1 HMI Polarization calibration.

1.1 Polarization Calibration Unit: PCU.

HMI will measure a linear combination of the solar Stokes vector in the following way,

$$\mathrm{L}_{\mathrm{hmi}} = \mathcal{O}\mathrm{I}_{\mathrm{sun}} \tag{1}$$

where $\mathcal{O}$ is the modulation matrix. In order demodulate and obtain the Stokes vector, the demodulation matrix $\mathcal{D}$ must be known and applied:

$$\mathrm{I}_{\mathrm{hmi}} = \mathcal{D}\mathrm{L}_{\mathrm{hmi}} = \mathcal{D}\mathcal{O}\mathrm{I}_{\mathrm{sun}} \tag{2}$$

Taking advantage that $\mathcal{D}\mathcal{O} = \mathbb{1}$ we will calibrate the modulation matrix $\mathcal{O}$ such that the product $\mathcal{D}\mathcal{O} = \mathbb{1}$ is known to a certain accuracy given by the error matrix $\mathcal{E}$,

$$\mathcal{E} = \begin{pmatrix} 0.01 & 0.1 & 0.1 & 0.1 \\ 0.001 & 0.01 & 0.01 & 0.01 \\ 0.001 & 0.01 & 0.01 & 0.01 \\ 0.001 & 0.01 & 0.01 & 0.01 \end{pmatrix} \tag{3}$$

For the determination of the modulation matrix $\mathcal{O}$ a Polarization Calibration Unit (PCU) has been designed at the High Altitude Observatory. The PCU consist of quarter waveplate retarder and a linear polarizer that can separately or both be inserted into the light beam (see Fig. 1).

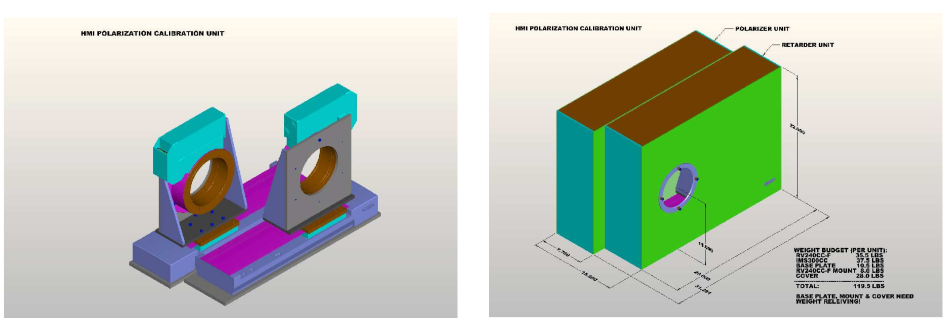

Figure 1: Schematic of the Polarization Calibration Unit (PCU).

During calibration the PCU is placed between the lamp and the telescope entrance. The light emitted by the lamp: $\mathrm{I}_{\mathrm{lamp}} = (i, q, u, v)$ goes through the PCU and HMI, being finally detected at the CCD as $\mathrm{L}_{\mathbf{cal}}$:

$$\mathrm{L}_{\mathbf{cal}} = \underbrace{\mathcal{D}\mathcal{M}_{\mathrm{pcu}}}_{\mathcal{O}'}\mathrm{I}_{\mathrm{lamp}} = \mathcal{O}'\mathrm{I}_{\mathrm{lamp}} \quad (4)$$

where $\mathcal{M}_{\mathrm{pcu}}$ is the Polarization Calibration Unit matrix (see Sect.1.2). $\mathrm{I}_{\mathrm{lamp}}$ can be either produced by a lamp or sunlight fed into the instrument by a heliostat. In order to avoid large input polarizations the lamp is preferred.

1.2 System modeling.

The matrix $\mathcal{O}'$ (Eq. 4) considers:

$$\mathcal{O}' = \underbrace{\mathcal{M}_{\mathrm{hmi}} \times \mathcal{M}_{\mathrm{tel}}}_{\mathcal{O}} \times \mathcal{M}_{\mathrm{pcu}} \quad (5)$$

- Three waveplates in HMI (with nominal retardances of $r_1 = \lambda/2$; $r_2 = \lambda/4$ and $r_3 = \lambda/2$) and a linear analyzer placed at an angle $\theta = \pi/2$. It is assumed to have an ideal transmission of 1. (See appendix A.1 for definitions).

$$\mathcal{M}_{\mathrm{hmi}} = P(\pi/2, 1) \times [\Pi_{j=3}^{1} W(\theta_j + \mathrm{d}\theta_j, r_j + \mathrm{d}r_j)] \quad (6)$$

- Telescope lenses at the entrance window: treated as a retarder W with an angle θ_{tel} and a retardance r_{tel}.

$$\mathcal{M}_{\mathrm{tel}} = W(\theta_{\mathrm{tel}}, r_{\mathrm{tel}}) \quad (7)$$

- PCU elements: retarder W with a nominal retardance $r_{\mathrm{pcu}} = \lambda/4$ and a linear polarizer P with an angle θ_{l} and a transmission t_{l}.

$$\mathcal{M}_{\mathrm{pcu}} = W(\theta_{\mathrm{pcu}} + \mathrm{d}\theta_{\mathrm{pcu}}, r_{\mathrm{pcu}} + \mathrm{d}r_{\mathrm{pcu}}) \times P(\theta_{\mathrm{l}}, t_{\mathrm{l}}) \quad (8)$$

1.3 Calibration procedure.

The observation $\mathrm{L}_{\mathrm{cal}}$ (Eq. 4) is defined by:

- The position of the 6 rotating elements: the HMI waveplates angles (θ_1,θ_2 and θ_3), the PCU retarder angle θ_{pcu} and the PCU linear polarizer angle θ_{l}

- 15 unknowns (meant to represent the imperfections of the different optical devices): 3 for the lamp (q, u, v), 6 for HMI (3 mount errors $\mathrm{d}\theta_j$ and 3 retardance errors $\mathrm{d}r_j$), 2 for the telescope lenses (angle θ_{tel} and retardance r_{tel}) and 4 for the PCU (mount error and retardance error in the PCU retarder: $\mathrm{d}\theta_{\mathrm{pcu}}$ and $\mathrm{d}r_{\mathrm{pcu}}$, transmissions of the PCU retarder and linear polarizer: $t_{\mathrm{r}}, t_{\mathrm{l}}$).

By knowing the angles of the 6 rotating 6 elements, and assuming some initial values for the 15 unknowns, *synthetic observation* $\mathrm{L}_{\mathrm{cal}}^{\mathrm{syn}}$ can be constructed and compared with the actual observation read at the CCD: $\mathrm{L}_{\mathrm{cal}}^{\mathrm{obs}}$. If our modeling of the optical elements is correct (Sect. 1.2), any mismatch between these two can be ascribed to the a wrong choice of the 15 unknowns. Their original values are then changed according to a Non-linear squares fitting algorithm until convergence.

To properly constraint the values of the 15 unknowns two different sequences of observations have been used. The shortest contains 69 observations (including 8 darks), while the long sequence contains 139 (also including 8 darks). The convergence of the fitting algorithm has been thoroughly tested. An analysis of the Jacobian of the model unknowns reveals that the mount errors of the three HMI waveplates cannot be determined individually, but rather the following linear combinations of them: $2\mathrm{d}\theta_3 - \mathrm{d}\theta_2$ and $2\mathrm{d}\theta_1 - \mathrm{d}\theta_2$. In practice we have procedeed to determine $\mathrm{d}\theta_2$ independently and obtaining the mount errors of the other first and third waveplates with respect to the middle ope/

Once the imperfections of the optical elements are characterized, the modulation matrix $\mathcal{O} = \mathcal{M}_{\mathrm{hmi}}\mathcal{M}_{\mathrm{tel}}$ can be constructed. As an example, two different calibration runs (taken at Lockheed in June 18 and 19, 2006) have been used to infer two different modulation matrices: $\mathcal{O}_1$ and $\mathcal{O}_2$. Equation 9 shows an example of the $\mathcal{DO}$ matrix, where $\mathcal{O} = \mathcal{O}_1$ and $\mathcal{D} = \mathcal{O}_2^{-1}$.

$$\mathcal{DO} = \begin{pmatrix} 1 & 0.0009 & 0.0065 & 0.0004 \\ 0 & 1.0001 & 0.0053 & 0.0096 \\ 0 & -0.0067 & 0.9985 & 0.0109 \\ 0 & -0.0123 & -0.0002 & 1.0004 \end{pmatrix} \tag{9}$$

The goal of the polarization calibration is to determine the elements of $\mathcal{DO}$ within the uncertainties $\mathcal{E}$ (Eq. 3). Note that the first column of the $\mathcal{DO}$ matrix (cross talk from Stokes I into Q,U and V) can be obtained very accurately in orbit. The rest of the elements should be determined previous to flight. According to Eq. 9 we are close to meet the specifications.

With the accrued experience (calibrations March and June 2006) we have identified a number of problems that once dealt with should allow us to determine even more accurately the elements of the calibration matrix. The most critical are fluctuations of the lamp intensity and image motions. To solve these issues, we plan to add a field stop (larger than the solar image) to check for image motions during the observing sequence. At the same time we will monitor the lamp intensity. Small modifications (new observations with different configurations of the PCU and HMI elements) to the original observing sequences might be also needed.

A.1 Matrix for polarizer and retarder elements.

The matrix of a linear polarizer P with an angle θ and a transmission t is given by:

$$P(\theta, t) = \frac{t}{2} \begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \cos 2\theta \sin 2\theta & 0 \\ \sin 2\theta & \cos 2\theta \sin 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{10}$$

Matrix of a retarder W with at an angle θ and retardance r is given by:

$$W(\theta, r) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\theta + \sin^2 2\theta \cos r & \cos 2\theta \sin 2\theta (1 - \cos r) & -\sin 2\theta \sin r \\ 0 & \cos 2\theta \sin 2\theta (1 - \cos r) & \sin^2 2\theta + \cos^2 2\theta \cos r & \cos 2\theta \sin r \\ 0 & \sin 2\theta \sin r & -\sin r \cos 2\theta & \cos r \end{pmatrix} \tag{11}$$