Physics 780. Problem set 3

October 29, 2018
Due: Tueaday, November 13, 2018

Problem 3.1. The solar corona mostly consists of fully ionized hydrogen plasma with density $n = 10^8$ cm^{-3}, temperature $T = 10^6$ K. Assuming that a typical magnetic field strength in coronal loops B =10 G estimate:

(a) the Larmor frequency ω_L;

(b) the characteristic collision time τ;

(c) the electrical conductivity along and across magnetic field lines.

Problem 3.2. For typical plasma conditions of the corona: $n = 10^8$ cm^{-3}, $T = 10^6$ K, B =10 G, and the chromosphere: $n = 10^{12}$ cm^{-3}, $T = 10^4$ K, B =100 G, calculate:

(a) the adiabatic sound speed, $c_S = \sqrt{\gamma P/\rho}$;

(b) the Alfven speed $v_A = B/\sqrt{4\pi\rho}$;

(c) the speed of fast MHD waves traveling across the magnetic field lines.

Problem 3.3. The typical size of sunspots is L = 10 Mm, electrical conductivity $\sigma = 10^9\ s^{-1}$.

(a) Calculate a characteristic decay time of sunspots due magnetic diffusion;

(b) Compare your estimate with a lifetime of a typical sunspot. The observed lifetime of sunspots can be significantly shorter than the diffusion decay time. Why?

Problem 3.4. For observations of X2.1 flare of September 6, 2011, from the HMI instrument on SDO it was found that the mean magnetic field changed from 700 G to 450 G. The flare energy release was about 3.5×10^{31} erg (Liu, et al, 2014, ApJ, 795:128). Estimate the plasma volume in which the magnetic energy was released. What is a characteristic size (in Mm) of the flare energy release site?

Problem 3.5.

A simple model of the solar dynamo can be described in a local Cartesian system, x, y, z, (where x corresponds to the radius, y to the azimuthal (ϕ) direction and z to the direction of increasing latitude) by the following equations:

$$\frac{\partial A}{\partial t} = \eta \frac{\partial^2 A}{\partial x^2} + \alpha B \tag{1}$$

$$\frac{\partial B}{\partial t} = \eta \frac{\partial^2 B}{\partial x^2} + \Omega \frac{\partial A}{\partial x}, \tag{2}$$

where A and B represent the poloidal and toroidal components of the magnetic field, $\boldsymbol{B} = B(x, z)\boldsymbol{e}_y + \nabla \times [A(x, z)\boldsymbol{e}_y]$, $\Omega = \frac{\partial v_y}{\partial x}$ is the rotational shear, η is the turbulent diffusion coefficient, and α is the turbulent helicity.

(a) Consider propagating wave-like solutions of the form: $A = A_0 e^{i(kx - \omega t)}$, $B = B_0 e^{i(kx - \omega t)}$ and find a pure periodic (marginally stable) solution, and the condition for the growth of magnetic field in terms of the dynamo number, $N_d = \dfrac{|\alpha \Omega|}{\eta^2 k^3}$.

(b) **(extra credit)** Consider the dynamo at the base of the solar convection zone where $\Omega \approx 4 \times 10^{-7}$ s^{-1}. The solar dynamo wave has a period of 22 years, it propagates equatorward, and its half wavelength corresponds to about 40^o in latitude. Assuming that the solar dynamo is marginally sustained, estimate the turbulent diffusion coefficient, η, and the helicity coefficient, α.