}} \quad \left(\mathbf{SI} : E_E \simeq \frac{e^2}{8\pi\epsilon_0 n^{-1/3}}\right)$$

When $E_K \gg E_E$ the plasma is *weakly coupled* or *ideal*.

The condition for ideal plasma is:

$$T \gg \frac{e^2 n^{1/3}}{2} \quad \left(\mathbf{SI} : T \gg \frac{e^2 n^{1/3}}{8\pi\epsilon_0} \right)$$

Practical formula:

$$T_{\rm eV} \gg 0.72 \cdot 10^{-7} n_{\rm cm^{-3}}^{1/3}$$

In quantum plasma the kinetic energy is not equal to temperature. When the density increases, the plasma becomes degenerate, so that there cannot be more than two electrons at the same point with the spins "up" and "down". If the distance between particles $d \sim n^{-1/3}$ than each electron is confined in a box of the size $\Delta x \sim d \sim n^{-1/3}$. According to the uncertainty principle a particle confined in a box of size Δx has a momentum $p \sim \frac{\hbar}{\Delta x}$, and kinetic energy $E_K = p^2/m$. Then

$$E_K \simeq \frac{\hbar^2 n^{2/3}}{2m_e}$$

This is Fermi energy of degenerate electron gas. In this case, the condition for ideal weakly coupled plasma is

$$E_K \gg \frac{e^2 n^{1/3}}{2} \quad \left(\mathbf{SI} : \frac{e^2 n^{1/3}}{8\pi\epsilon_0}\right)$$
$$\frac{\hbar^2 n^{2/3}}{2m} \gg \frac{e^2 n^{1/3}}{2} \quad \left(\mathbf{SI} : \frac{e^2 n^{1/3}}{8\pi\epsilon_0}\right)$$
$$n \gg n_* \equiv \left(\frac{me^2}{\hbar^2}\right)^3 \equiv a_B^{-3} = 6.75 \cdot 10^{24} \text{ cm}^{-3}$$

Here a_B is Bohr's radius.

If $n \ll n_*$ and $T \gg T_* \equiv e^2 n_*^{1/3}$ the plasma is classical weakly coupled. If $n \ll n_*$ and $T \ll T_*$ then the plasma is quantum strongly coupled.

$$T_* = \frac{1}{2}e^2 n_*^{1/3} = \frac{me^4}{2\hbar^2} = \text{Ry} = 13.6\text{eV}$$

Ry is the energy of the lowest state of hydrogen atom.

Problem 1. (due by Tuesday, January 16) Draw the T-n diagram using a computer and indicate the location of your favorite plasma.

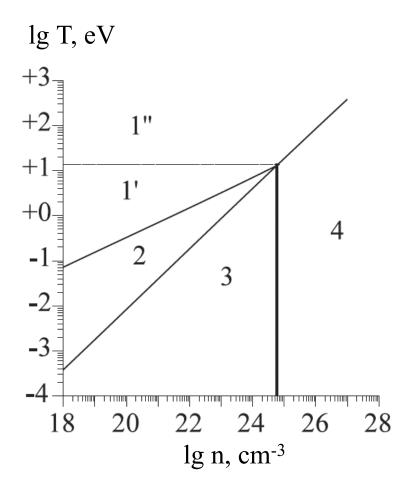


Figure 1: Plasma classification. 1- ideal classical plasma(1' - low-temperature plasma, 1" - hightemperature plasma, 2 - classical strongly coupled plasma, 3 - quantum strongly coupled plasma, 4 quantum weakly coupled plasma

Quasineutrality

Consider a plasma with n electrons and n ions in a unit volume. Let us assume that because of thermal motions in a region of size l electrons and ions are separated, so that the number of electrons is n_e and the number of ions is n_i . Then the density difference between electrons and ions is:

$$\delta n = n_i - n_e$$

This results is electric potential ϕ which can be estimated from Poisson equation:

$$\nabla^2 \phi = -4\pi e \delta n \quad \left(\mathbf{SI} : -\frac{e \delta n}{\epsilon_0}\right)$$

Estimate the second derivative as

$$\nabla^2 \phi \sim \frac{\delta \phi}{l^2}$$

Then

$$\delta\phi \approx 4\pi e\delta nl^2 \quad \left(\mathbf{SI}:\frac{e\delta l^2}{\epsilon_0}\right)$$

The electrostatic energy of particles $e\delta\phi$ cannot be much greater than their kinetic energy T because a greater electrostatic potential will slow down the particles and stop the separation of charges. Thus, assume

$$e\delta\phi\sim T$$

Then,

$$\frac{\delta n}{n} \sim \frac{T}{4\pi n e^2 l^2} \equiv \frac{r_d^2}{l^2}$$

where

$$r_d = \sqrt{\frac{T}{4\pi n e^2}} \quad \left(\mathbf{SI} : \sqrt{\frac{\epsilon_0 T}{n e^2}}\right)$$

is Debye's radius. This is a characteristic distance of the separation of charges in plasma under thermal motions.

If $l \gg r_d$ then the plasma is quasi-neutral. If $l \leq r_d$ then such a gas is a system of individual charged particles rather than a plasma.

Practical formula for Debye's radius:

$$r_d[cm] = 740 \sqrt{\frac{T_{\rm eV}}{n_{cm^{-3}}}}$$

Debye's radius is a characteristic scale of charge separation in plasmas. The corresponding characteristic time is:

$$t \sim \frac{r_d}{v_e} \sim \left(\frac{T}{4\pi n e^2}\right)^{1/2} \left(\frac{m_e}{T}\right)^{1/2} \sim \left(\frac{m_e}{4\pi n e^2}\right)^{1/2} \equiv \omega_p^{-1}.$$
$$\omega_p = \left(\frac{4\pi n e^2}{m_e}\right)^{1/2} \quad \left(\mathbf{SI} : \left(\frac{n e^2}{\epsilon_0 m_e}\right)^{1/2}\right)$$

is Langmuir (plasma) frequency.

$$\omega_p[sec^{-1}] = 5.6 \cdot 10^4 \sqrt{n_{cm^{-3}}}$$

If a fluctuation in plasma occurred on the scale $l \gg r_d$ then it will not dissipate during the characteristic time ω_p^{-1} , due to thermal motion, and thus the plasma will oscillate with the period of $2\pi/\omega_p$. These are so-called Langmuir oscillations which represent oscillations of electrons relative to ions.

Let's make more accurate estimates. Consider a sheet of electrons moving along axis x perpendicular the sheet plane.

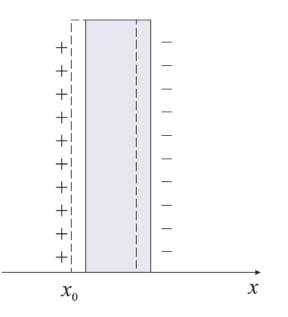


Figure 2: Oscillations of a sheet of electrons in plasma

Suppose this sheet moved to the right to distance $x - x_0$ from its equilibrium position x_0 (when the electron density was uniform and equal the density of ions n_0). Because of this on the left side of the sheet there is an excess of the positive charge of $en_0(x - x_0)$ for unit area. As a result, there is electric field of

$$E = 4\pi\Delta q = 4\pi e n_0 (x - x_0)$$

Then the equation of motion of electrons in this field is

$$m\ddot{x} = -4\pi e^2 n_0 (x - x_0)$$

The solution is harmonic oscillation with frequency

$$\omega_p = \sqrt{\frac{4\pi e^2 n_0}{m}} \quad \left(\mathbf{SI} : \sqrt{\frac{e^2 n_0}{\epsilon_0 m}}\right)$$

Debye shielding

Charged particles in plasma attract particles of the opposite charge increasing the concentration of the opposite changes around them. As a results of this "shielding" the electrostatic potential decreases more rapidly with distance compared to the Coulomb law. Estimate the effect of this shielding. Consider a probe charge q in a plasma and calculate the distribution of the electrostatic potential ϕ around it. Assume for simplicity that ions are single charged particles. Then the distribution of electron and ions follows Boltzmann's law:

$$n_e = n_0 \exp{\frac{e\phi}{T_e}}$$
$$n_i = n_0 \exp{-\frac{e\phi}{T_i}}$$

If $|e\phi| \ll T$ then

$$n_e \approx n_0 \left(1 + \frac{e\phi}{T_e} \right)$$

 $n_i \approx n_0 \left(1 - \frac{e\phi}{T_i} \right)$

Thus,

$$\delta n = n_i - n_e = -n_0 e\phi \left(\frac{1}{T_i} + \frac{1}{T_e}\right)$$

Then Poisson's equation for potential ϕ is written as:

$$\Delta \phi = -4\pi e \delta n = \frac{\phi}{r_d^2}$$

where

$$\frac{1}{r_d^2} = 4\pi e^2 n_0 \left(\frac{1}{T_i} + \frac{1}{T_e}\right)$$

In the spherical coordinates, the solution of the

Poisson equation:

$$\frac{1}{r^2}\frac{d}{dr}r^2\frac{d\phi}{dr} = \frac{\phi}{r_d^2}$$

is

$$\phi = \frac{q}{r} \exp{-\frac{r}{r_d}}$$

The integration constant is chosen to obtain the original Coulomb potential q/r at $r \to 0$.

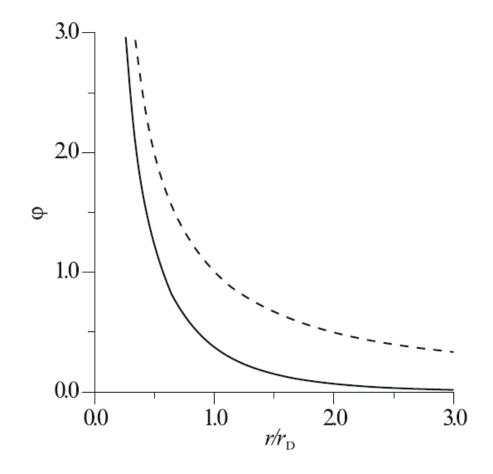


Figure 3: Debye (solid curve) and Coulomb (dashed curve) potentials in units q/r_d .

Consider potential ϕ as a sum of the Coulomb potential q/r and a screening potential ϕ_{scr} . Then

$$\phi_{scr} = \frac{q}{r} \exp\left(-\frac{r}{r_d}\right) - \frac{q}{r}$$

At $r \to 0$

$$\phi_{scr}(0) = -\frac{q}{r_d}$$

This is potential provided by all plasma particles at the location of our probe charge q. Therefore, the energy of interaction between the probe particle and the plasma is

$$q\phi_{scr}(0) = -\frac{q^2}{r_d}$$

Every charged particle in plasma interacts with all other particles. We can now the total energy of these interactions by summing energies for each particle and dividing by 2 because the interaction between pairs are counted twice in this procedure. Therefore, for the electrostatic energy of a unit volume of plasma we get:

$$w = -\frac{1}{2}\sum_s \frac{1}{r_d} e_s^2 n_s$$

where subscript s = e, i. In the case, $e_i = e$ we get

$$w = -\frac{ne^2}{r_d}$$

The energy for one particle is

$$w_e = -\frac{e^2}{2r_d} = -\frac{T}{12N_d}$$

where

$$N_d = \frac{4}{3}\pi r_d^3 n$$

is the number of particles in the Debye sphere.

The electrostatic energy is small compared to the kinetic thermal energy, $w_e \ll T$ when

$$N_d \gg 1$$
$$N_d = \frac{4}{3}\pi r_d^3 n \equiv \Lambda$$

is called *plasma parameter*.

When $\Lambda \gg 1$ the plasma is weakly coupled. In this case, a typical particle interacts with all other particles within its Debye sphere but this interaction does not cause sudden changes in its motion. Such plasma behaves almost like ideal gas. In strongly coupled plasma, electrostatic interaction between particles is dominant, their potential energies are much higher than the kinetic energies. Such plasma behaves more like a fluid.

On scales larger than Debye's radius and longer than the plasma period, plasma shows *collective* behavior. The statistical properties of this behavior are controlled by the plasma parameter Λ .

Parameters ω_p , r_d and Λ are the most fundamental characteristics of plasma.

Collisionality

Collisions between charged particles in plasmas are controlled by the long-range Coulomb force and modified by the Debye shielding and collective effects. The collision frequency ν is the rate at which particles change their momentum because the scattering on other particles. The typical collision time is defined as a time for particle to deviate from their original trajectory by 90 degrees.

From the kinetic theory of gases we know that the collision rate is determined as

$$\nu = \frac{1}{nv\sigma}$$

where n is the plasma density, v is the particle velocity, and σ is the collision cross section.

In gases, the cross-section is determined by the size of particles. In plasma, the cross-section can be estimated from the size of the Debye's sphere:

$$\sigma\simeq r_d^2$$

Then, the collision rate is:

$$\nu \simeq \frac{1}{nvr_d^2} \simeq \frac{r_d}{nvr_d^3} \simeq \frac{\omega_p}{\Lambda}$$

Thus, in weakly coupled plasmas $(\Lambda \gg 1)$:

 $\nu \ll \omega_p$

and collisions do not interfere with plasma oscillations. Whereas in strongly coupled plasmas:

 $\nu \gg \omega_p$

and collision prevent oscillations.

More precise formula for the collision frequency:

$$\nu \simeq \frac{\ln \Lambda}{\Lambda} \omega_p$$

where $\ln \Lambda$ is called "Coulomb logarithm".

The mean free path is estimated as $\lambda = v/\nu$. When λ is greater than the characteristic scales in plasma (e.g. the size of plasma volume or gradients) than collisions are not important, and plasma called "collisionless". Otherwise, it is "collisional" and often described in MHD approximation.