

Understanding Solar Eruptions with *SDO/HMI*

Measuring Photospheric Flows, Testing Models, and Steps Towards
Forecasting Solar Eruptions

Work Supported by: LWS Tools & Methods, LWS TR&T Strategic
Capability, and HGI

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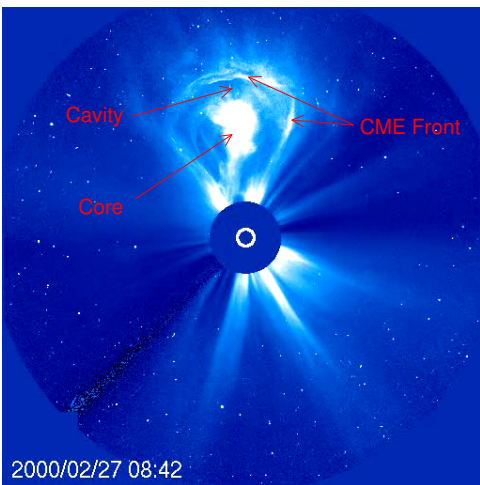
⁴Space Sciences Laboratory, University of California, Berkeley

⁵Colorado Research Associates/NorthWest Research Associates Inc

⁶Stanford University

CMEs and Space Weather

Corona Mass Ejection (CME)

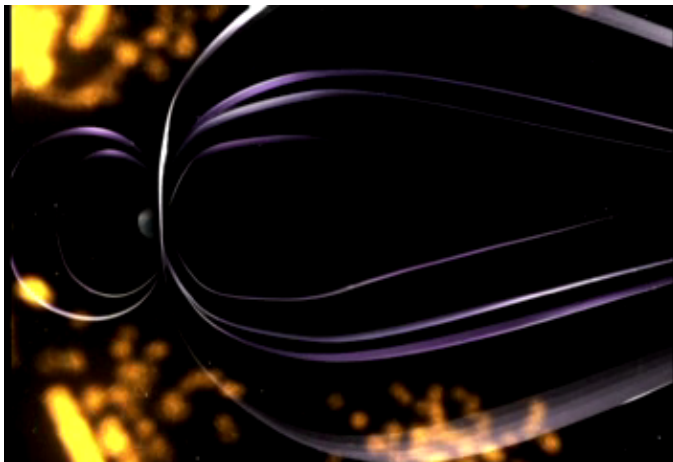


CME Energetics

- Mass: $10^{13} - 10^{17}$ g of plasma
- Kinetic: $10^{27} - 10^{32}$ ergs
- Magnetic: $10^{27} - 10^{32}$ ergs
- Solar Energetic Particles (SEPs): 1 – 10% of KE
- Associated Flare (sometimes) similar to KE

CMEs and Space Weather

CMEs impact Earth



www.nasa.gov/mpg/160602main_what_is_a_cme_NASA%20WebV_1.mpg

CMEs and Space Weather

CMEs impact Earth

Civilian Infrastructure:

- Electrical power grids, oil pipelines, polar aviation routes, satellite- and long-line communication systems, space tracking, navigation systems, and satellite operations
- Direct economic consequences \$200-400 million dollars a year (Horne, 2003)

NASA Operations:

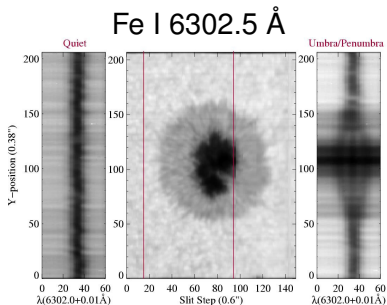
- Space Shuttle, satellite, and International Space Station (ISS) operations
- Dangers and unpredictability of solar eruptions operationally constrain a manned mission to Mars (Foullon et al., 2005)

Solar Magnetic Fields: Magnetograms

- Widely accepted that energy stored in the coronal magnetic field drives CMEs and flares.
 - Vector measurements of the coronal magnetic field?
Rare and uncertain
- State of the photospheric magnetic field provides limited predictive capabilities (Leka & Barnes, 2007)
 - Examined 1200 photospheric vector magnetograms
 - “[W]e conclude that the state of the photospheric magnetic field at any given time has limited bearing on whether that region will be flare productive”
- Dynamics and time-history of the photospheric magnetic field key to understanding the energization and initiation of solar eruptions
 - Plasma flow properties
 - Poynting flux - energy budget of the corona

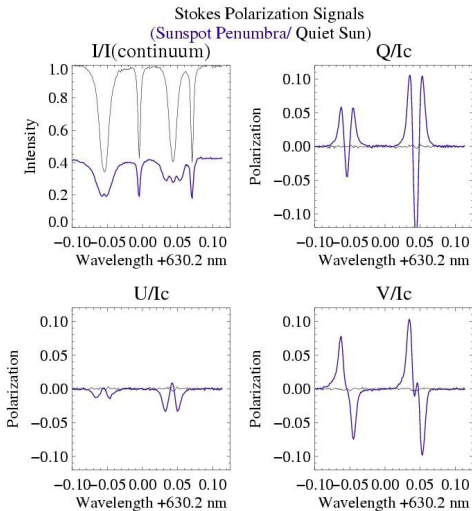
Solar Magnetic Fields: Magnetograms

How are Vector Magnetograms Measured?: Adapted from Leka *et al.* (2009)



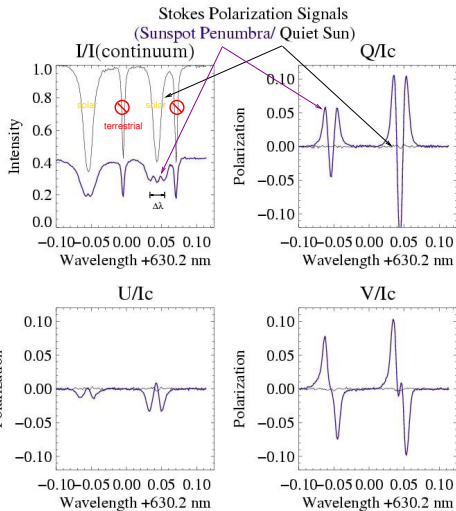
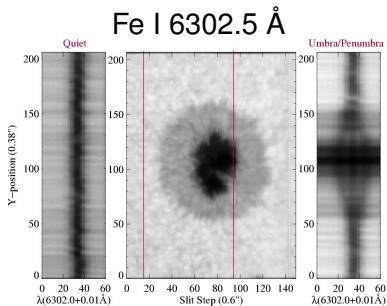
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- Splitting proportional to $|B|$



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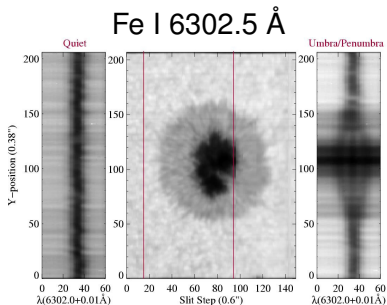
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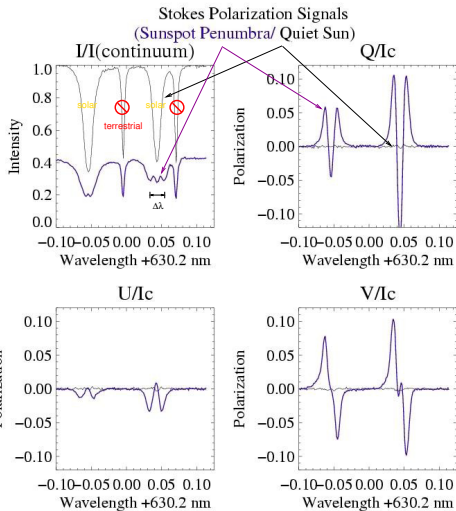
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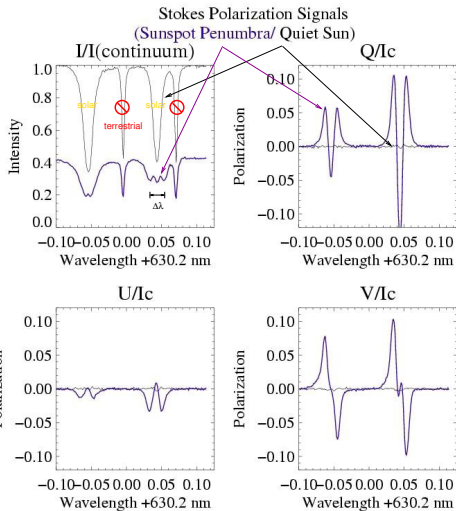
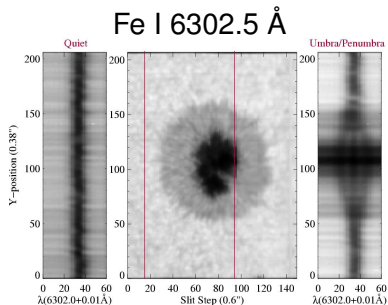
Weak field approximations:

- $B_{\parallel} \propto V$
- $B_{\perp} \propto \sqrt{Q^2 + U^2}$
- $\Phi \approx n\pi + \tan^{-1}(U/Q)$ (azimuthal ambiguity)
- $\gamma = \tan^{-1}(B_{\parallel}/B_{\perp})$ (inclination)



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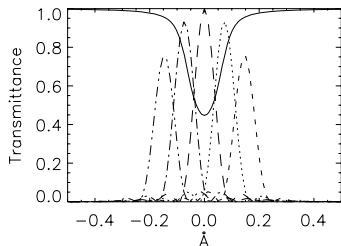
General:

- Spectra fit to a Milne-Eddington atmosphere to determine B_{\parallel} and B_{\perp} (Borrero *et al.*, 2007)
- 180° ambiguity in B_{\perp} resolved by minimizing currents and $\nabla \cdot \mathbf{B}$ via simulated annealing (Metcalf *et al.*, 2006; Leka *et al.*, 2009)

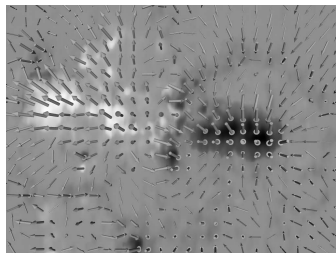
Solar Magnetic Fields: Magnetograms

Solar Dynamics Observatory/Helioseismic Magnetic Imager

- HMI will sample 5-6 points along the Stokes profiles I, Q U, V of the Ni I 6768 absorption line
- Cadence of science quality ambiguity resolved vector magnetograms 10 – 15 minutes (available after 24 hours)
- full disk 4096×4096 pixels or 1" resolution



(Norton et al., 2006)

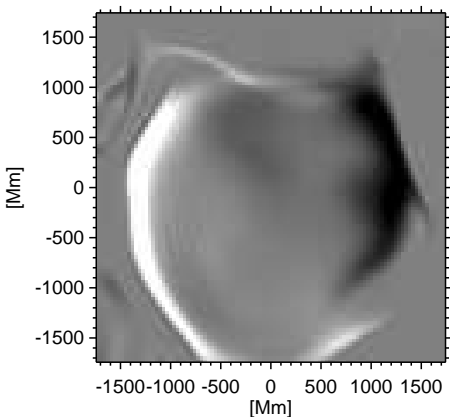


(Keller et al., 2008)

Solar Magnetic Fields: Magnetograms

What is a “Neutral-Line?”

Synthetic Magnetogram of the Vertical Magnetic Field

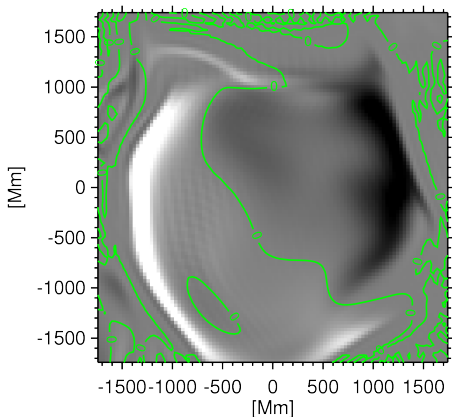


- White/Black positive/negative vertical flux
- Green neutral line ($B_z = 0$)
- *Sometimes* we get vector fields:
 $\mathbf{B} = \mathbf{B}_h + B_z \hat{\mathbf{z}}$

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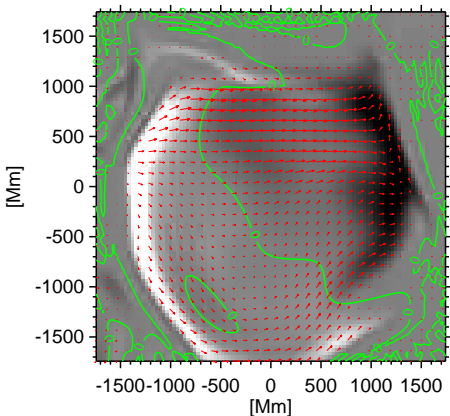


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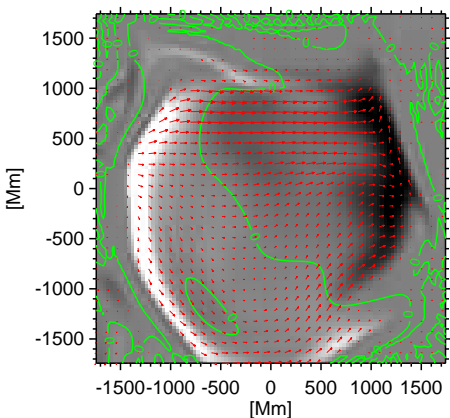


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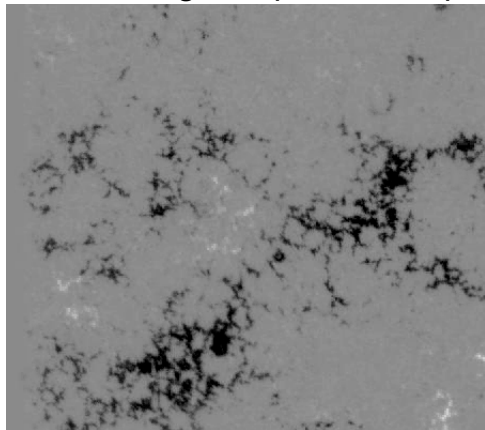
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Flux Emergence (Hinode/SOT)



Photospheric Flows: Why are They Important?

Photospheric plasma flows estimated from a sequence of vector magnetograms can be used to:

- 1 Test CME initiation models that require neutral line magnetic footpoint shearing in the photosphere
 - Major open questions in Solar Physics:
“How, why, and when do CMEs and flares erupt?”
- 2 Quantify the magnitude and timing of Poynting and helicity fluxes in active regions – free energy and structure in the corona
- 3 Provide boundary conditions for MHD simulations of the corona evolution — first principles predictive space weather models

Photospheric Flows: Why are They Important?

Understanding and Testing: Flows and CME Initiation Models

Breakout Model

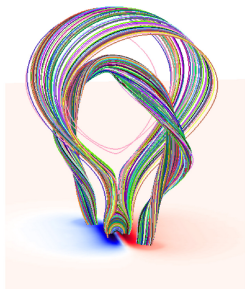
- Magnetic topology: Quadrupole: Dipolar active region in the Sun's dipolar magnetic field
- Unsigned Flux: constant (no emergence/submergence/cancelation)
- Flows parallel to the magnetic neutral line (twisting)
- Shears across the neutral line

(Antiochos, 1998; Antiochos et al., 1999)

(DeVore & Antiochos, 2008)

Photospheric Flows: Why are They Important?

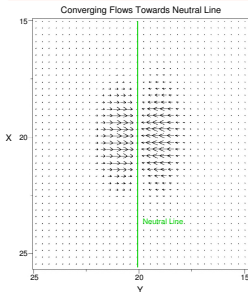
Understanding and Testing: Flows and CME Initiation Models



Flux-Cancellation Model

- Magnetic topology: Dipole or Quadrapole
- Driving:
 - Phase#1 Energize corona with twisting (like Breakout)
 - Phase#2 Initiation:
 - Unsigned Flux: decreasing (cancellation)
 - Converging flows towards the neutral line

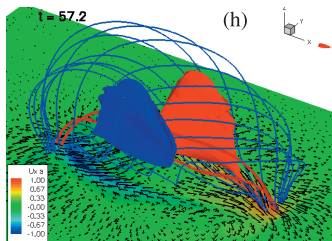
(Amari et al., 2003)



Photospheric Flows: Why are They Important?

Understanding and Testing: Flows and CME Initiation Models

Flux-Emergence Model



- Magnetic topology: Dipole
- Unsigned Flux: increasing (emergence)
- Diverging flows away from the neutral line

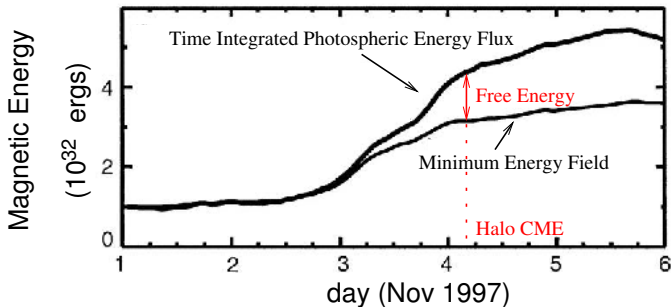
(Manchester, 2001; Manchester et al., 2004)

Photospheric Flows: Why are They Important?

Forecasting: Eruptions from Flows

Coronal Energy Budget

- $\frac{d\Delta E}{dt} = \int_{S_p} d^2x (\mathbf{v}_h B_z - v_z \mathbf{B}_h) \cdot \mathbf{B}_h / (4\pi)$
- Compare to minimum energy corona: current free “potential” field $\mathbf{B} = -\nabla\Phi$ consistent with *just* the observed photospheric B_z
- CME energy budget $\sim 10^{32}$ ergs



(Kusano et al., 2002)

Photospheric Flows: Why are They Important?

Forecasting: Eruptions from Flows

Welsch et al. (2009) examine a large number of metrics that quantified the line-of-sight magnetograms $B \simeq B_z$ and horizontal plasma velocities in 46 active regions derived from two velocity estimation algorithms: FLCT and DAVE.

Some Phenomenological Results:

- Small active regions are the most dynamic, but least likely to flare
- Big active regions that evolve are most likely to flare

Quantitative Results: Most strongly associated with flaring

- Quasi-Poynting flux proxy $S = \int d^2x |\mathbf{v}_h| B_z^2$
Assumes $\mathbf{v}_h B_z^2 \sim (\mathbf{v}_h B_z - v_z \mathbf{B}_h) \cdot \mathbf{B}_h$, $B_z \propto \mathbf{B}_h$
- Unsigned flux near the neutral line $R = \Sigma W_{NL} |B_z|^2$

How Can We Estimate Photospheric Flows?

- 1 Doppler measurements (spectroscopy)
 - Provides line of sight velocity
 - Mixture of flows parallel and perpendicular to the photospheric magnetic field except near the neutral line where $\mathbf{B} = \mathbf{B}_h$
- 2 Optical flow methods (LCT, MEF, DAVE, DAVE4VM), etc, solve inverse problem: given time-history of \mathbf{B} calculate \mathbf{v}
 - Assume a motion model: For example the magnetic induction equation with the ideal Ohm's law $\mathbf{E} = -\mathbf{v} \times \mathbf{B}/c$
$$\partial_t B_z = -\nabla \cdot (\mathbf{v}_h B_z - v_z \mathbf{B}_h)$$
 - Ancillary assumption: Additional information about the **local flow structure** or **global flow properties** is required to resolve motion ambiguity
 - Answer depends on assumed model and ancillary assumptions

REPEAT: All optical flow methods involve a motion model and ancillary assumptions, **including local correlation tracking (LCT)**!

How Can We Estimate Photospheric Flows?

Simple illustration of motion ambiguity

Edge Moving in an Aperture

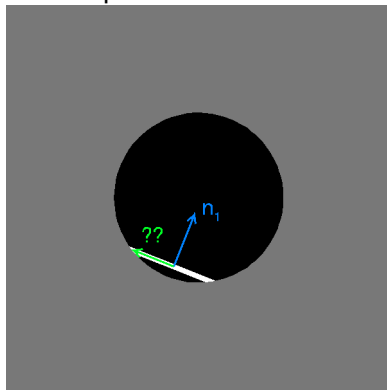
- 1 Assume a motion model:

$$\partial_t B_z = -\mathbf{v}_h \cdot \nabla_h B_z \text{ (advection)}$$

Two unknowns: \mathbf{v}_h

- 2 Assume local velocity profile:
 $\mathbf{v}_h = \mathbf{v}_0$ (rigid motion)
- 3 Correlate location of edge to infer motion from frame to frame

Aperture Problem



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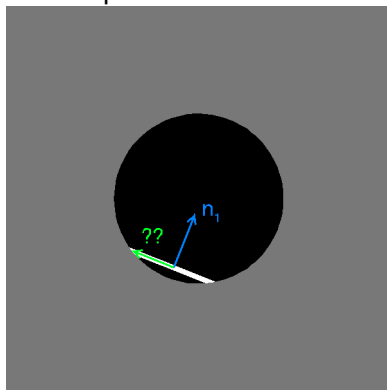
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- These are the assumptions underlying local correlation tracking (Schuck, 2005, 2006)

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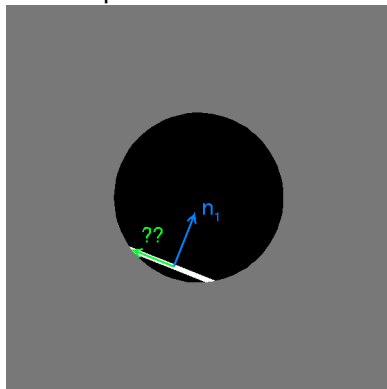
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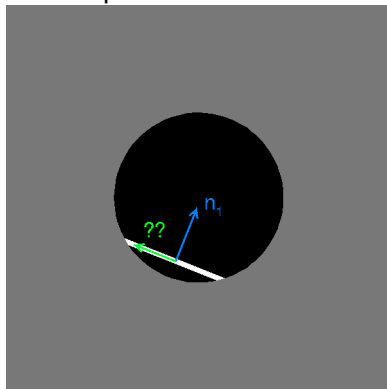
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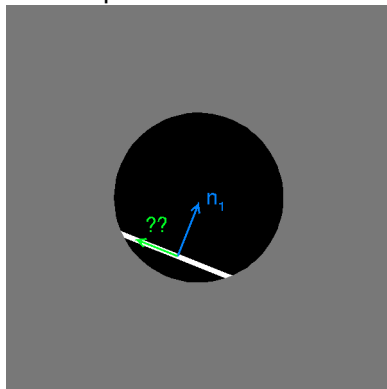
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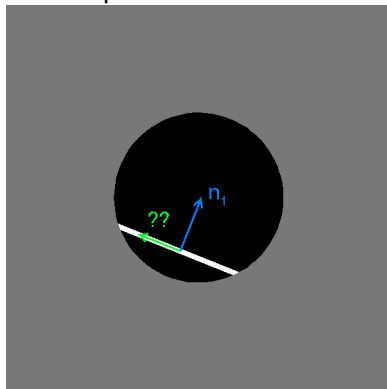
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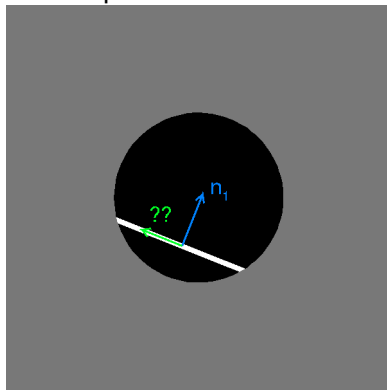
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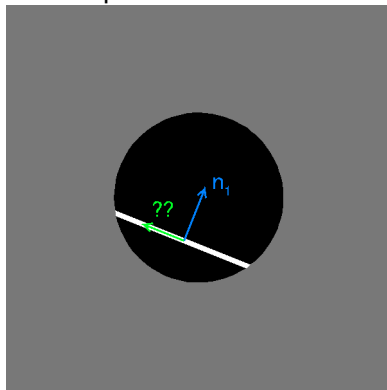
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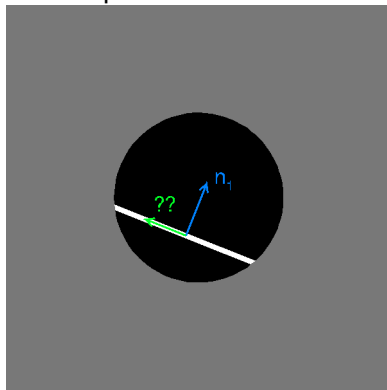
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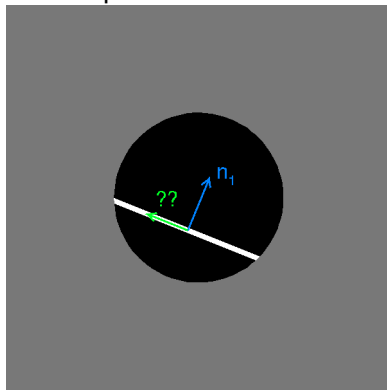
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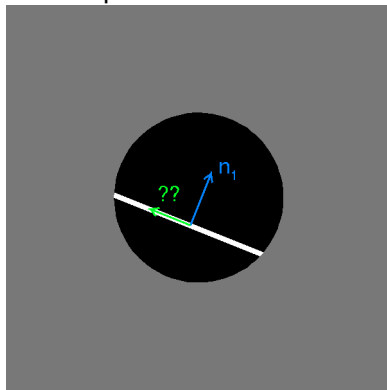
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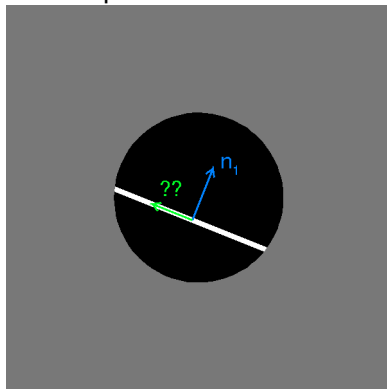
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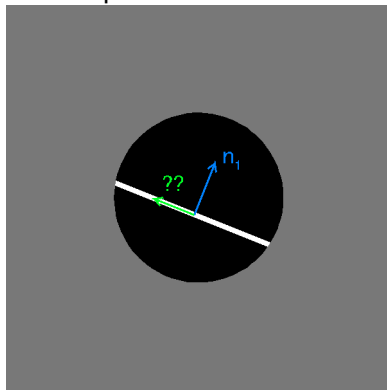
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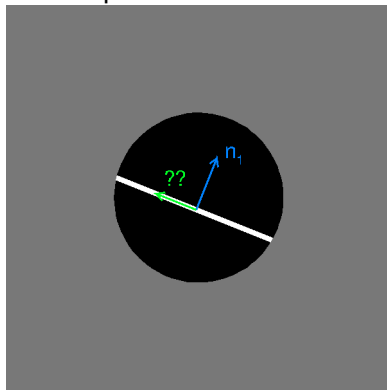
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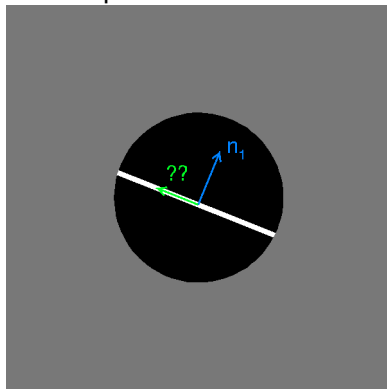
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- 3 Correlate location of edge to infer motion from frame to frame

In a small aperture there may not be enough structure to unambiguously determine \mathbf{v}_0 , i.e., we only determine

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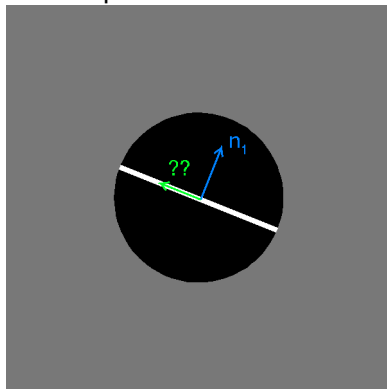
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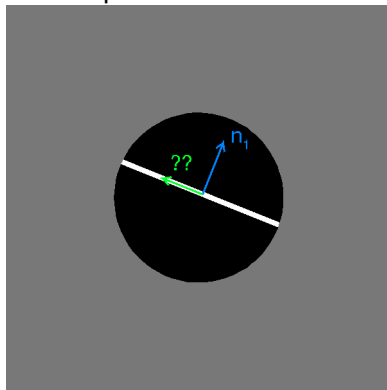
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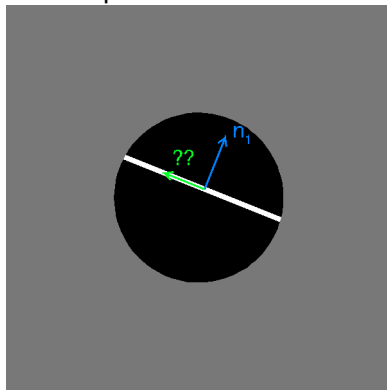
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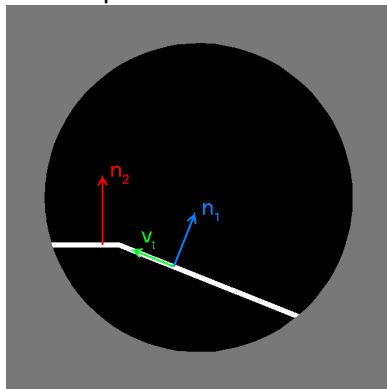
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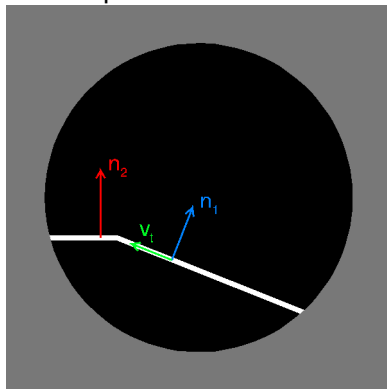
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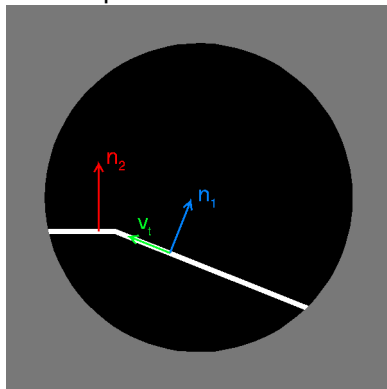
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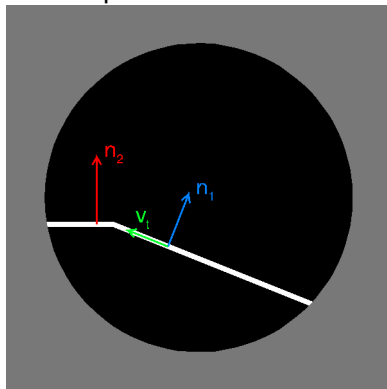
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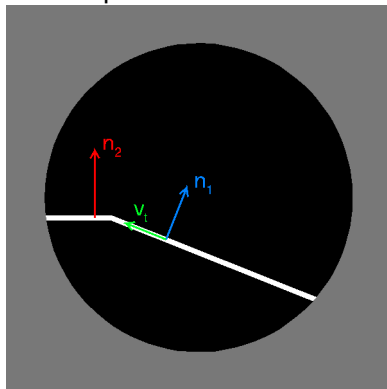
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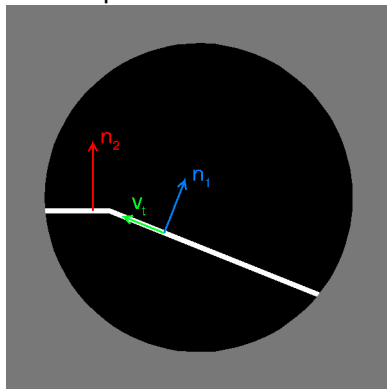
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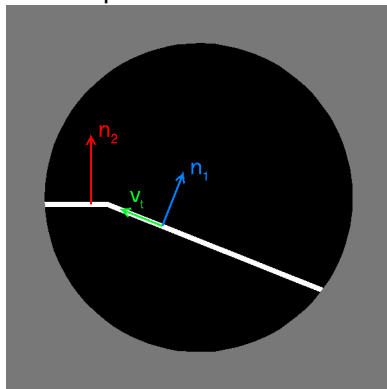
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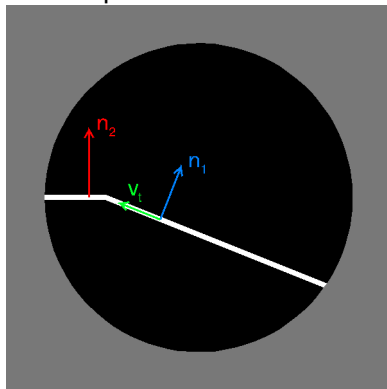
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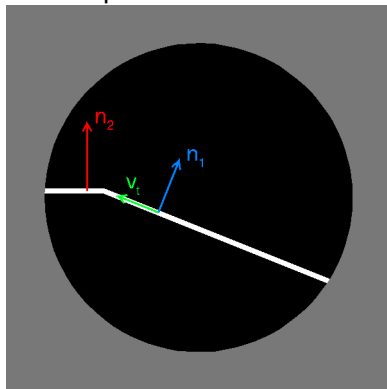
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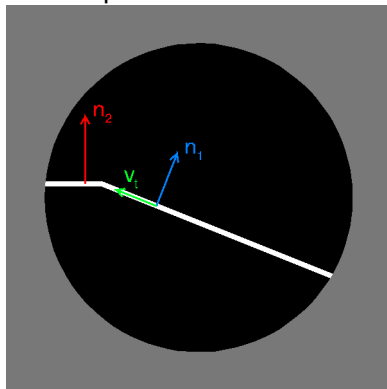
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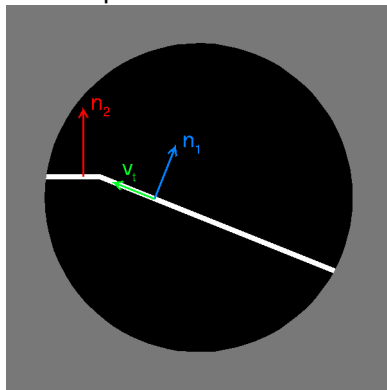
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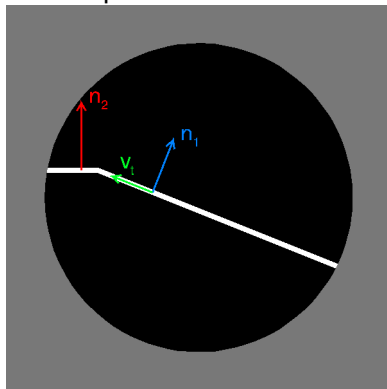
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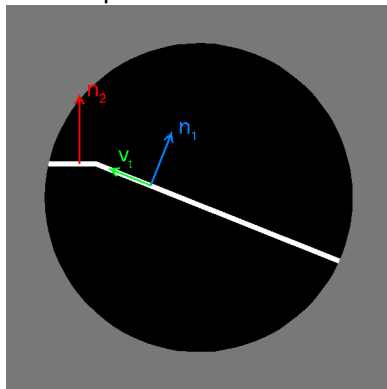
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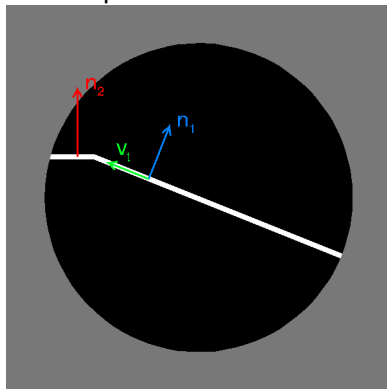
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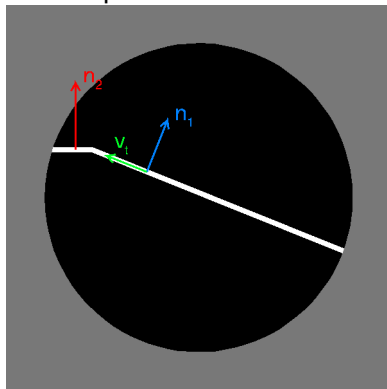
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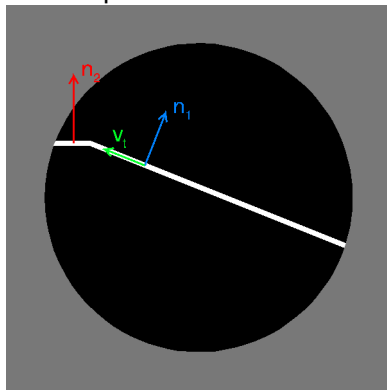
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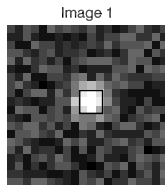
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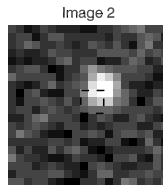
How Can We Estimate Photospheric Flows?

Local Correlation Tracking (LCT)

Template/Pattern Matching Algorithm

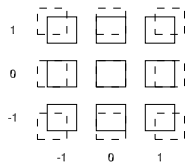


(a)

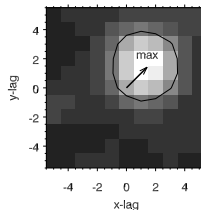


(b)

Sub-region Shifts



Cross-Correlation Matrix



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Local Correlation Tracking (LCT)

How does LCT work?

- Minimizes the functional:

$$C = \int d^2x w(\mathbf{X}_0 - \mathbf{x}) [B_n(\mathbf{x} + \mathbf{u}_0 \Delta t, t + \Delta t) - B_n(\mathbf{x}, t)]^2$$

- First order Taylor expansion:

$$C \approx \Delta t^2 \int d^2x \overbrace{w(\mathbf{X}_0 - \mathbf{x})}^{\text{Aperture}} \left[\overbrace{\partial_t B_n(\mathbf{x}, t) + \mathbf{u}_0 \cdot \nabla B_n(\mathbf{x}, t)}^{\text{Advection Equation}} \right]^2 \approx 0$$

- LCT attempts to find the velocity \mathbf{u} that minimizes the **advection operator** in the apodizing window. — **Not Induction Equation!**

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Démoulin & Berger Conjecture (2003)

Geometrical Interpretation of the
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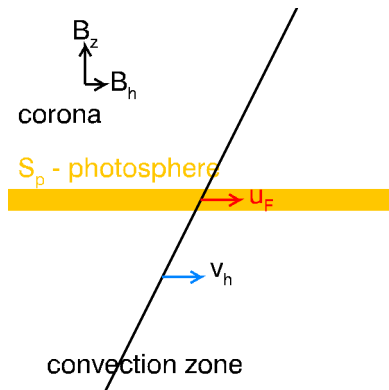
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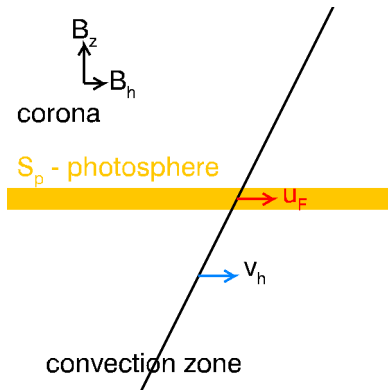
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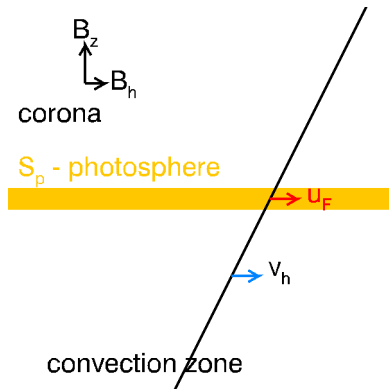
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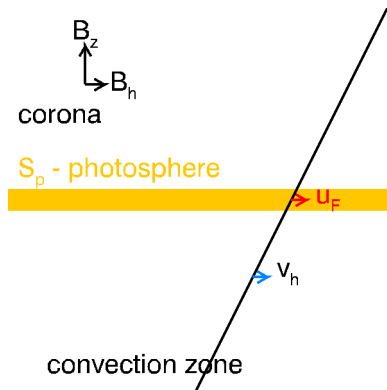
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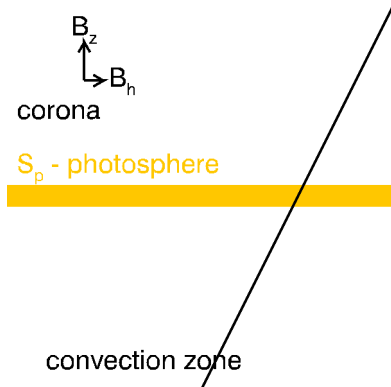
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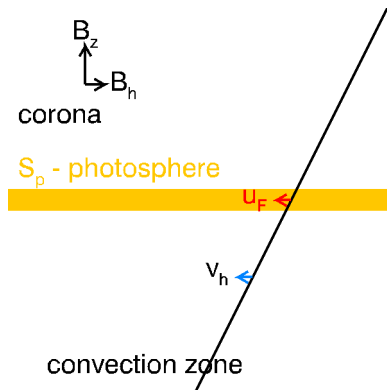
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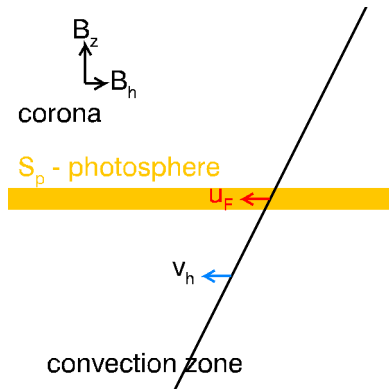
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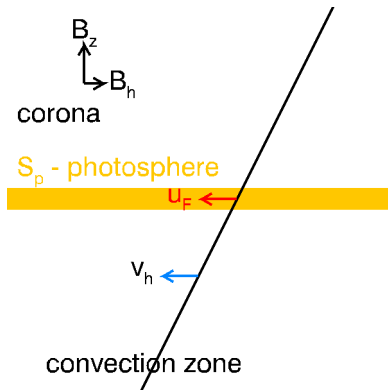
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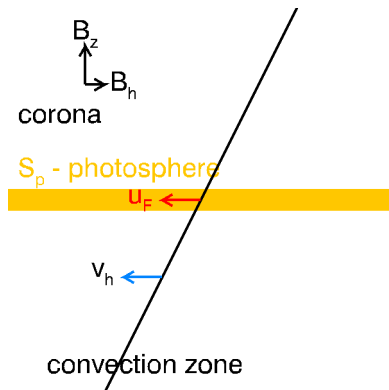
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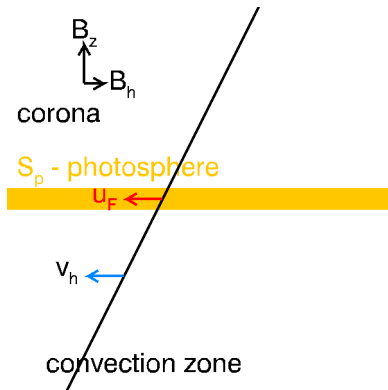
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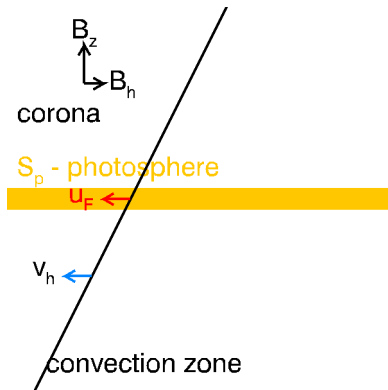
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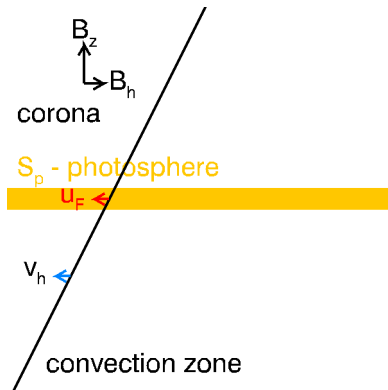
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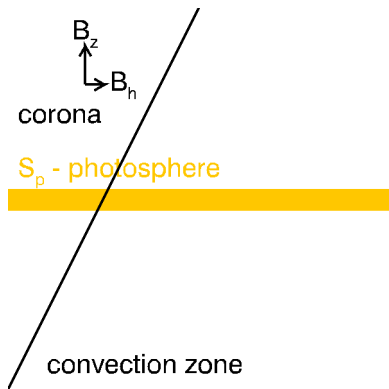
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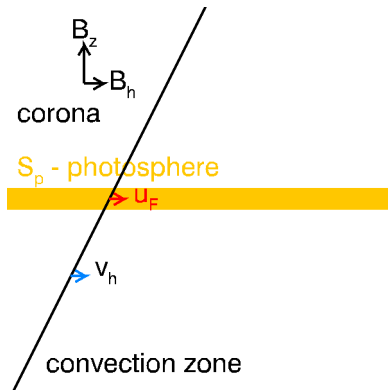
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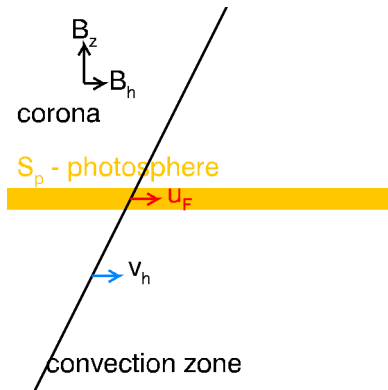
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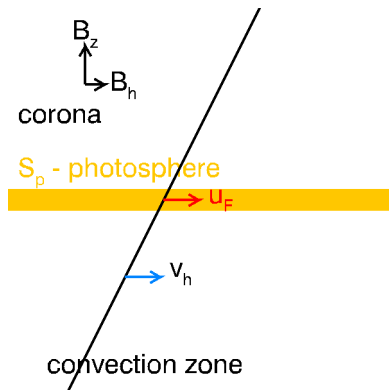
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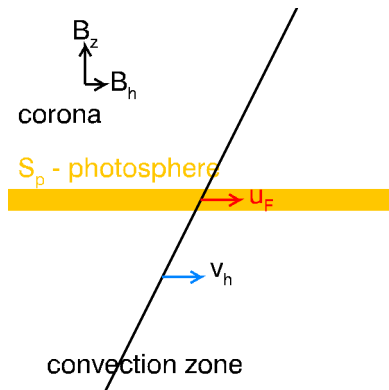
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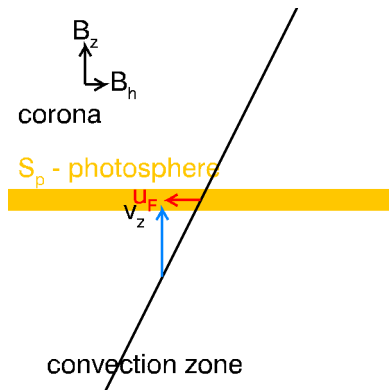
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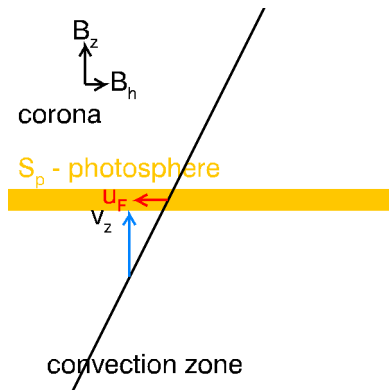
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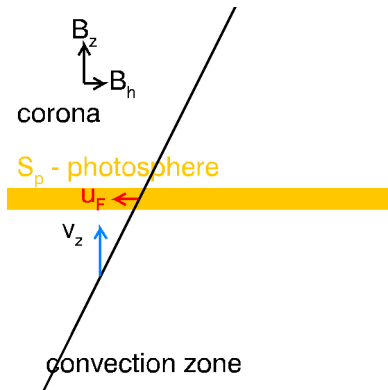
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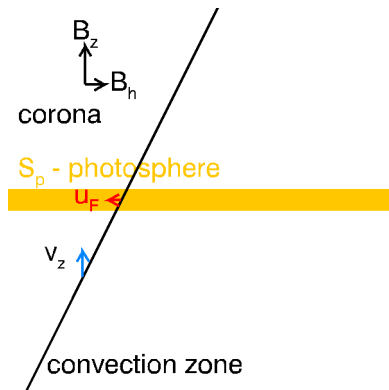
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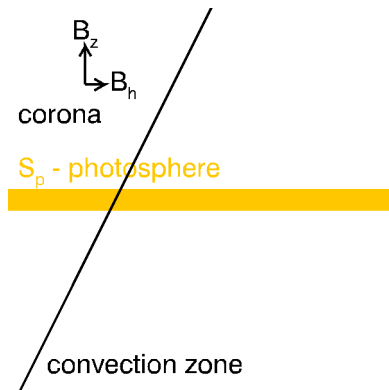
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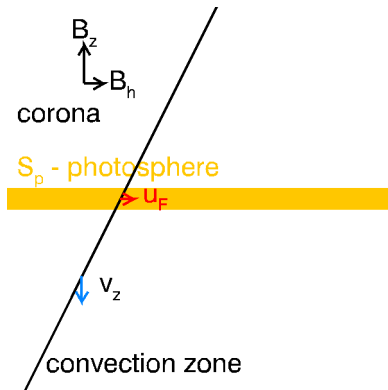
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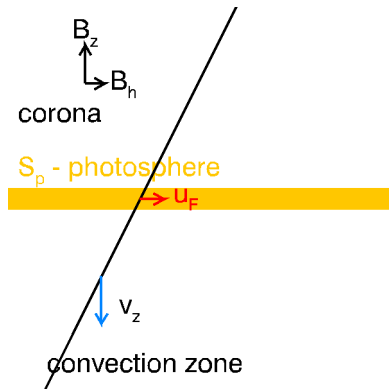
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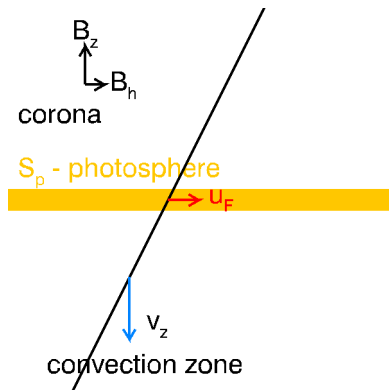
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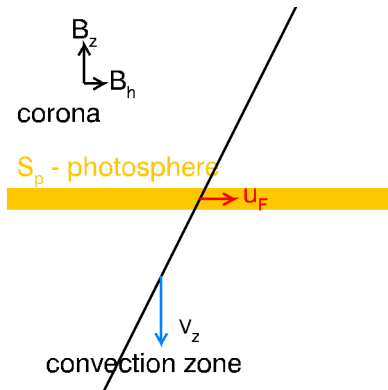
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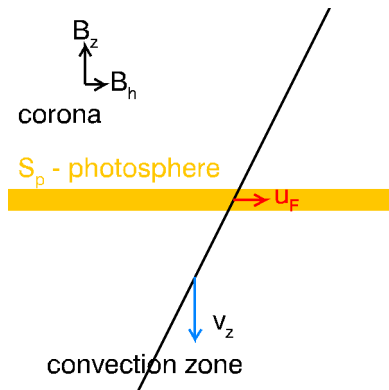
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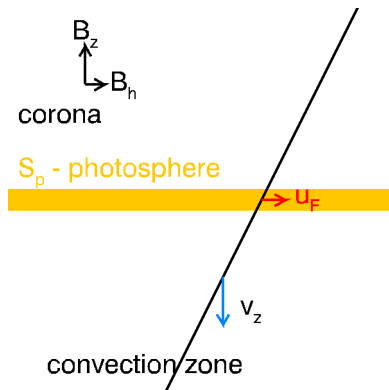
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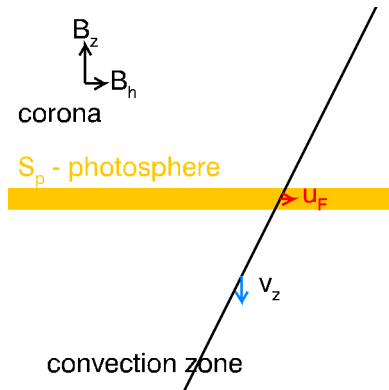
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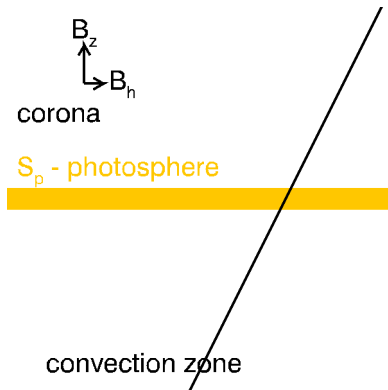
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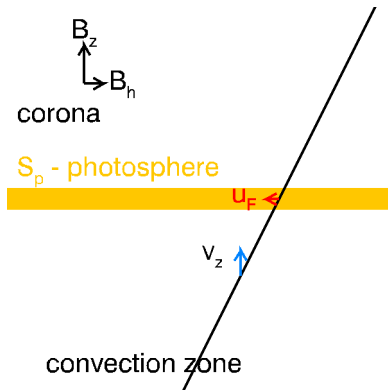
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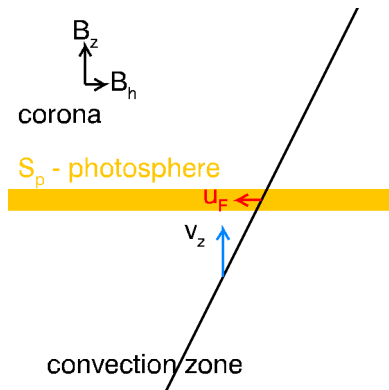
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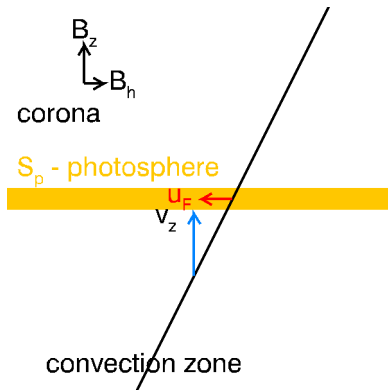
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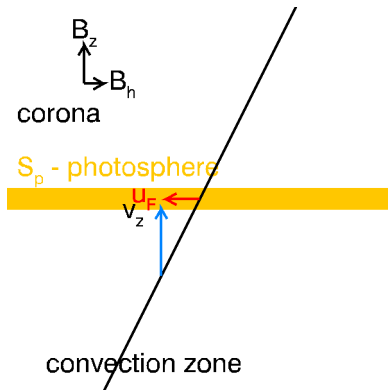
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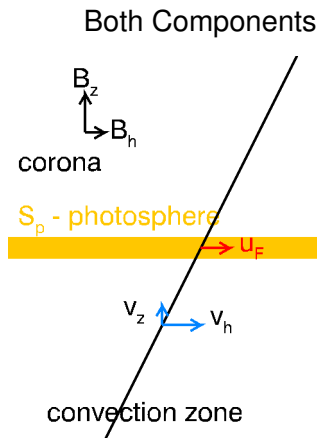
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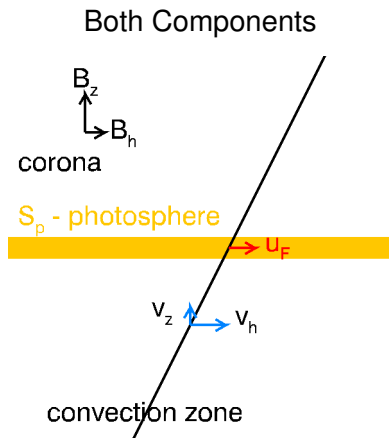
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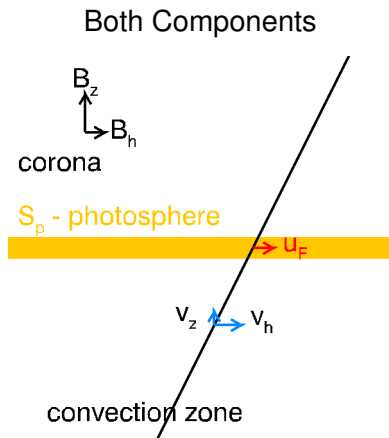
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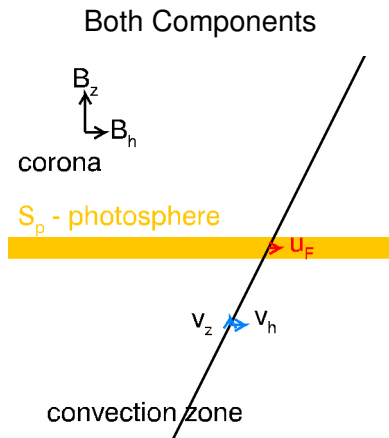
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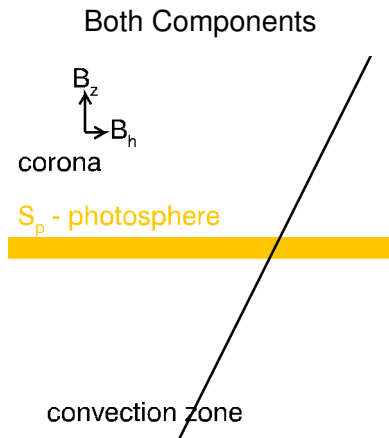
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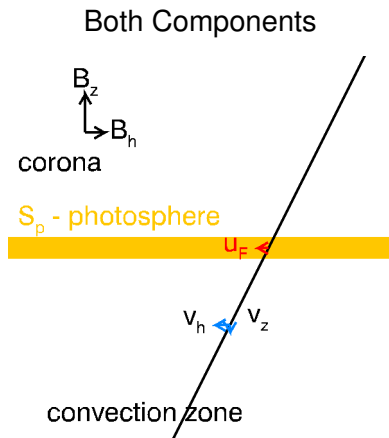
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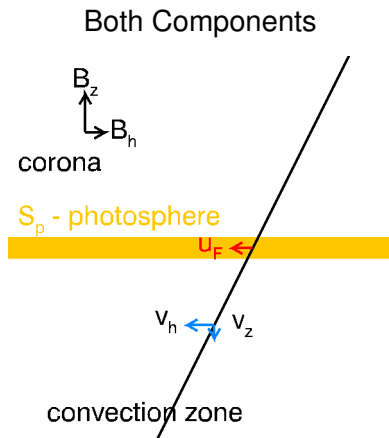
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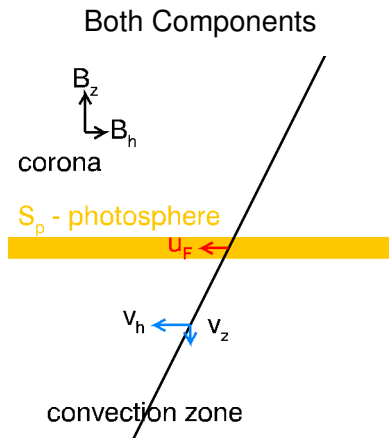
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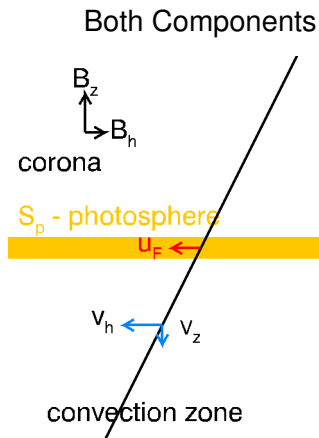
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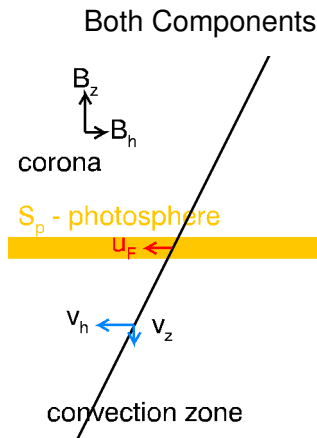
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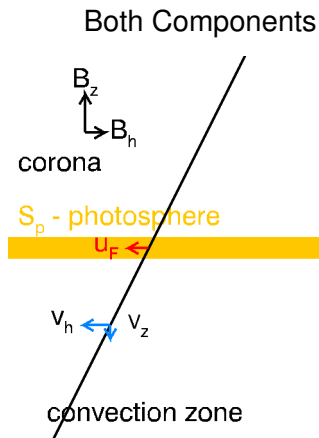
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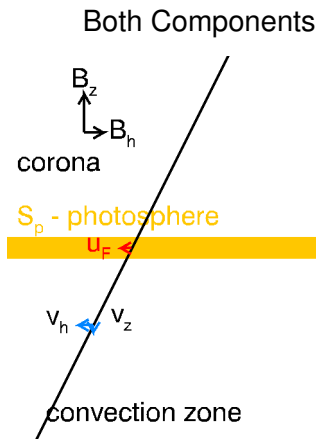
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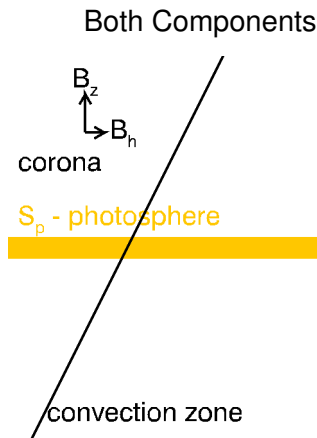
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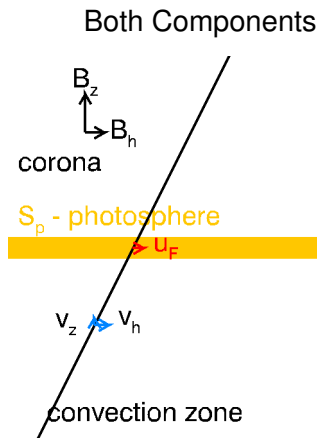
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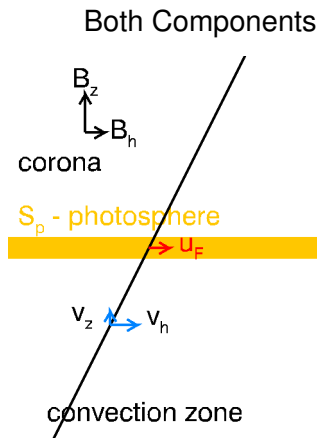
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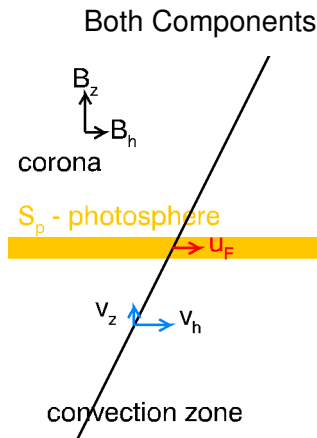
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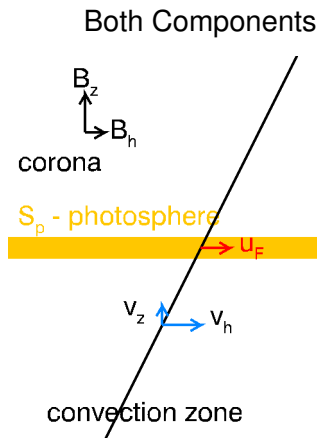
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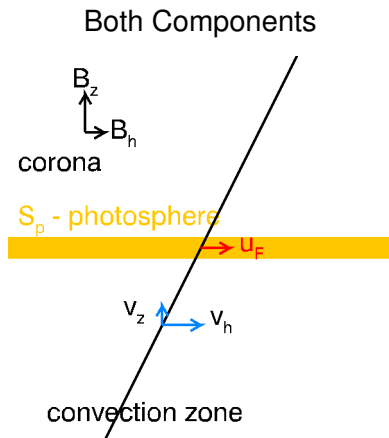
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Conjecture can be tested by comparing DAVE4VM and DAVE estimates against “ground truth” from MHD simulations



How Can We Estimate Photospheric Flows?

Differential Affine Velocity Estimator (DAVE)

$$C \approx \int dx^2 w(\mathbf{x} - \mathbf{x}_0) \left\{ \partial_t B_z(\mathbf{x}, t) + \nabla_h \cdot [B_z(\mathbf{x}, t) \hat{\mathbf{u}}_F] \right\}^2$$

$$\hat{\mathbf{u}}_F = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} + \begin{pmatrix} \hat{u}_x & \hat{u}_y \\ \hat{v}_x & \hat{v}_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad \nabla_h = (\partial_x, \partial_y)$$

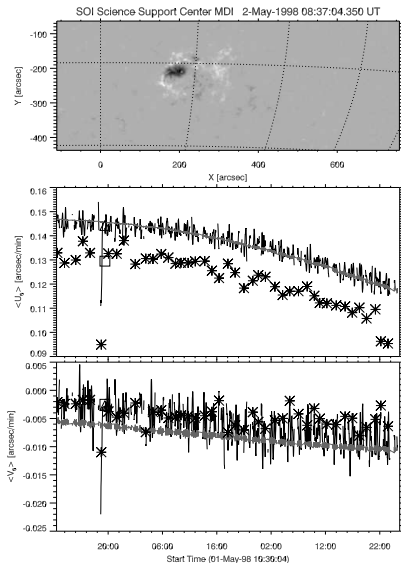
- Incorporates only vertical magnetic field component (line-of-sight)
- No explicit vertical flows
- Motivated by Démoulin & Berger's 2003 incorrect conjecture that the \mathbf{u}_F is the “magnetic footpoint velocity”
- Actually biased estimate of the horizontal plasma velocity $\mathbf{u}_F = \mathbf{v}_h$

Schuck (2006)

How Can We Estimate Photospheric Flows?

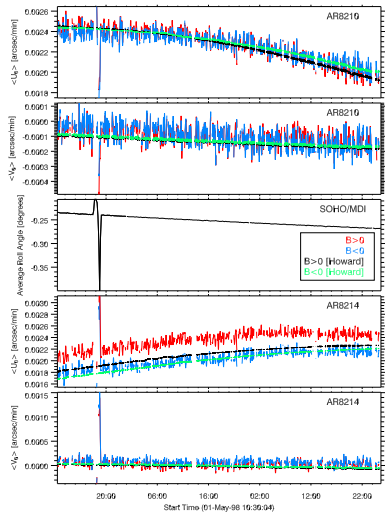
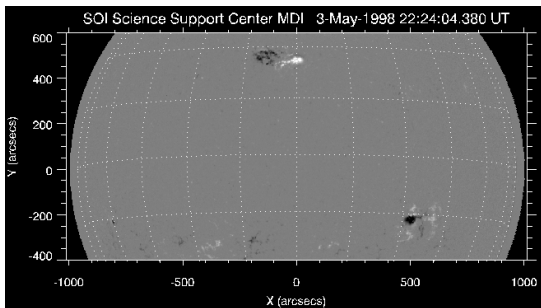
Application to MDI data AR8210

- 2600 MDI magnetograms (1-minute cadence)
- Velocities estimated for $|B_{\text{LOS}}| > 60$ G
- Mean V_x and V_y of the active region
- **Thick line** - Mean *synodic* differential velocity of the active region computed from Howard et al. (1990) and projected into the image plane.



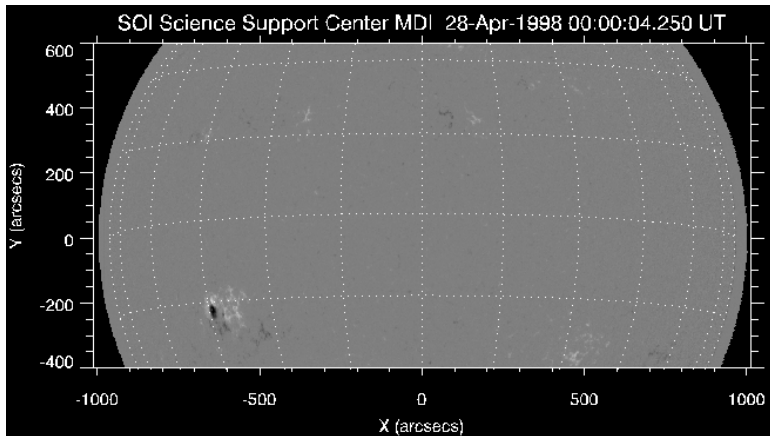
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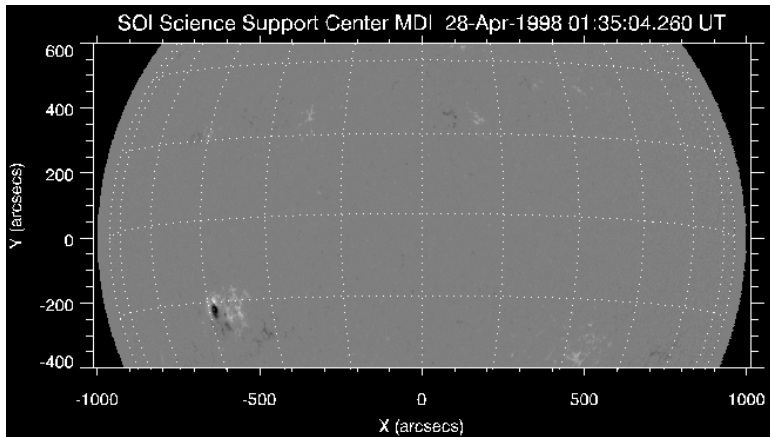
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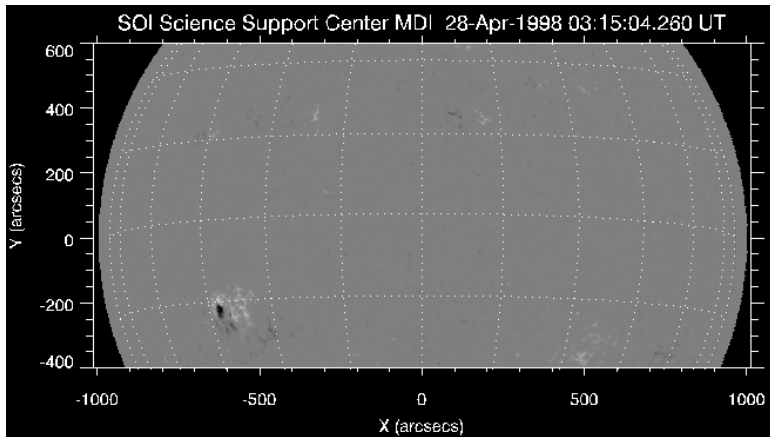
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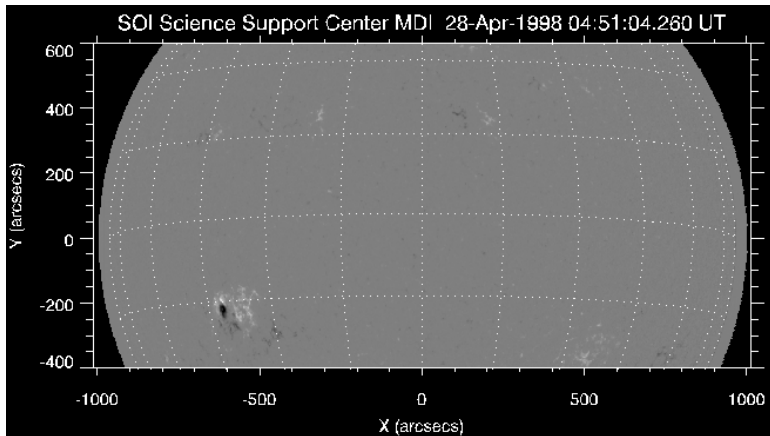
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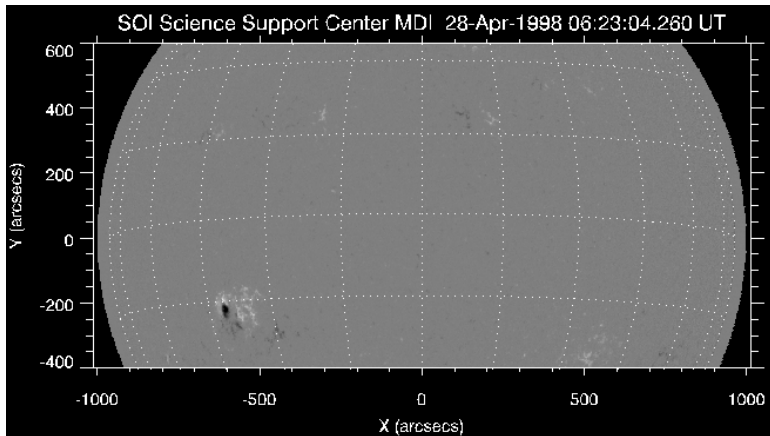
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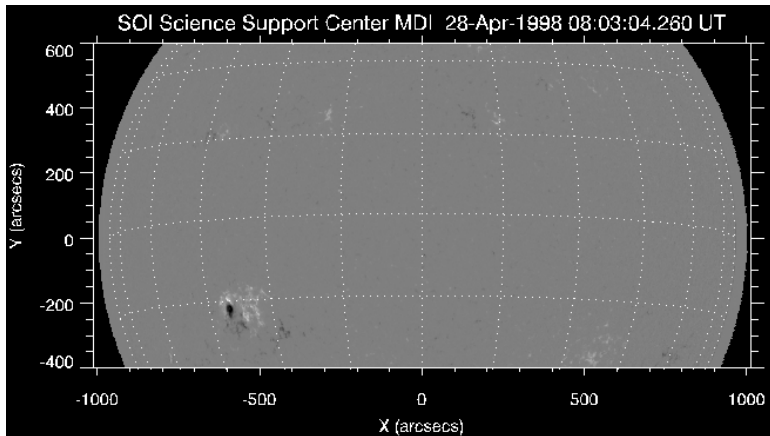
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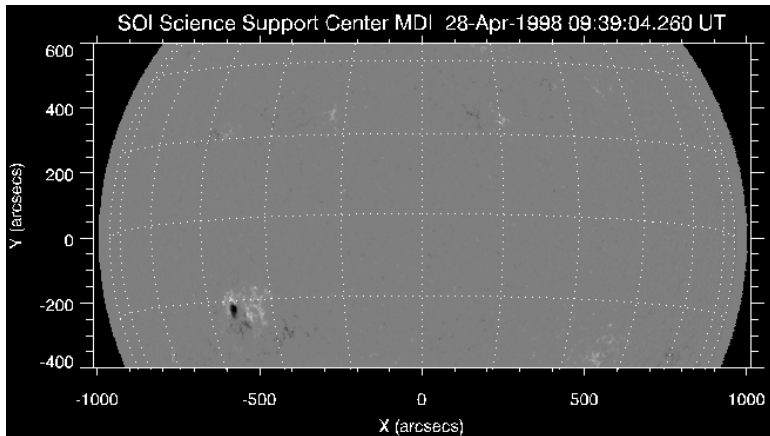
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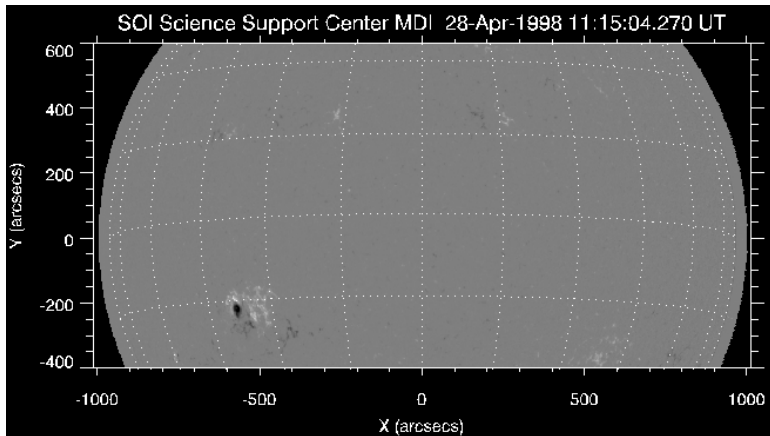
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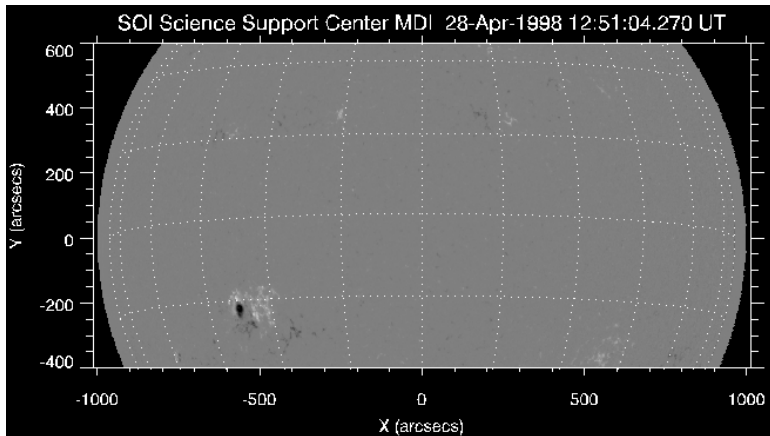
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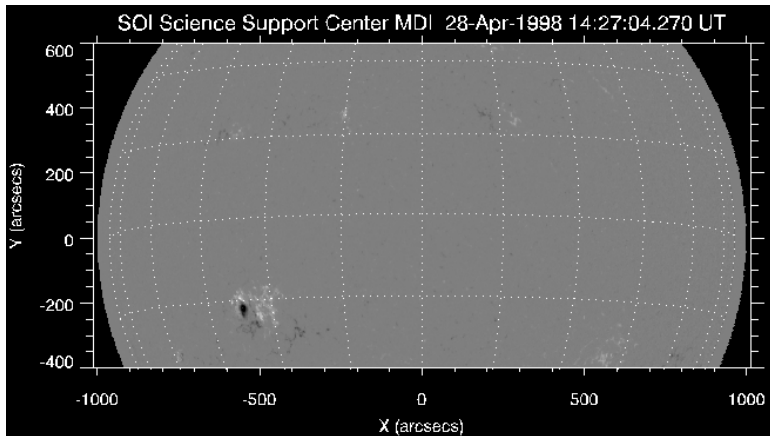
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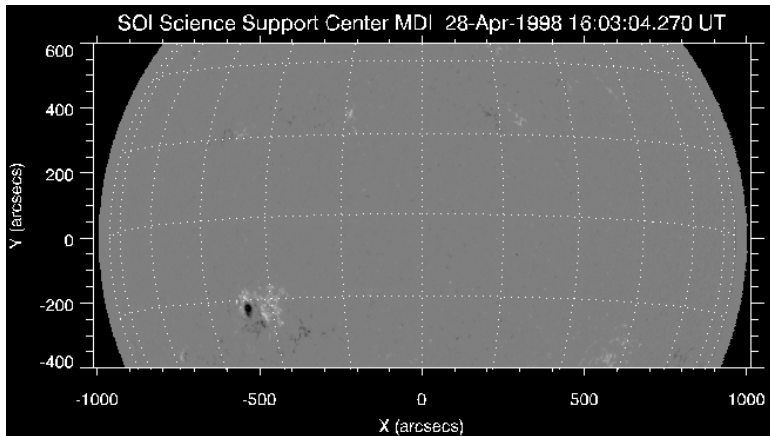
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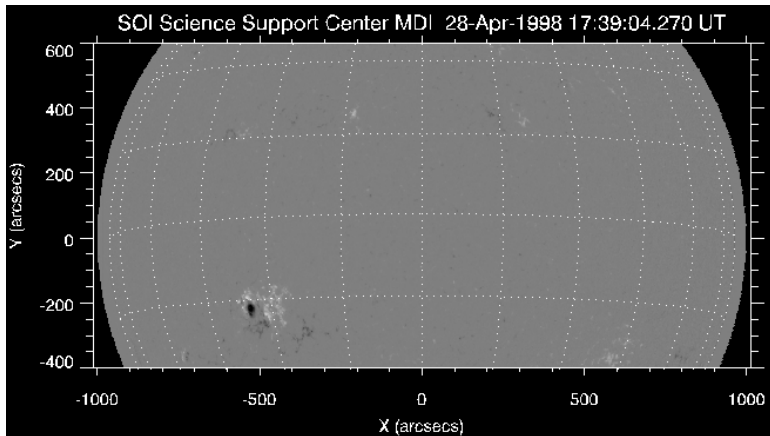
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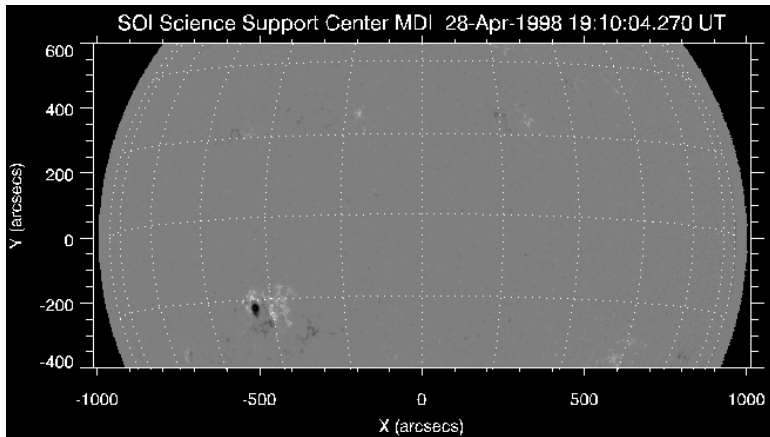
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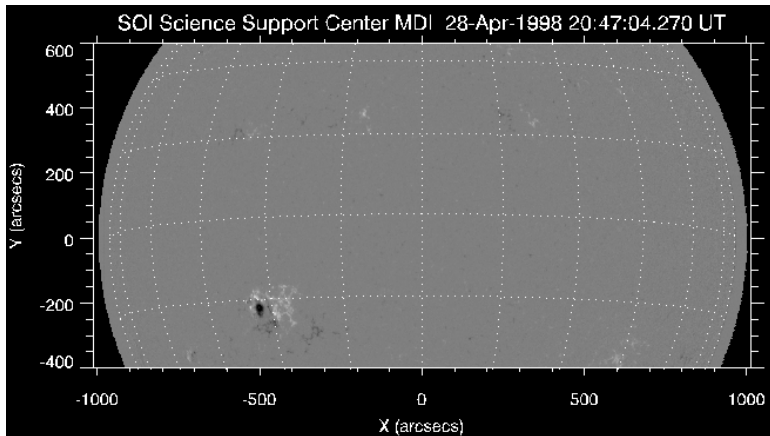
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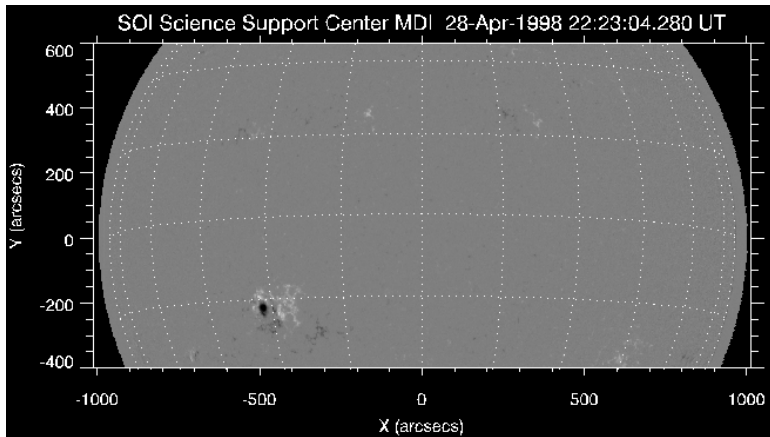
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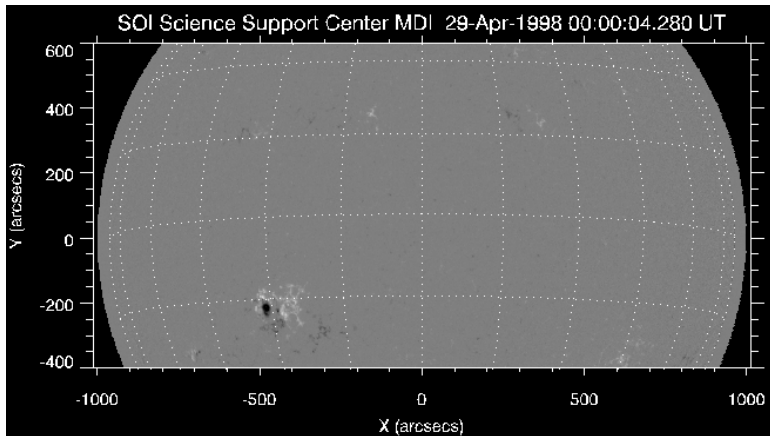
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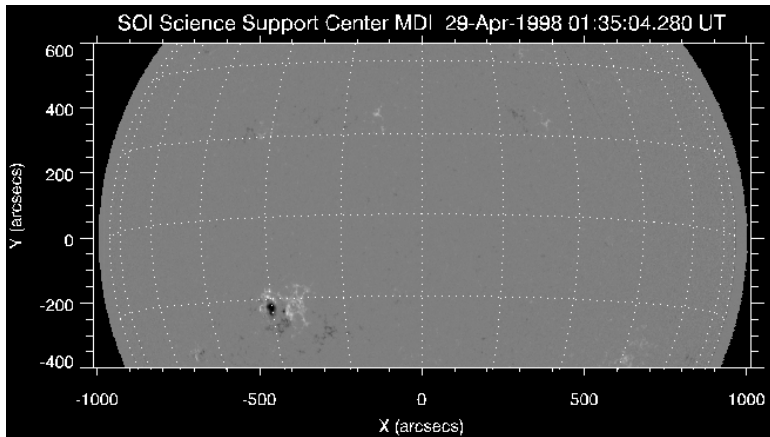
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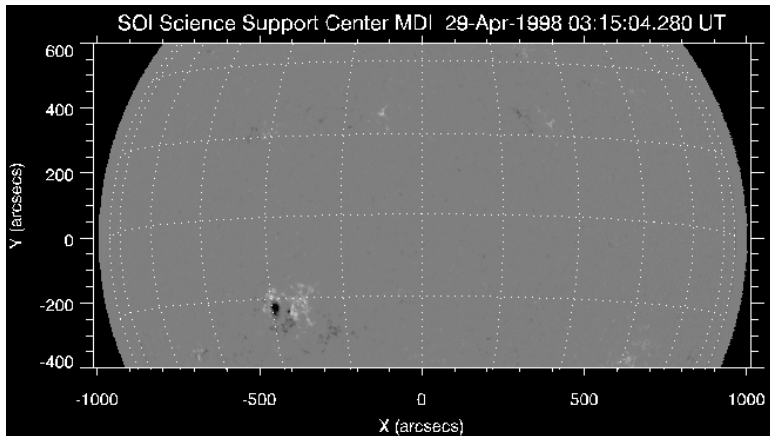
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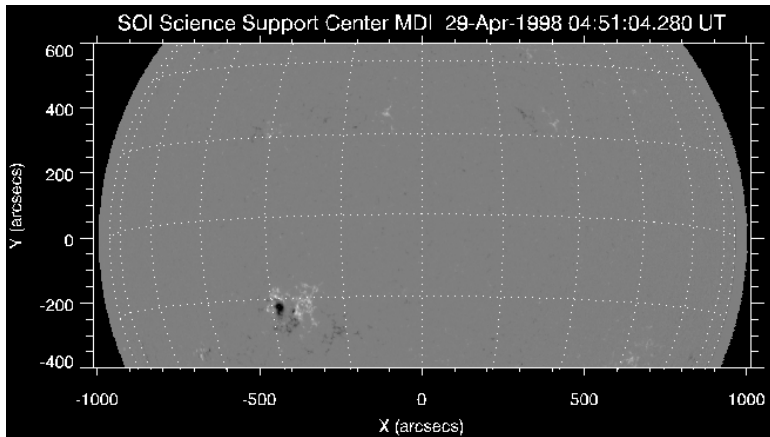
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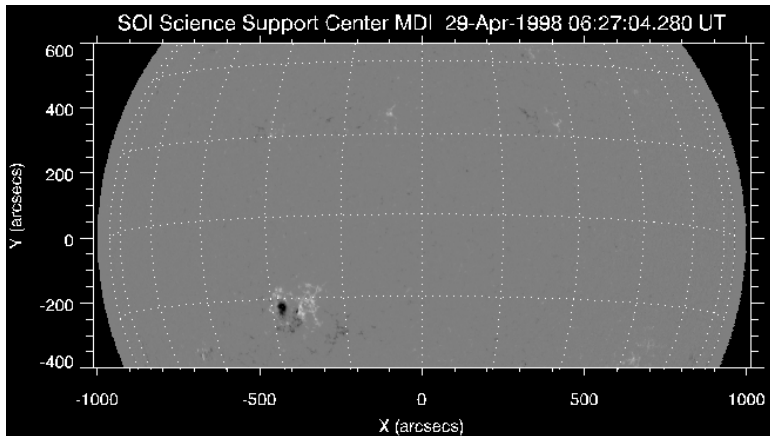
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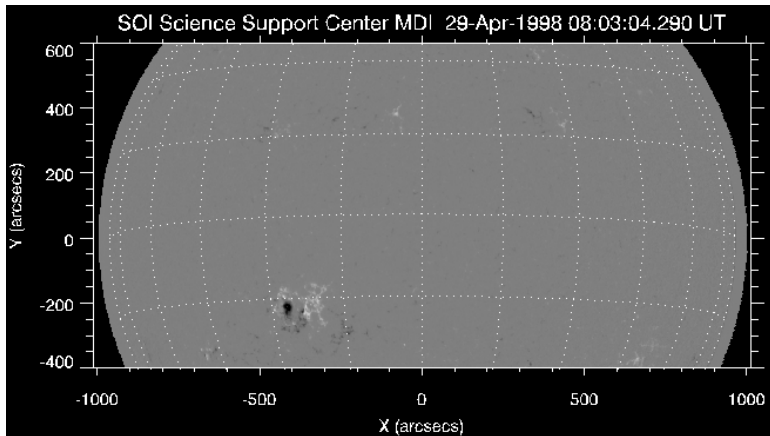
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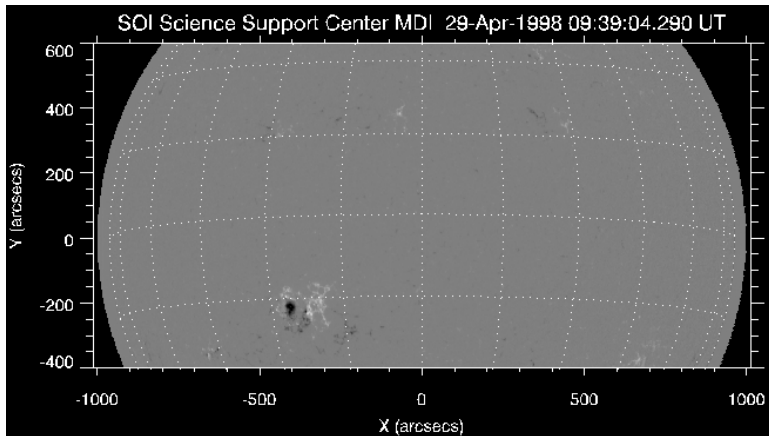
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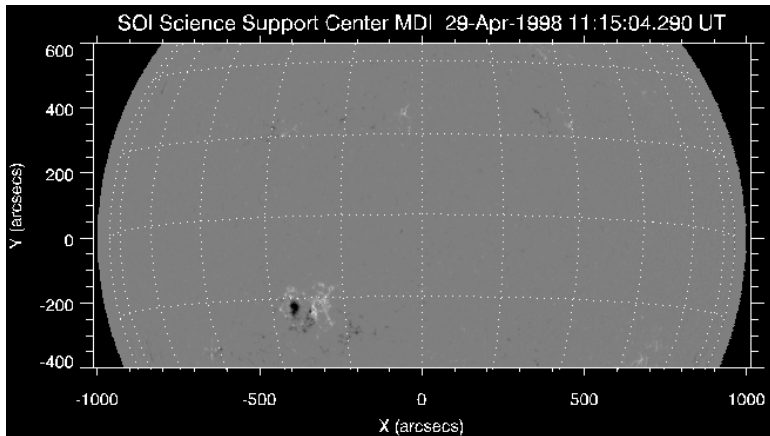
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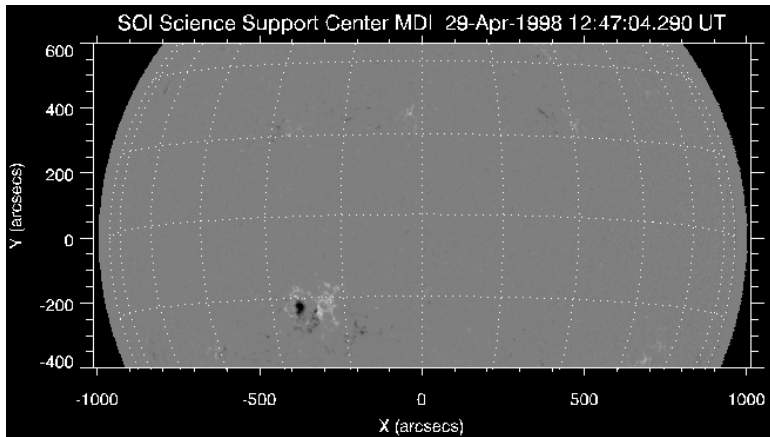
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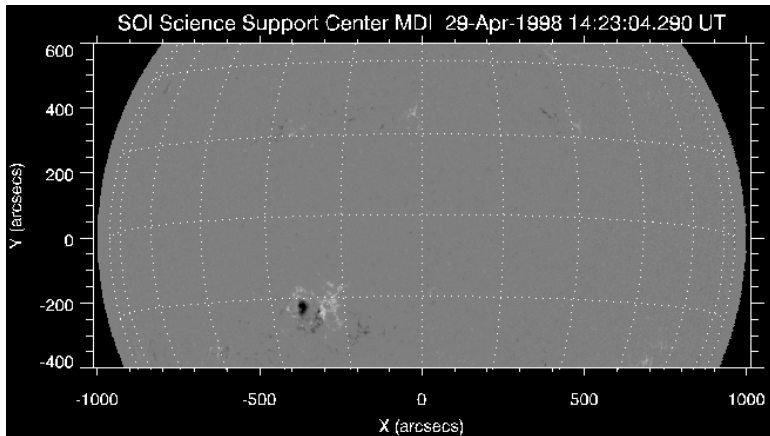
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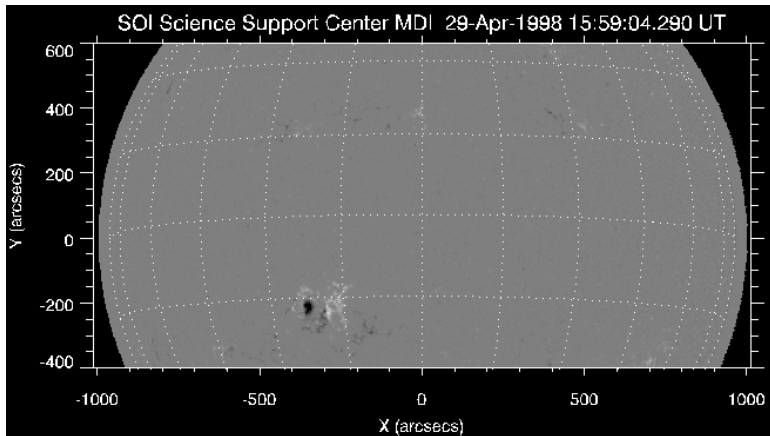
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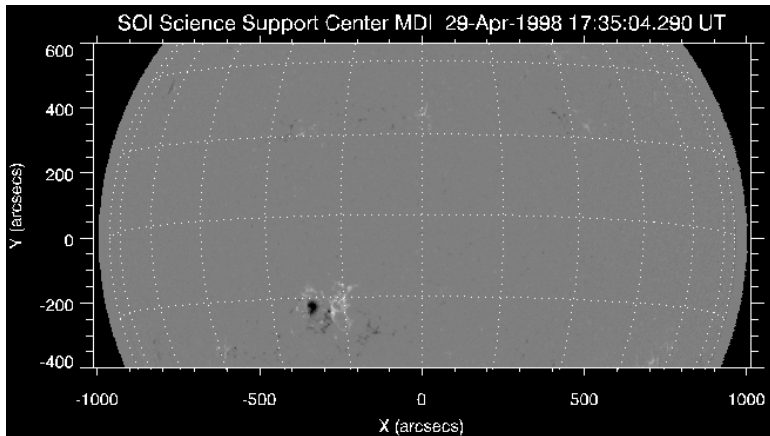
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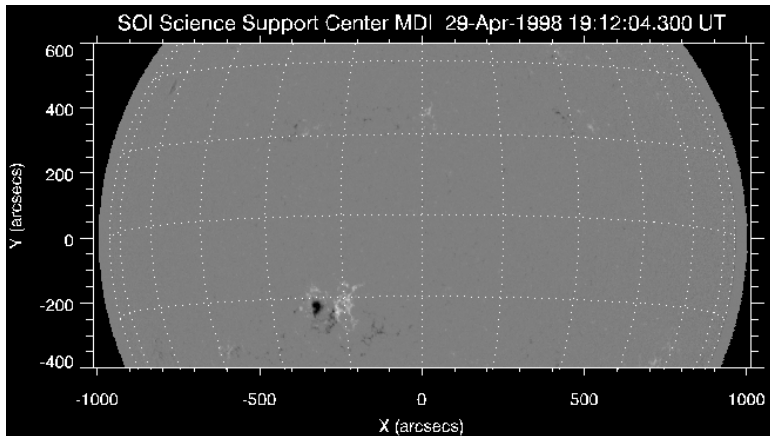
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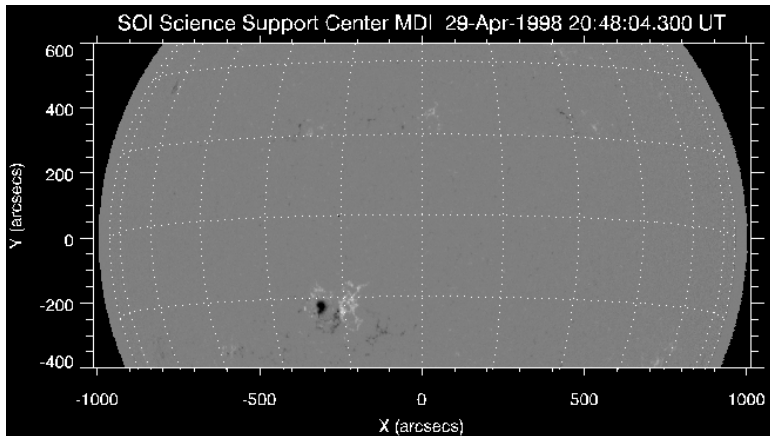
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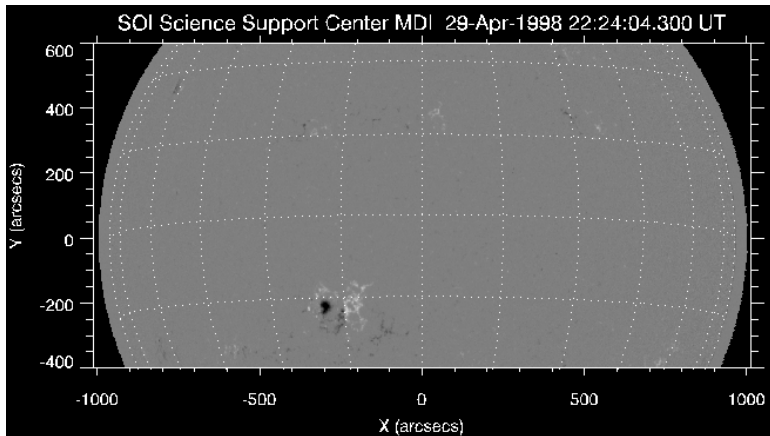
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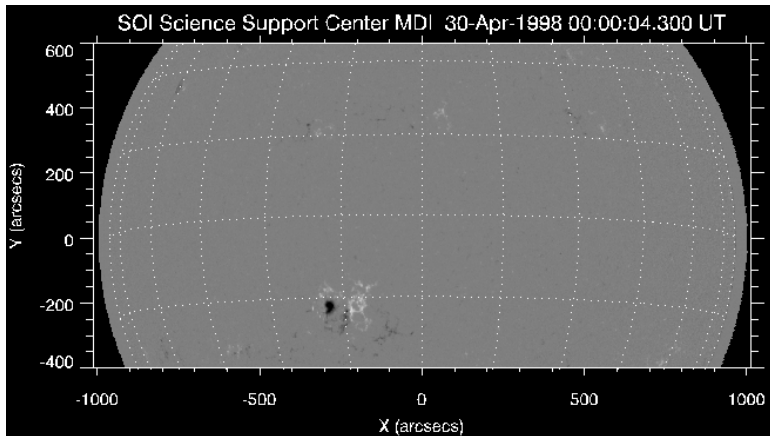
How Can We Estimate Photospheric Flows?

Application to MDI data AR8210



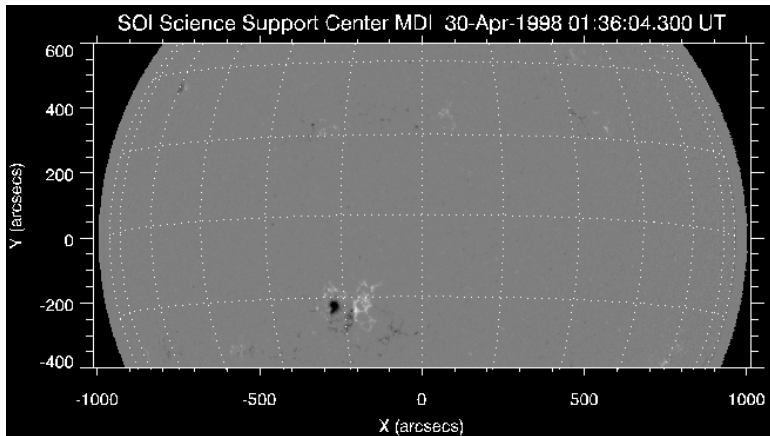
How Can We Estimate Photospheric Flows?

Application to MDI data AR8210



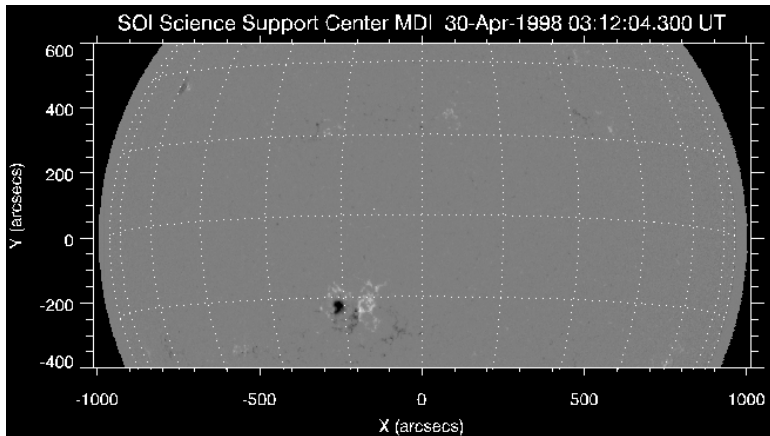
How Can We Estimate Photospheric Flows?

Application to MDI data AR8210



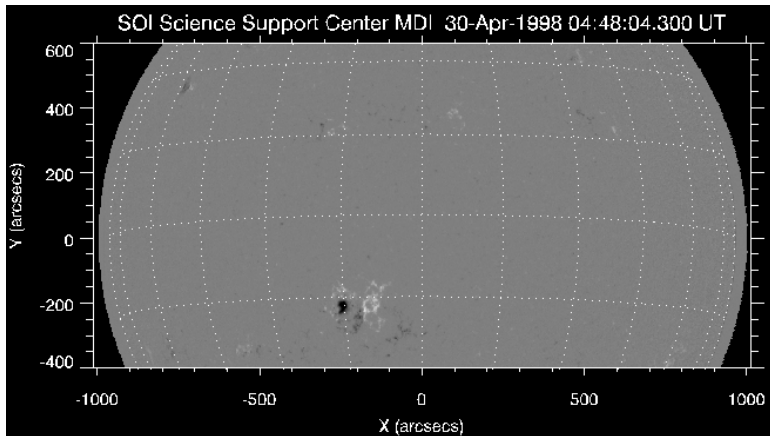
How Can We Estimate Photospheric Flows?

Application to MDI data AR8210



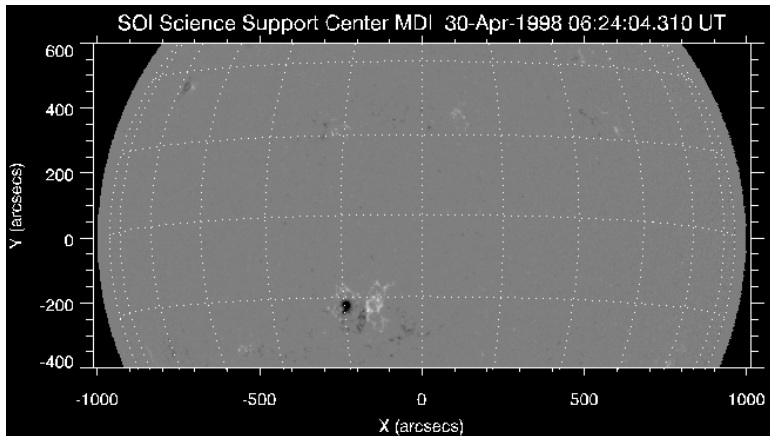
How Can We Estimate Photospheric Flows?

Application to MDI data AR8210



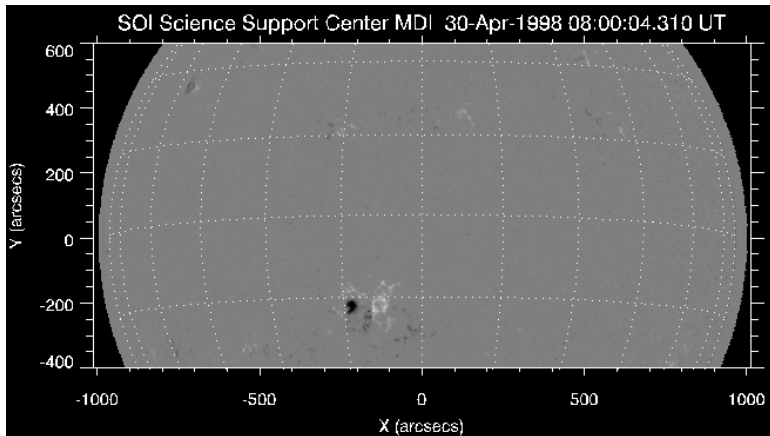
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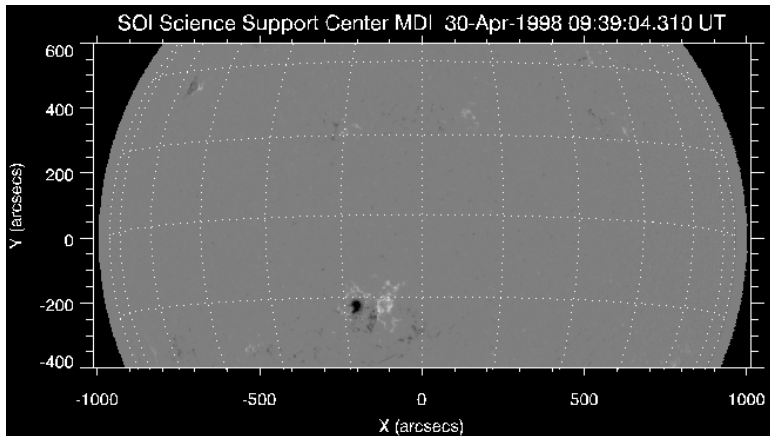
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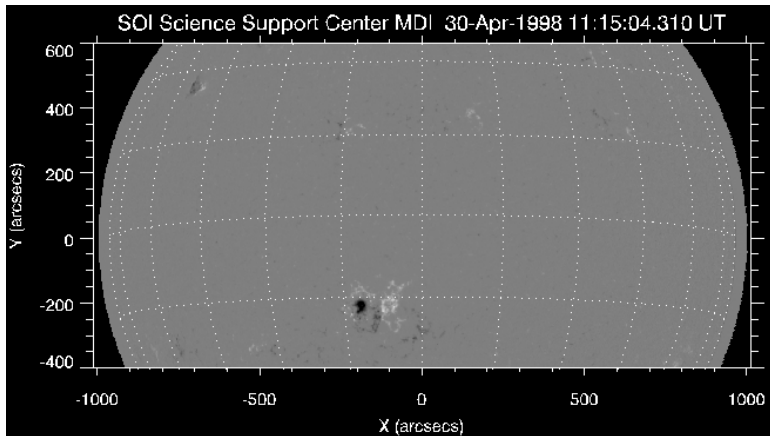
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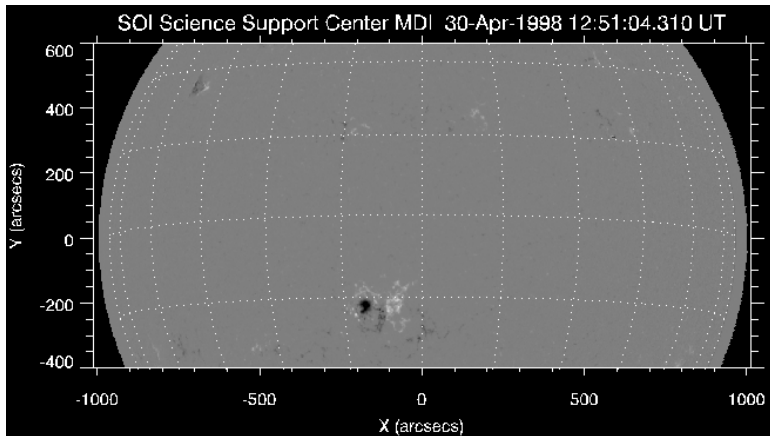
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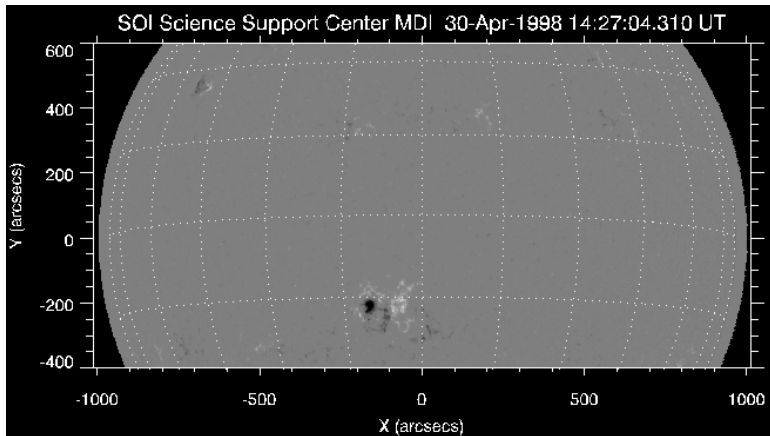
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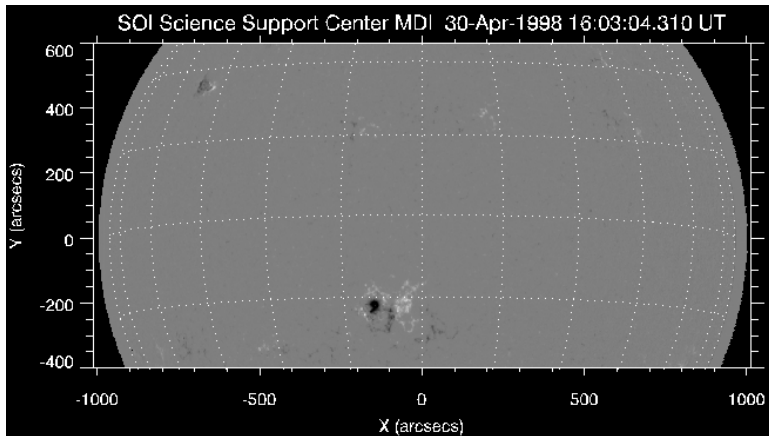
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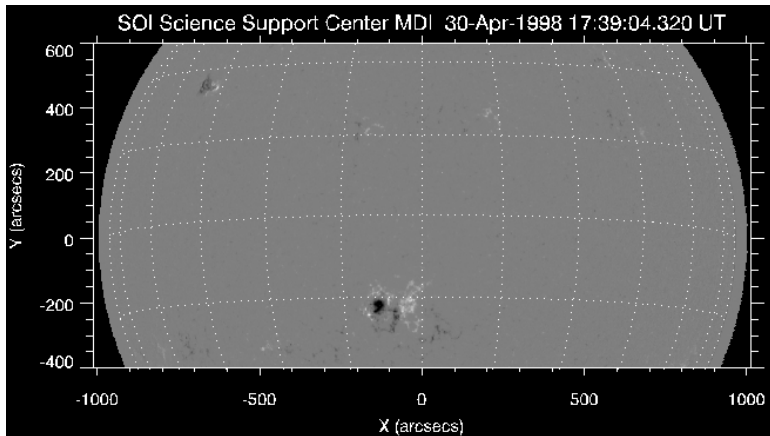
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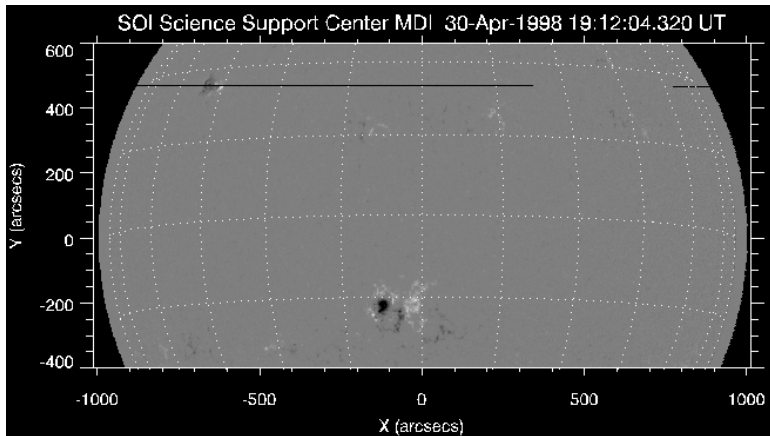
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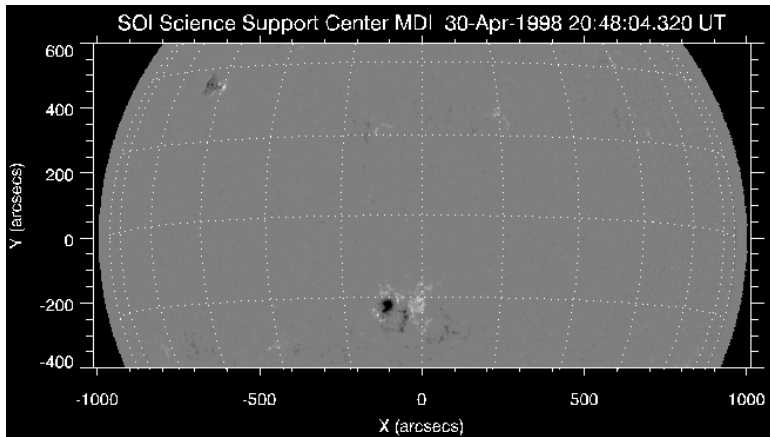
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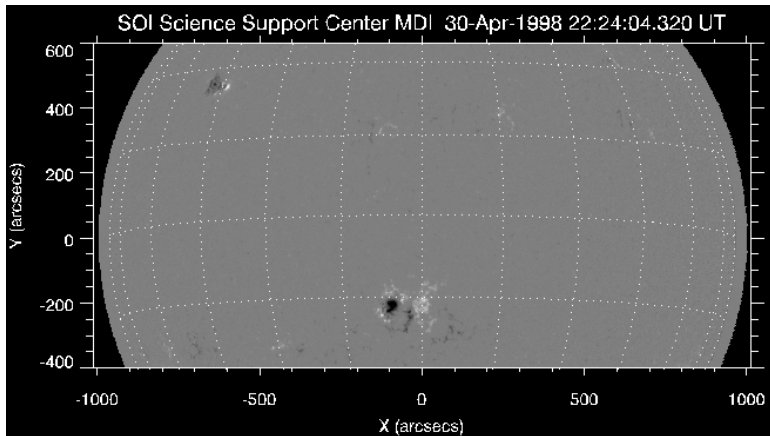
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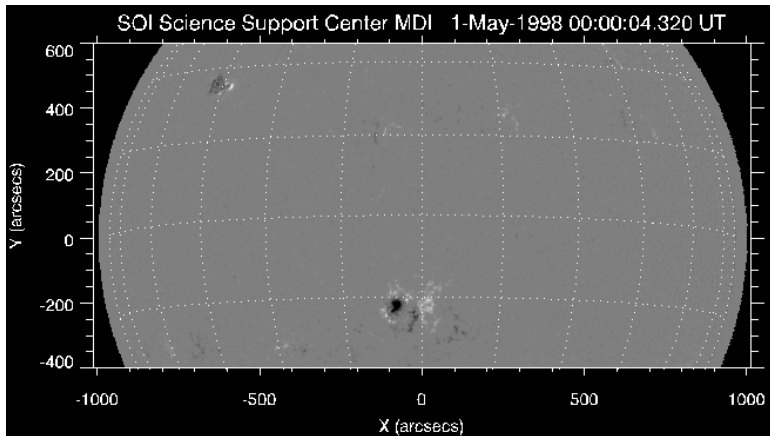
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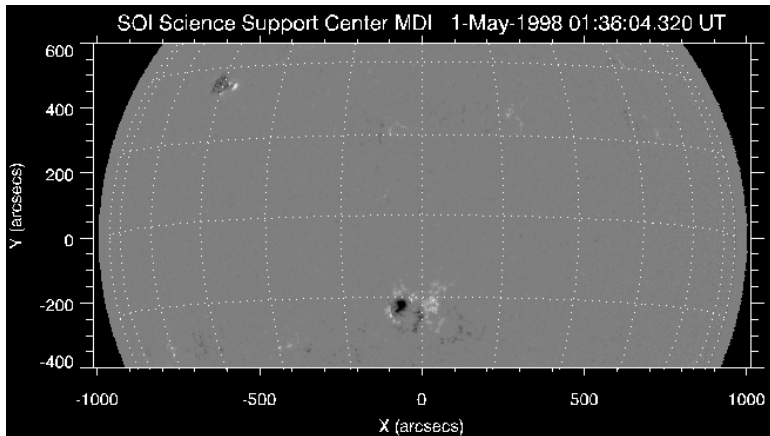
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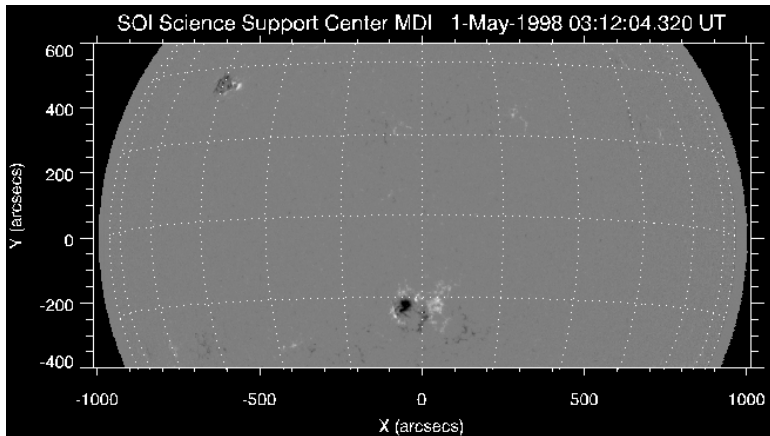
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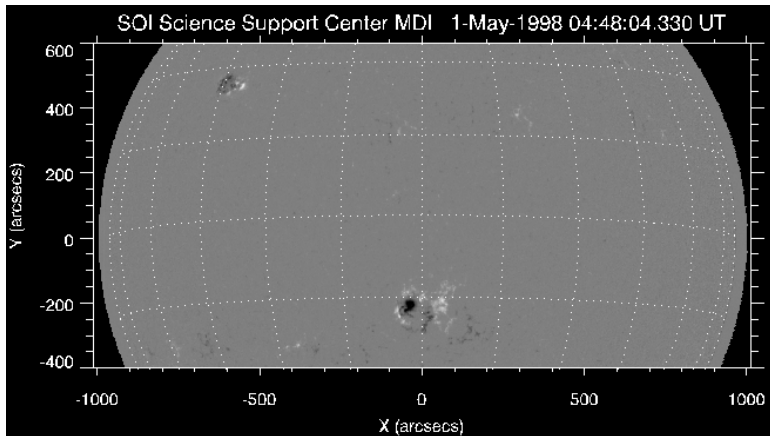
How Can We Estimate Photospheric Flows?

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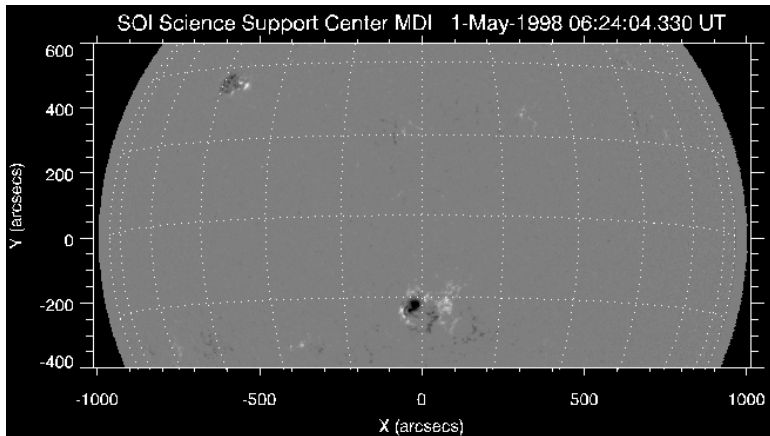
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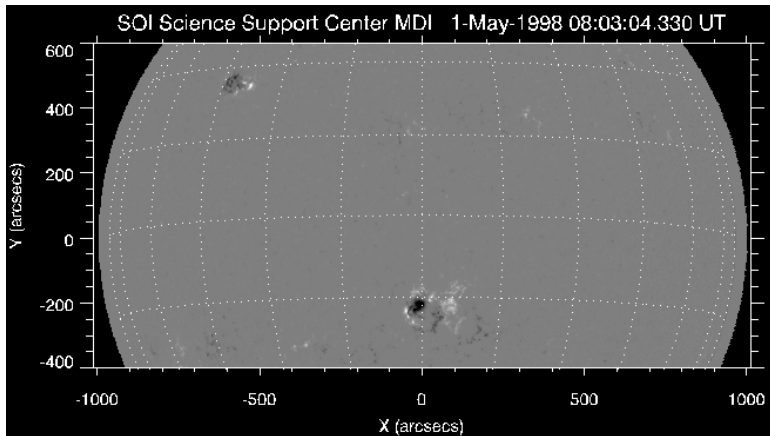
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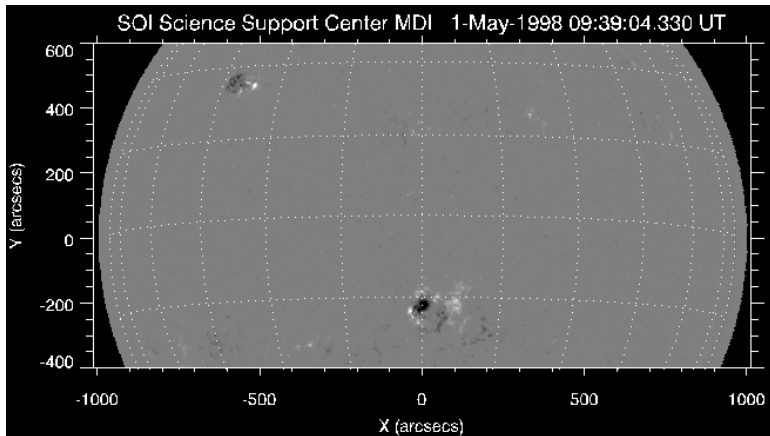
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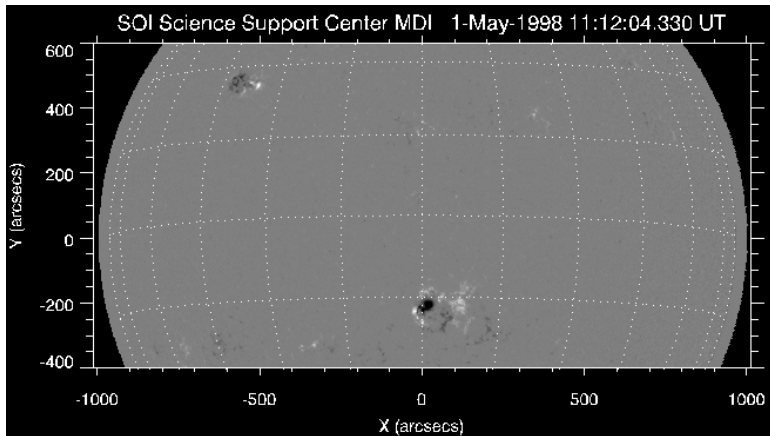
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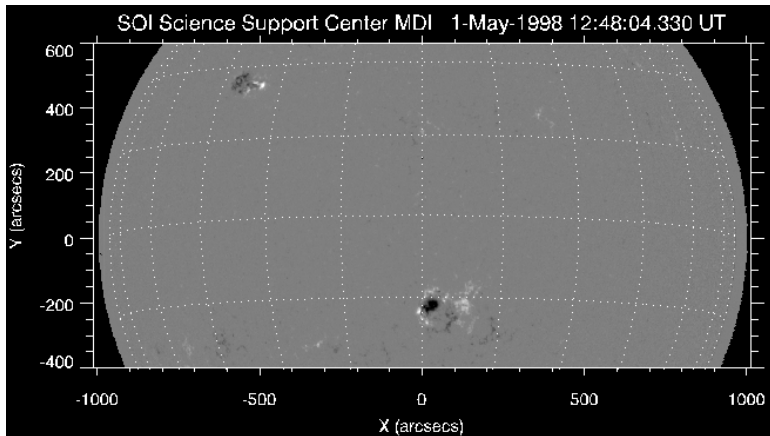
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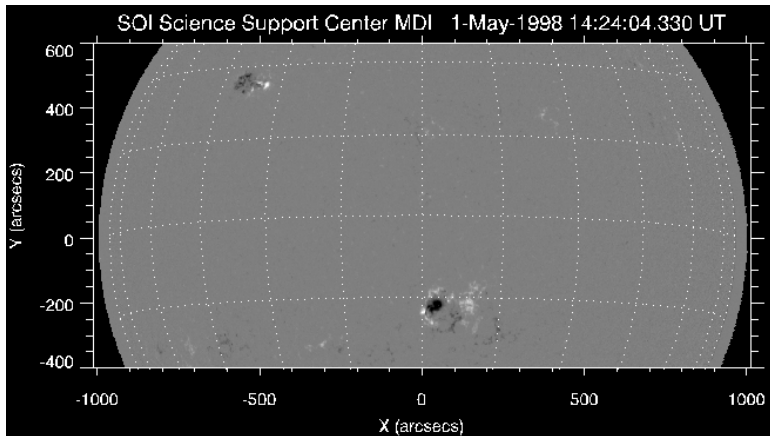
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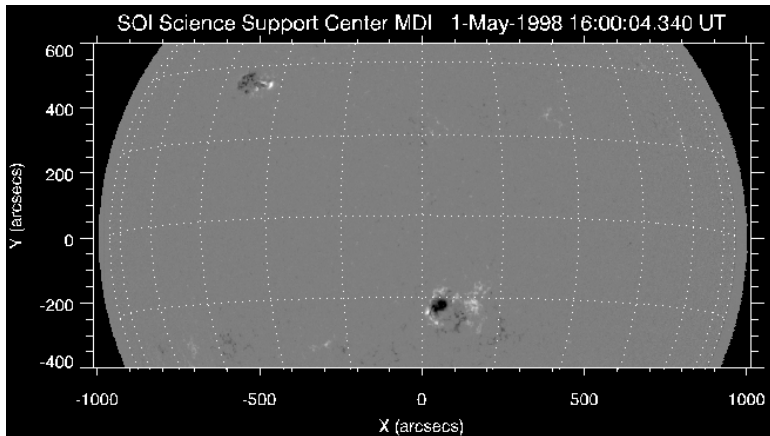
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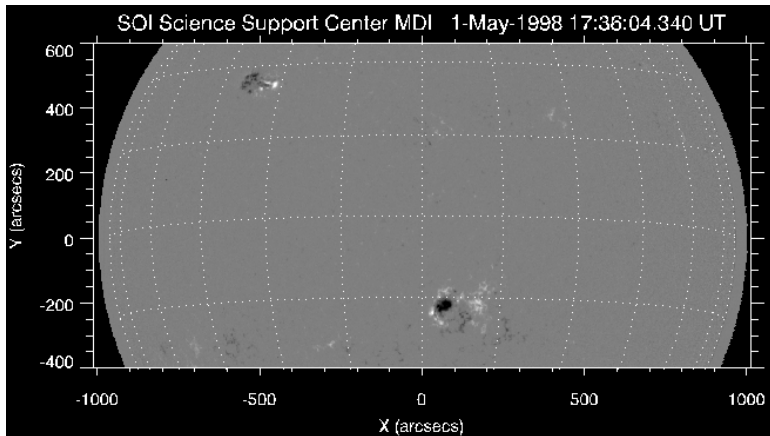
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Application to MDI data AR8210



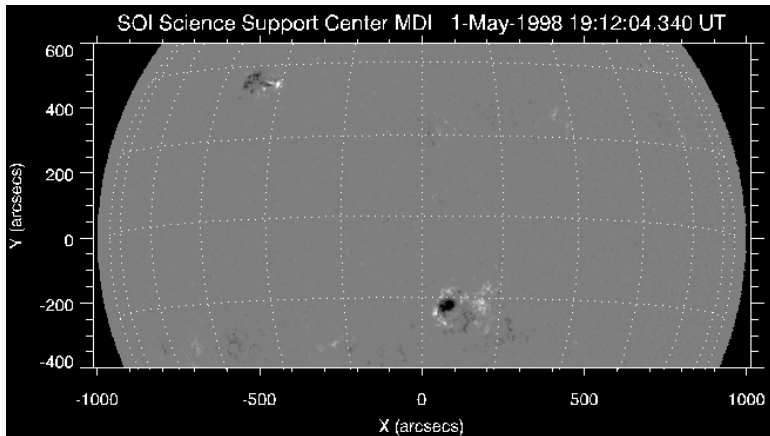
How Can We Estimate Photospheric Flows?

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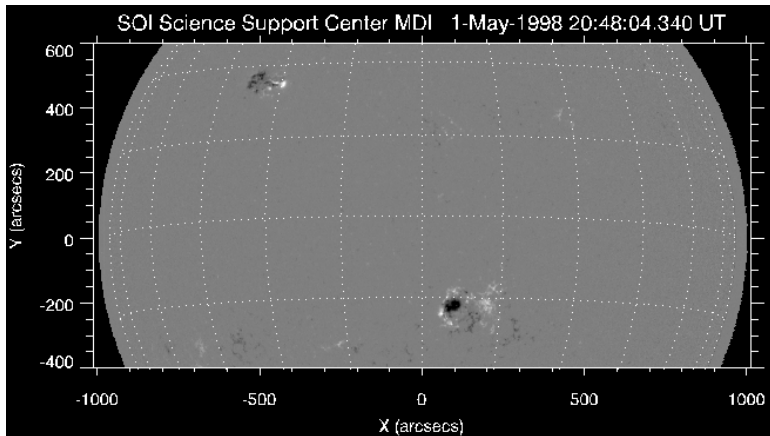
How Can We Estimate Photospheric Flows?

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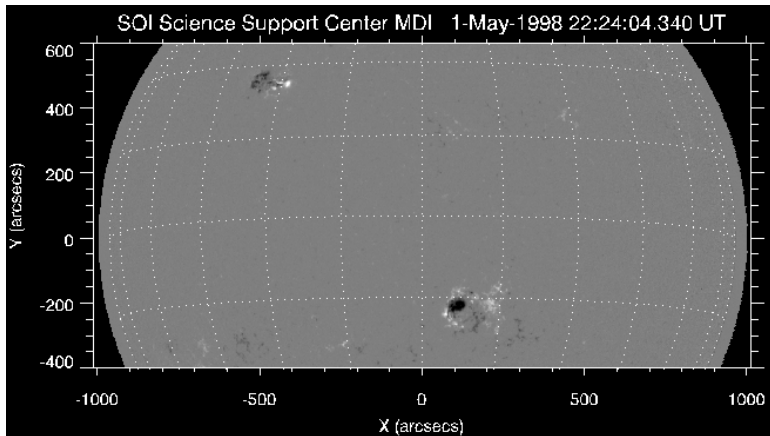
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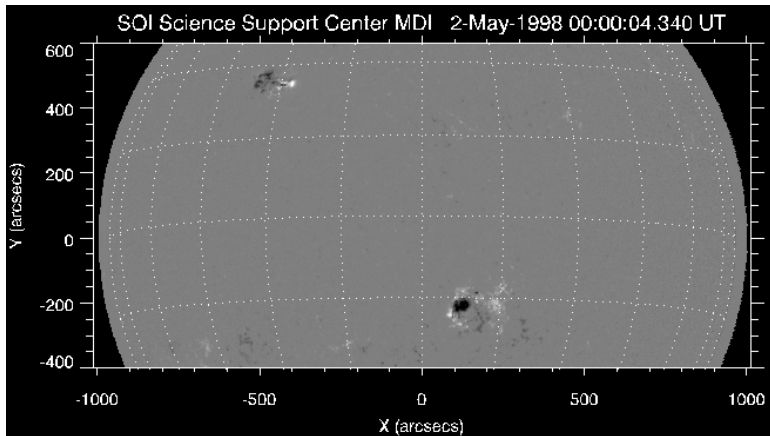
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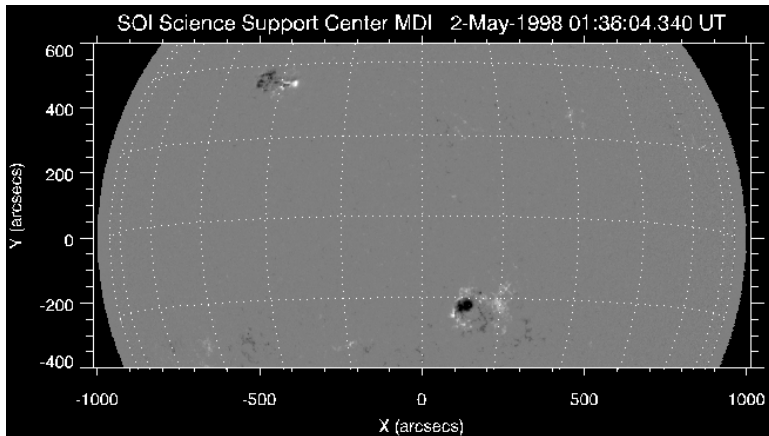
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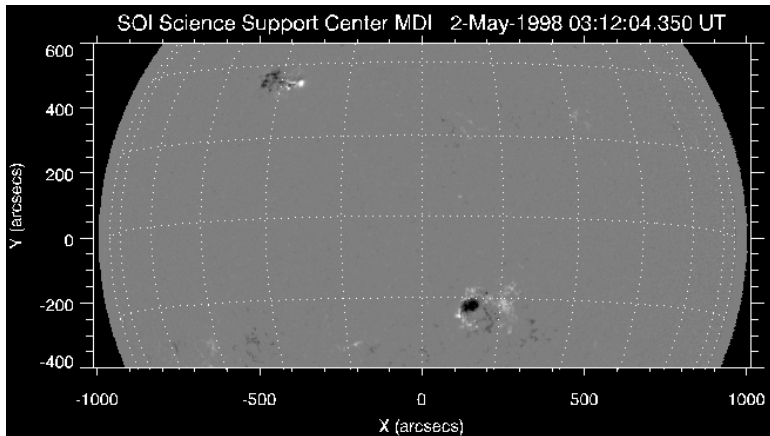
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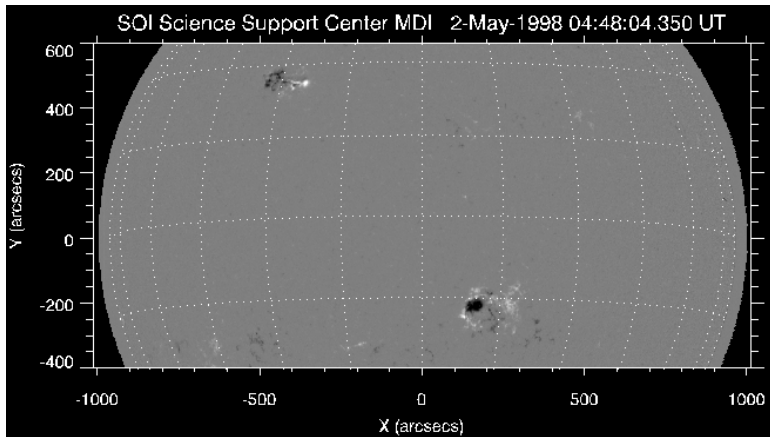
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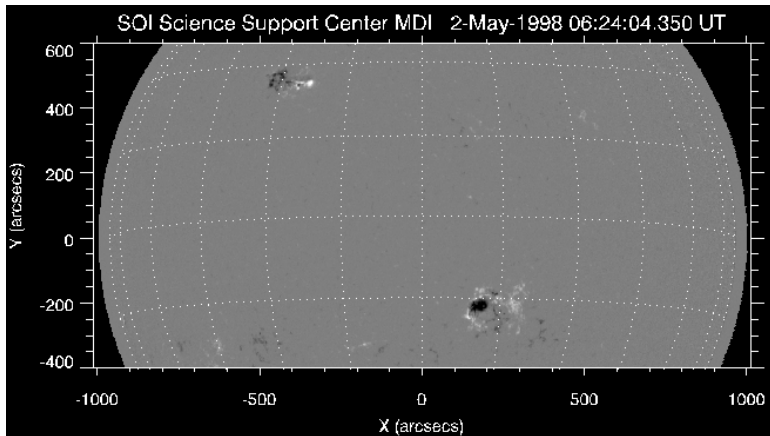
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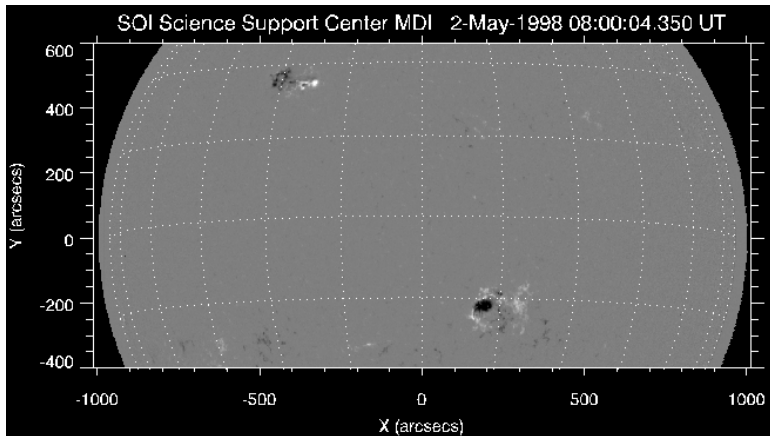
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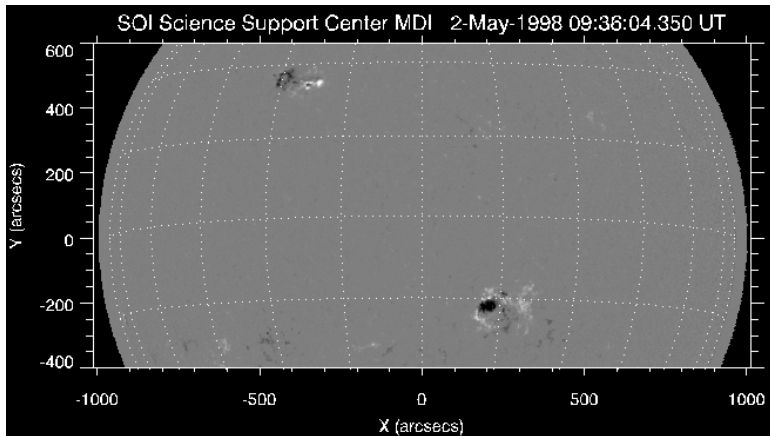
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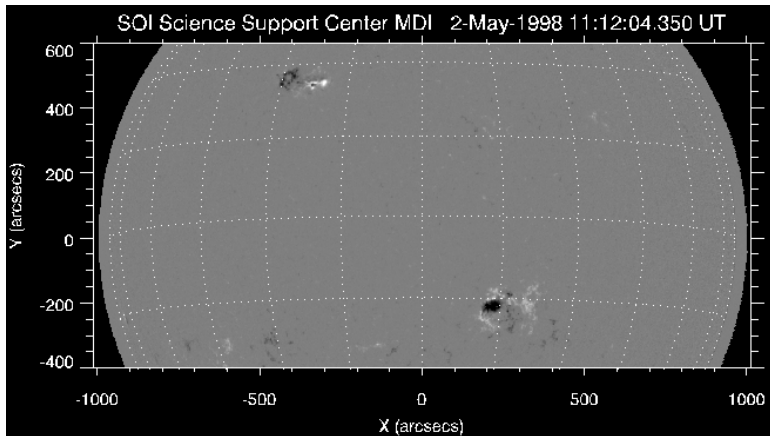
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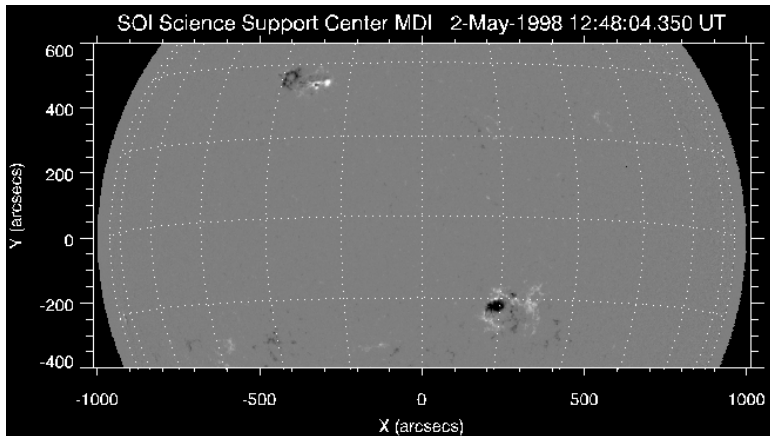
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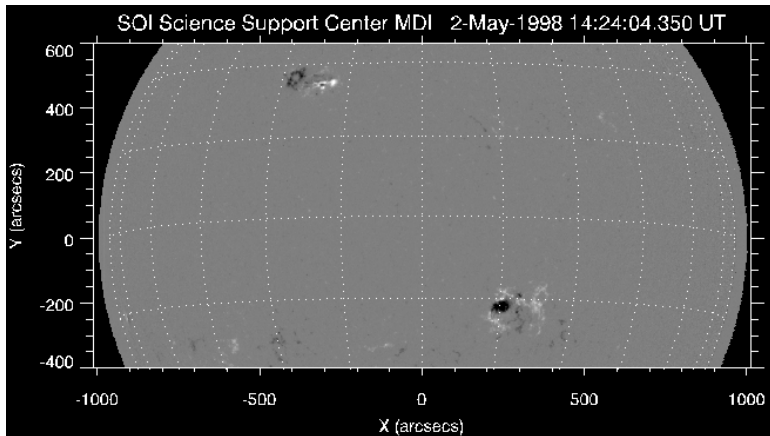
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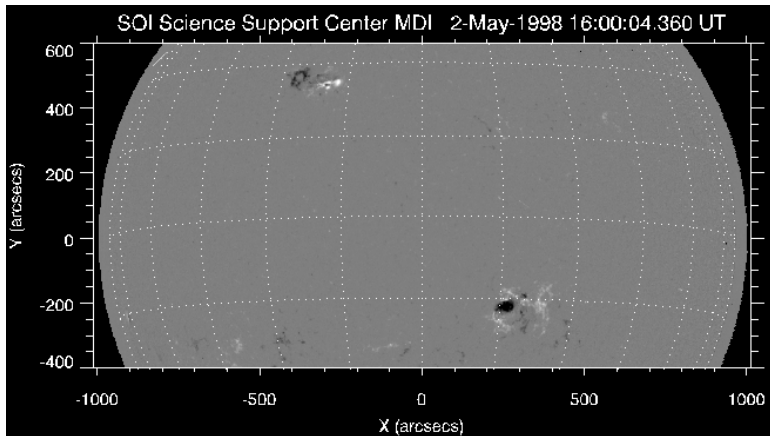
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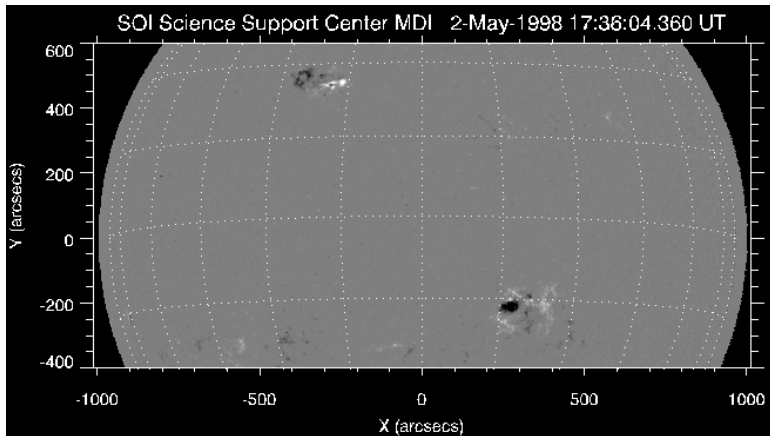
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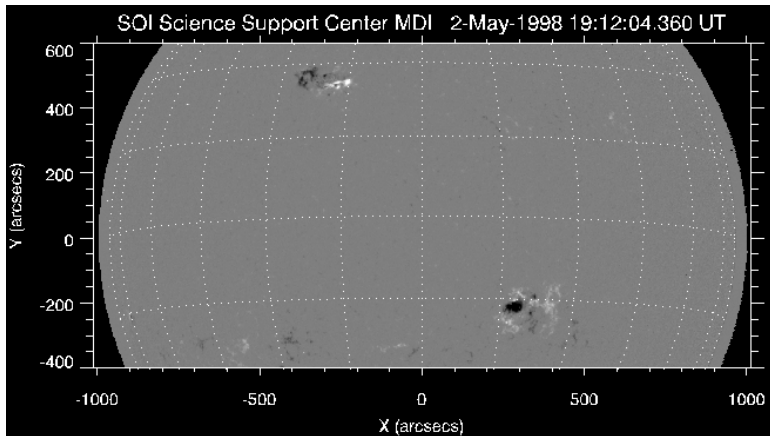
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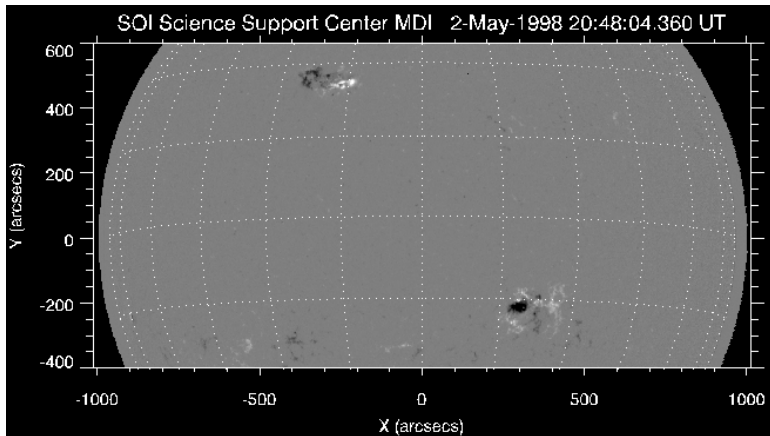
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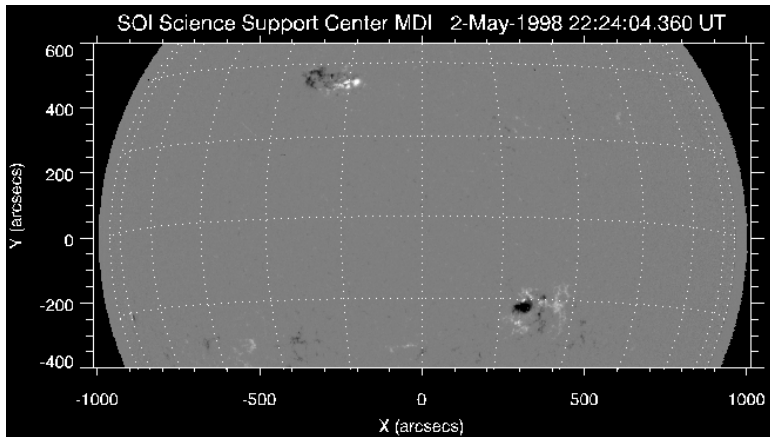
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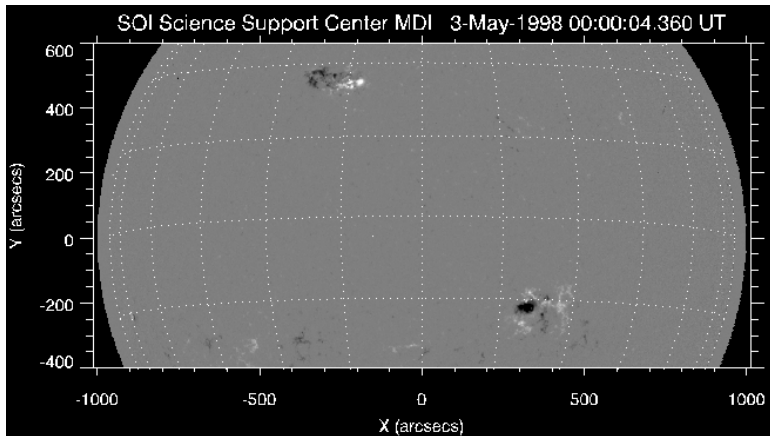
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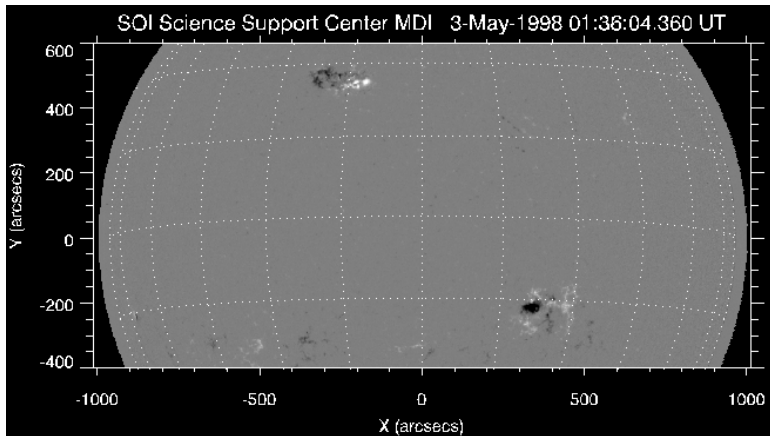
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Application to MDI data AR8210



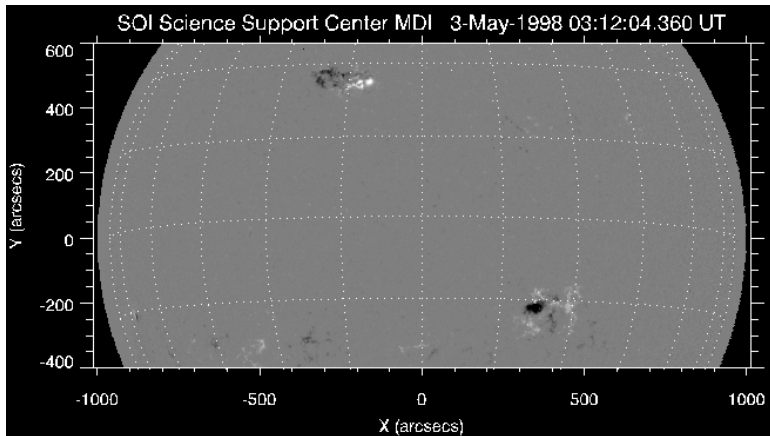
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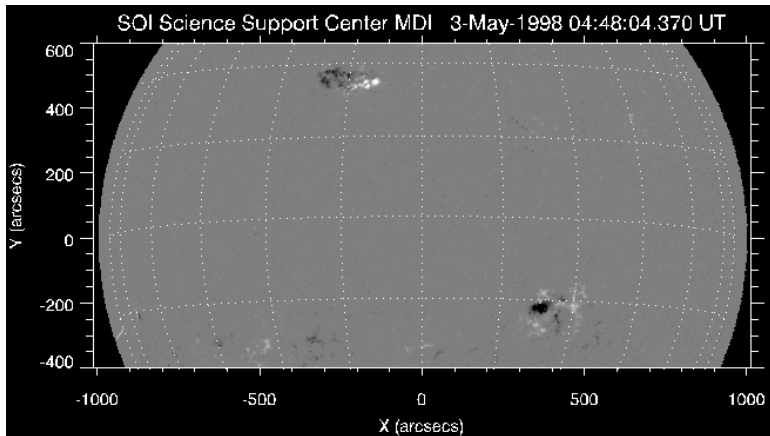
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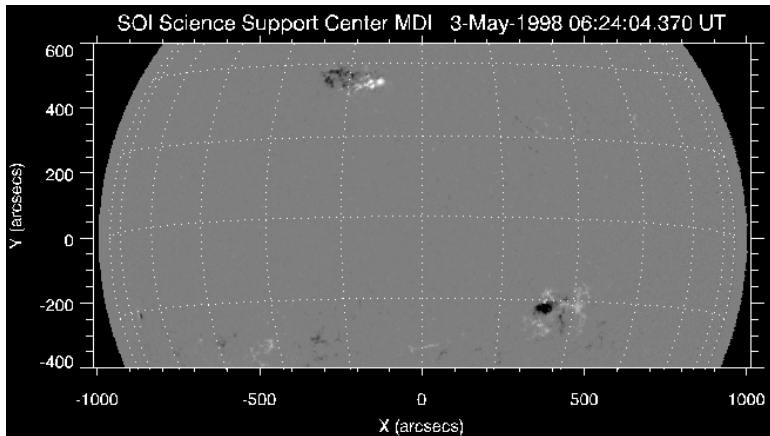
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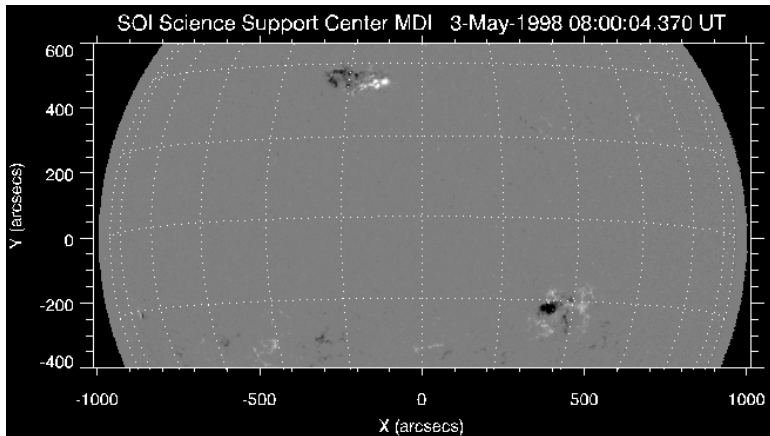
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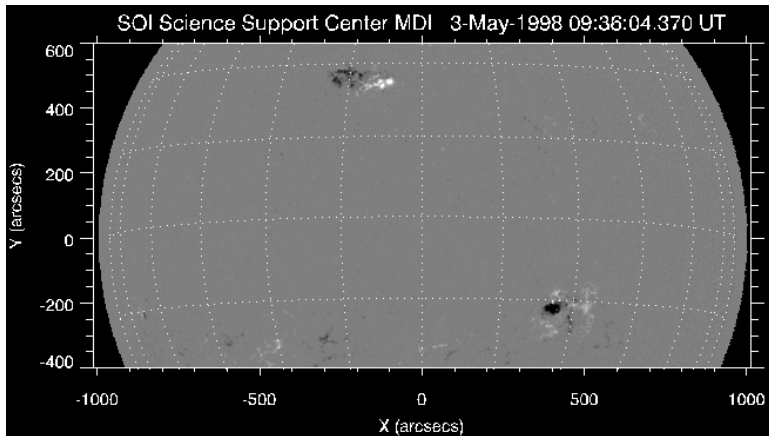
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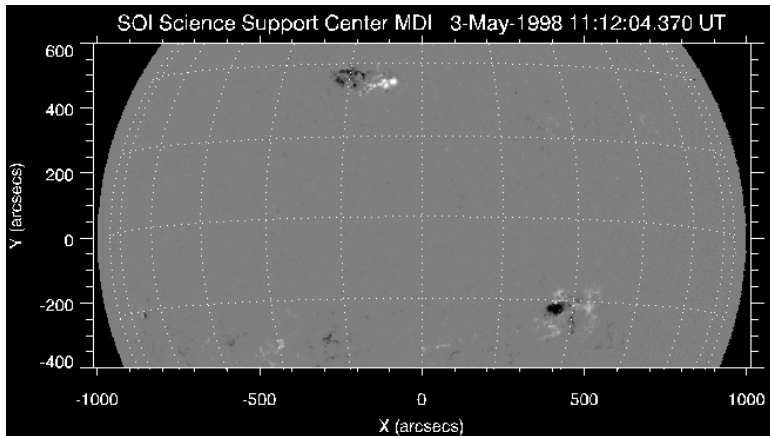
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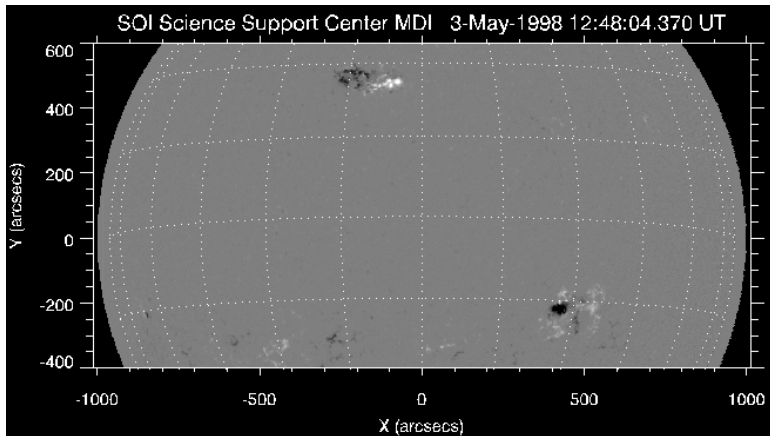
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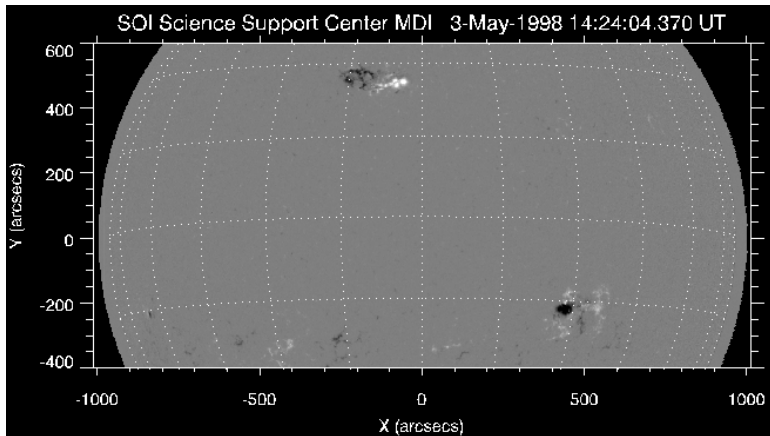
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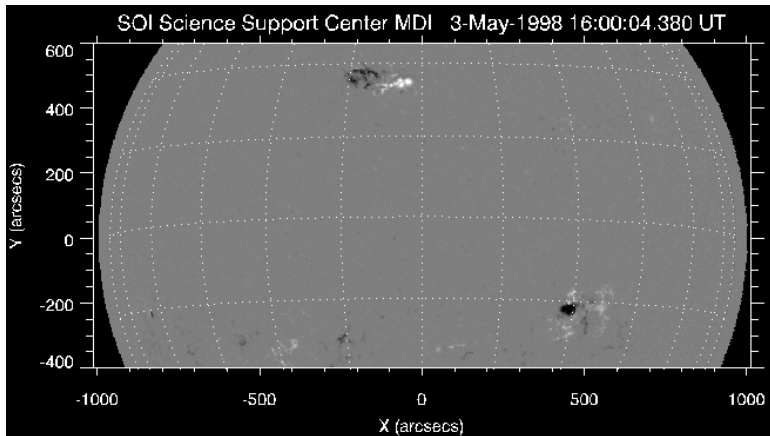
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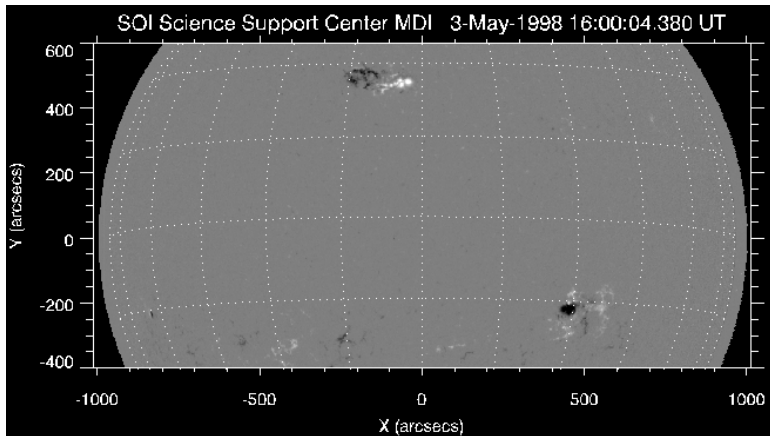
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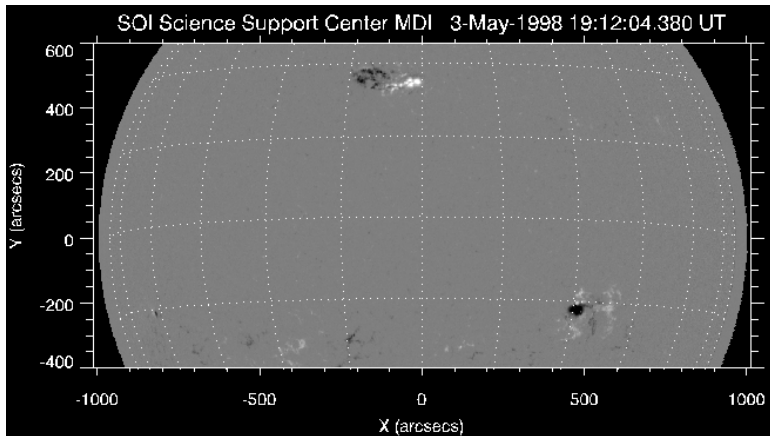
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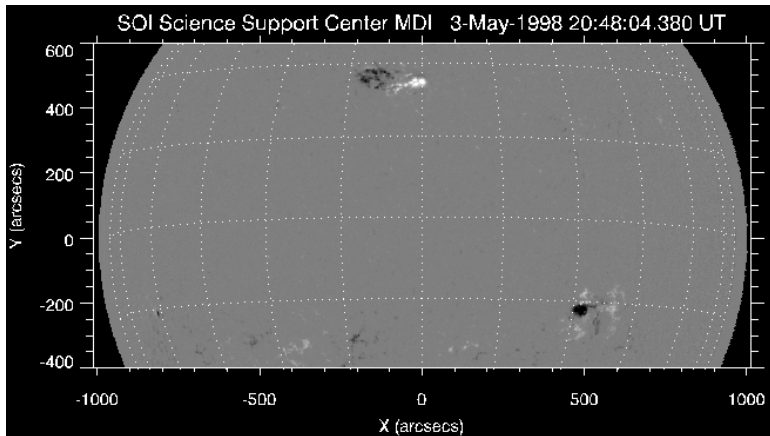
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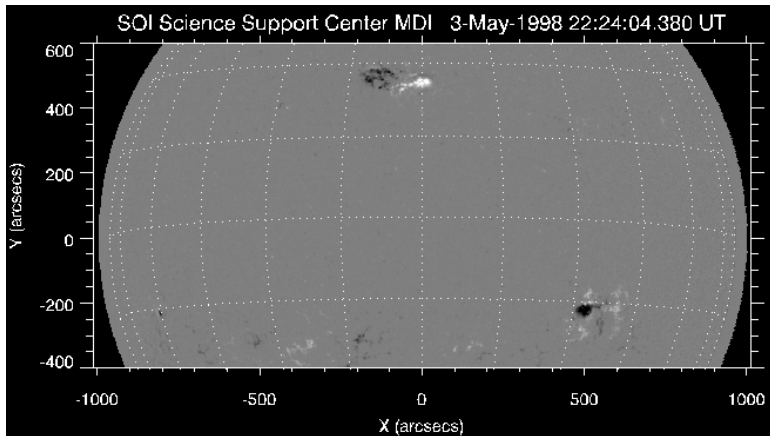
How Can We Estimate Photospheric Flows?

Application to MDI data AR8210



How Can We Estimate Photospheric Flows?

Application to MDI data AR8210



How Can We Estimate Photospheric Flows?

Differential Affine Velocity Estimator for Vector Magnetograms (DAVE4VM)

$$C \approx \int dx^2 w(\mathbf{x} - \mathbf{X}_0) \left\{ \partial_t B_z(\mathbf{x}, t) + \nabla_h \cdot [B_z(\mathbf{x}, t) \hat{\mathbf{v}}_h - \hat{v}_z \mathbf{B}_h(\mathbf{x}, t)] \right\}^2$$

$$\hat{\mathbf{v}} = \begin{pmatrix} u_0 \\ v_0 \\ w_0 \end{pmatrix} + \begin{pmatrix} \hat{u}_x & \hat{u}_y \\ \hat{v}_x & \hat{v}_y \\ \hat{w}_x & \hat{w}_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad \nabla_h = (\partial_x, \partial_y)$$

- Incorporates both vertical and horizontal magnetic field components
- 3D photospheric plasma velocities: explicit vertical flows
- Variational principle results in a least squares/total least squares estimator
 - Can incorporate magnetic field covariance matrices (uncertainties)

How Can We Estimate Photospheric Flows?

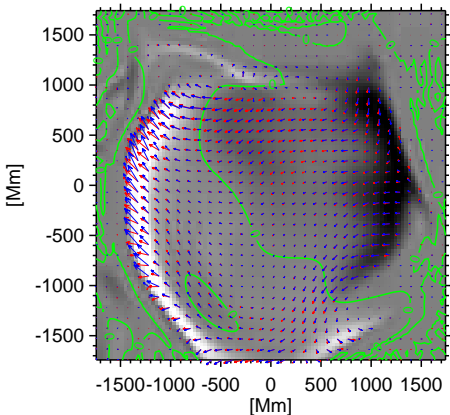
Validating DAVE4VM with an ANMHD Simulation

- Analactic Magnetohydrodynamics (ANMHD) (Fan et al., 1999; Abbett et al., 2000)
CCMC <http://ccmc.gsfc.nasa.gov/models/modelinfo.php?model=ANMHD>
- Simulation of a twisted flux rope rising through the turbulent convection zone
- Magnetic Reynolds Number: $Re_M \equiv 3500$ much more resistive than the photosphere $R_M \sim 10^5 - 10^6$
- Provides ground truth plasma velocities to compare with DAVE4VM and DAVE

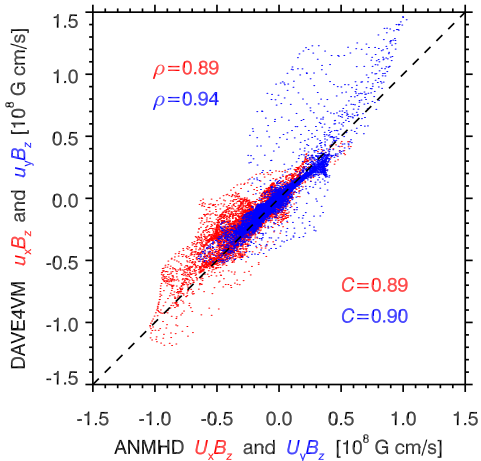
How Can We Estimate Photospheric Flows?

Validating DAVE4VM with an ANMHD Simulation

DAVE4VM $u B_z$ & **ANMHD** $U B_z$

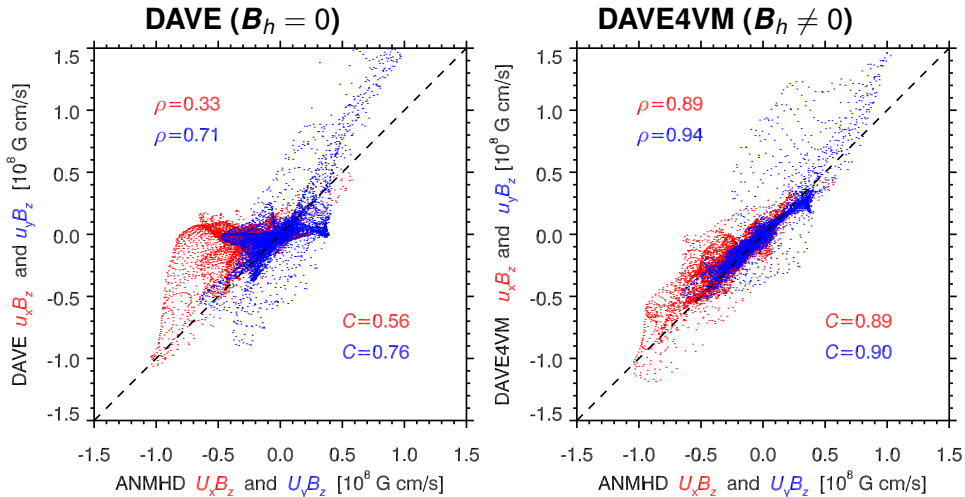


DAVE4VM ($B_h \neq 0$)



How Can We Estimate Photospheric Flows?

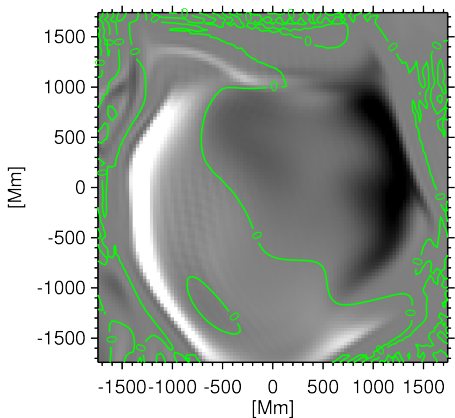
Validating DAVE4VM with an ANMHD Simulation



How Can We Estimate Photospheric Flows?

Validating DAVE4VM with an ANMHD Simulation

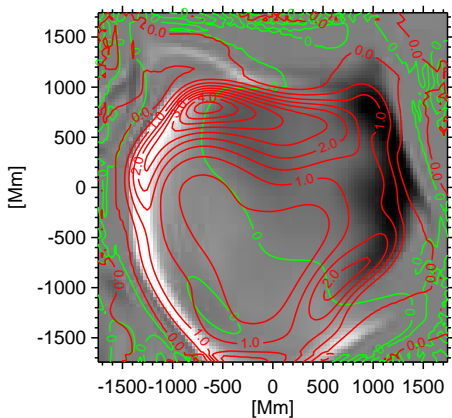
Neutral Line



How Can We Estimate Photospheric Flows?

Validating DAVE4VM with an ANMHD Simulation

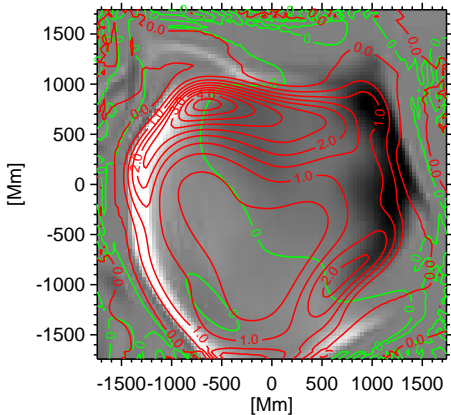
Neutral Line & Poynting Flux



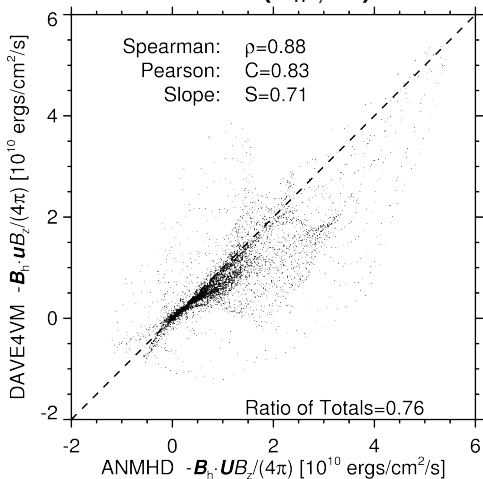
How Can We Estimate Photospheric Flows?

Validating DAVE4VM with an ANMHD Simulation

Neutral Line & Poynting Flux

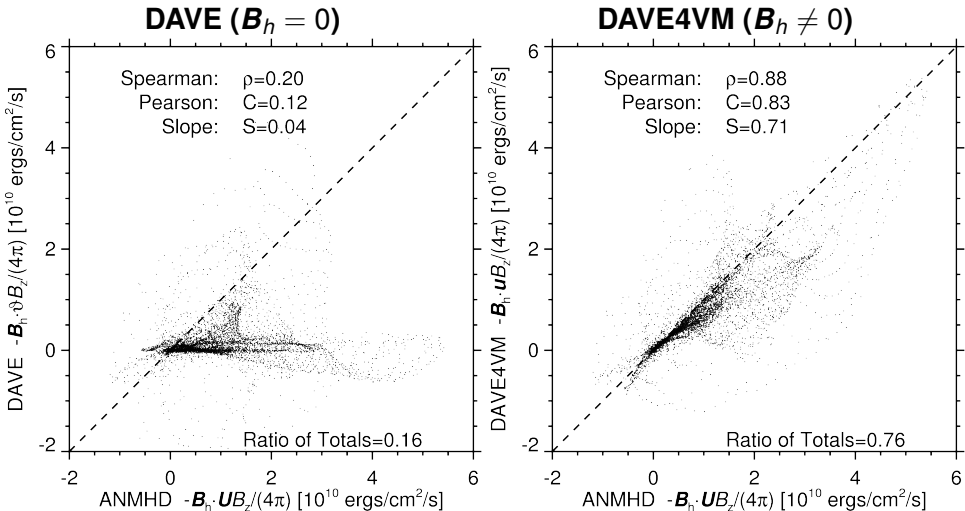


DAVE4VM ($B_h \neq 0$)



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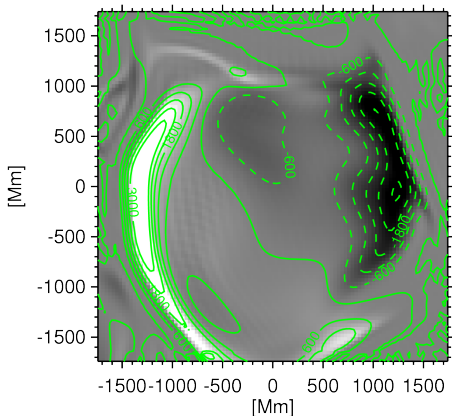
Validating DAVE4VM with an ANMHD Simulation



How Can We Estimate Photospheric Flows?

Validating DAVE4VM with an ANMHD Simulation

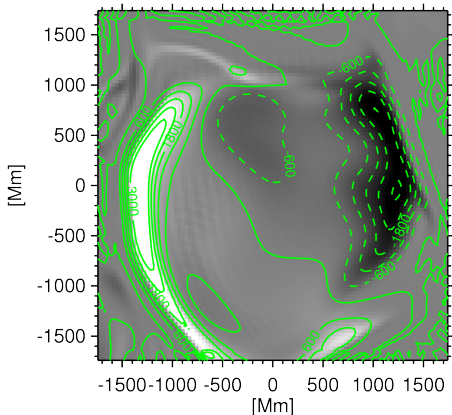
Flows \parallel & \perp to Neutral Lines



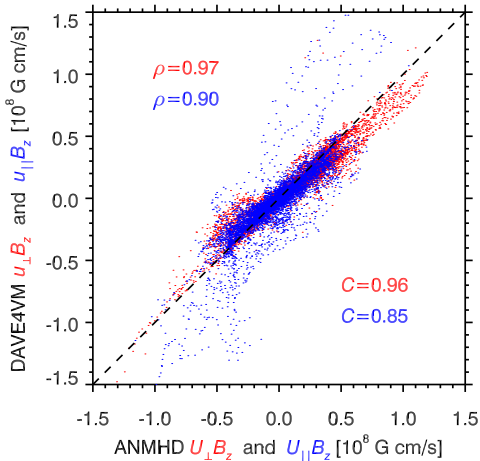
How Can We Estimate Photospheric Flows?

Validating DAVE4VM with an ANMHD Simulation

Flows \parallel & \perp to Neutral Lines

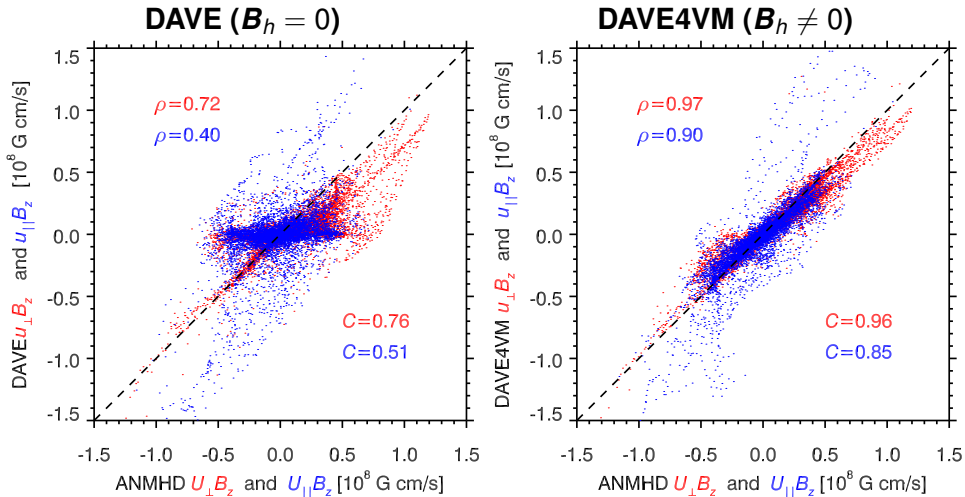


DAVE4VM ($B_h \neq 0$)



How Can We Estimate Photospheric Flows?

Validating DAVE4VM with an ANMHD Simulation

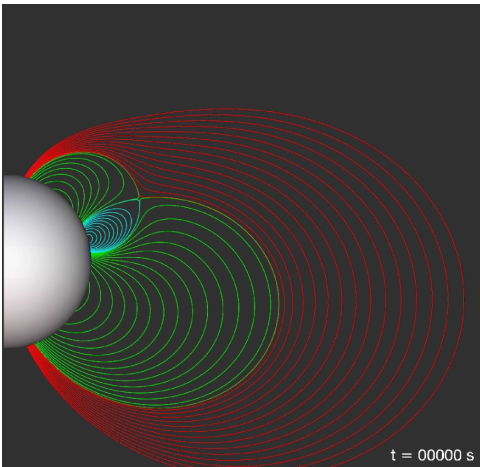


Challenges!

- 1 Must be able to produce an eruption in 3D spherical geometry
- 2 Must be able to reproduce to properties of a CME
- 3 Must be able to model the photosphere to the corona
- 4 Must be able to assimilate data

Data Driven Modeling of CMEs

ARMS can reproduce eruptions 3D spherical geometry

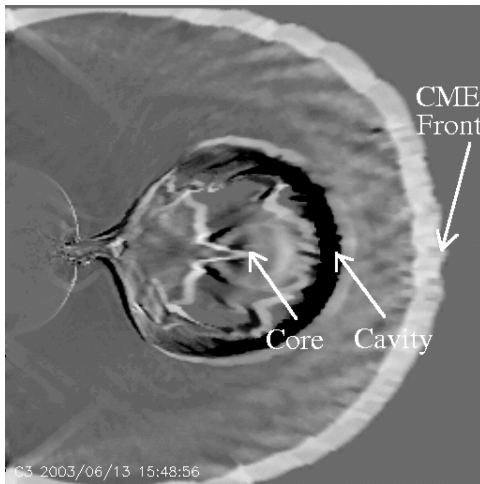
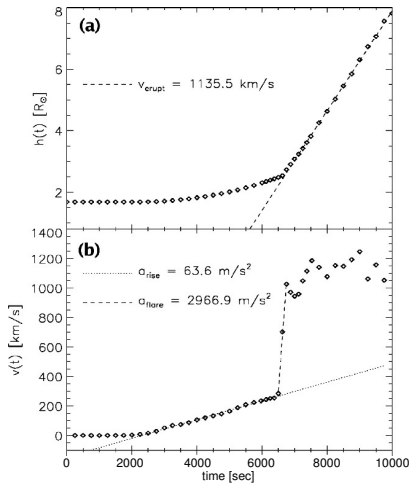


Lynch et al. (2008)

- ARMS - Adaptively Refined Magnetohydrodynamic Solver
- Flux corrected transport
- Highly Parallelized

Data Driven Modeling of CMEs

ARMS can reproduce a fast CME with a 3 part structure

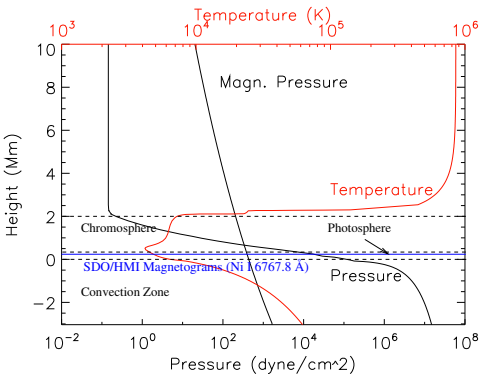


(Lynch et al., 2004)

Data Driven Modeling of CMEs

ARMS can model the photosphere to the corona

VAL-C Model of the Solar Atmosphere



(Vernazza et al., 1981)

- Plasma Pressure drops by 10^8
- Temperature increases by 10^3
- Magnetic Pressure drops by 15

- Plasma β indicates dominate forces

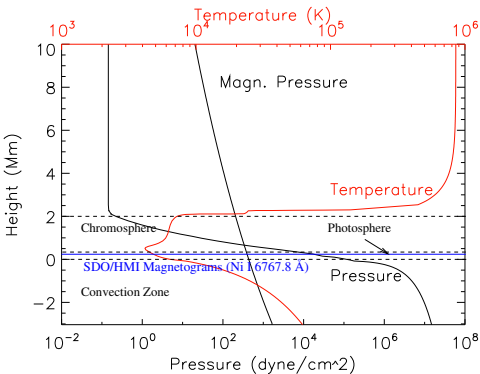
$$\beta \equiv \frac{4\pi P}{B^2}$$

- $\beta \gg 1$ plasma pressure dominant
- $\beta \ll 1$ magnetic forces dominant

Data Driven Modeling of CMEs

ARMS can model the photosphere to the corona

VAL-C Model of the Solar Atmosphere



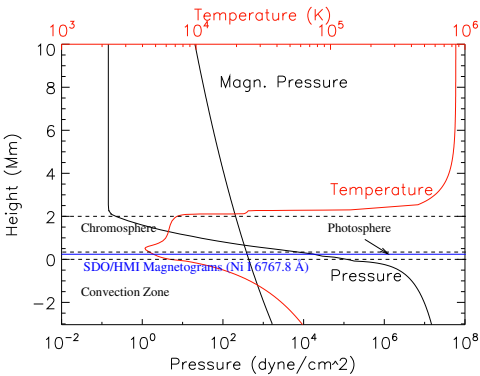
(Vernazza et al., 1981)

- Plasma Pressure drops by 10⁸
 - Temperature increases by 10³
 - Magnetic Pressure drops by 15
 - Plasma β indicates dominant forces
- $$\beta \equiv \frac{4 \pi P}{B^2}$$
- $\beta \gg 1$ plasma pressure dominant
 - $\beta \ll 1$ magnetic forces dominant

Data Driven Modeling of CMEs

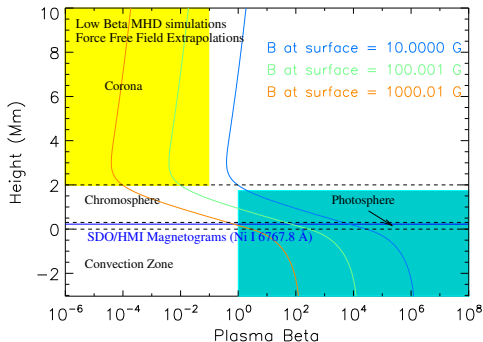
ARMS can model the photosphere to the corona

VAL-C Model of the Solar Atmosphere



(Vernazza et al., 1981)

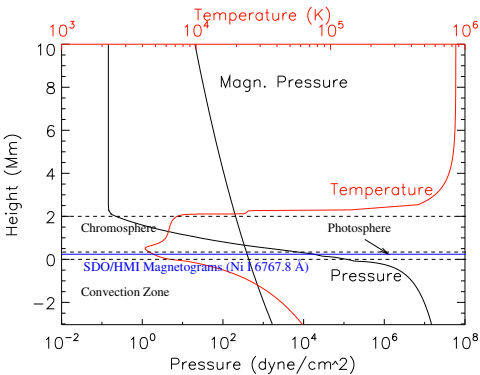
Plasma β



Data Driven Modeling of CMEs

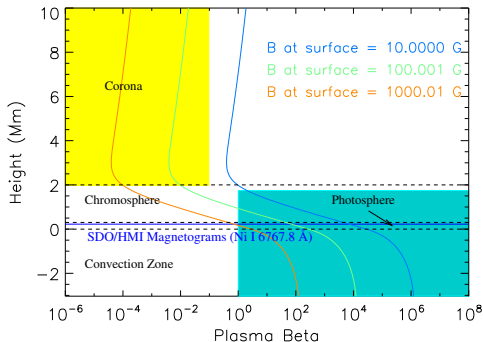
ARMS can model the photosphere to the corona

VAL-C Model of the Solar Atmosphere



(Vernazza et al., 1981)

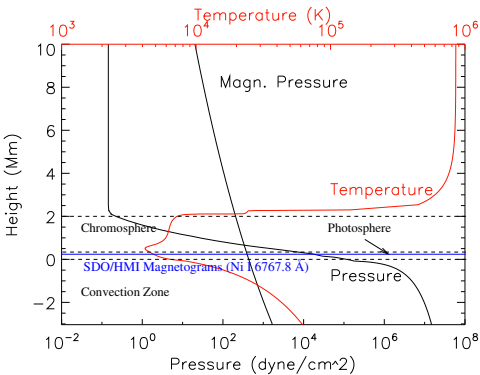
Plasma β



Data Driven Modeling of CMEs

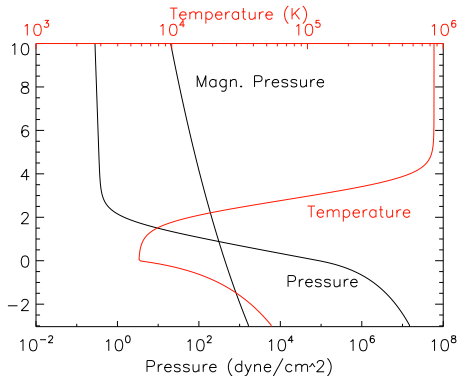
ARMS can model the photosphere to the corona

VAL-C Model of the Solar Atmosphere



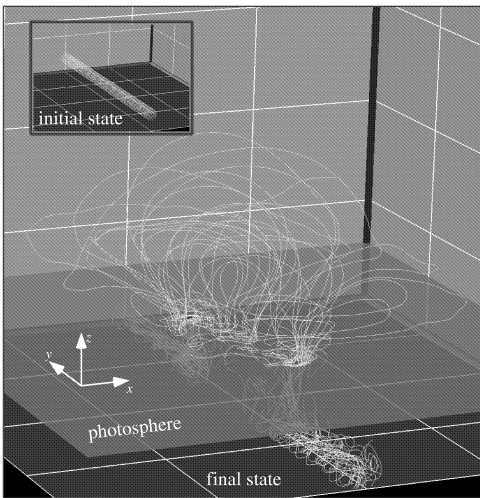
(Vernazza et al., 1981)

ARMS Model of the Solar Atmosphere



Data Driven Modeling of CMEs

ARMS simulation of flux emergence in a stratified atmosphere



.... And here we have a simulation using the ARMS model of the solar atmosphere!

- Magnetic flux rope emerging from the low β convection zone into the high β corona
- During emergence the flux rope expands and shear motions occur in the photosphere

(Magara et al., 2005)

Data Driven Modeling of CMEs

Assimilating data into ARMS: Challenges!

- Putting in all together: The pieces all work (3D eruption, CME properties, highly stratified solar atmosphere, DAVE4VM)
- Initializing the 3D simulation domain consistent with the vector magnetograms at the lower boundary (photosphere)
- Evolving the boundary consistent with the ideal magnetic induction equation and/or the numerical algorithms
- Merging observed magnetic fields and flows with quiet Sun and farside Sun

Despite the challenges, significant progress has been made!

- CME initiation theories have matured to the point that the first tests of these theories may be made with photospheric data.
- Optical flow techniques have matured to incorporate MHD.
 - DAVE4VM accurately estimates plasma velocities and Poynting flux from ANMHD synthetic vector magnetograms
- CME simulations have matured:
 - Produce fast CMEs with realistic properties.
 - Model the photosphere $\beta > 1$ to the corona $\beta \ll 1$.

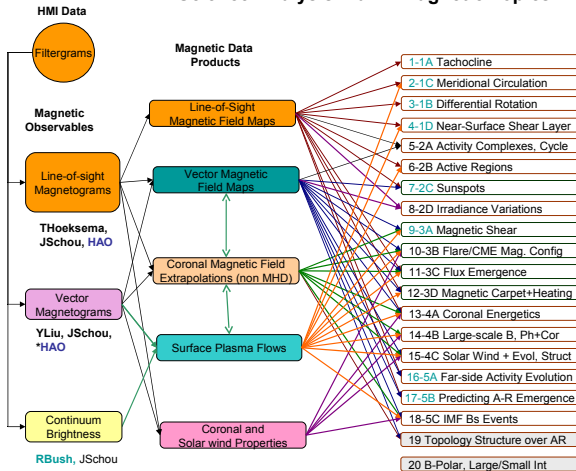
Poised for progress LWS forecasting goals:

The next step beyond phenomenological prediction of eruptions will require assimilation of SDO/HMI or BBSO observations into the latest MHD simulations!

Summary

DAVE4VM Will Provide Critical Science Products for SDO/HMI

HMI Science Analysis Plan – Magnetic Topics



Courtesy of the HMI Team

Summary

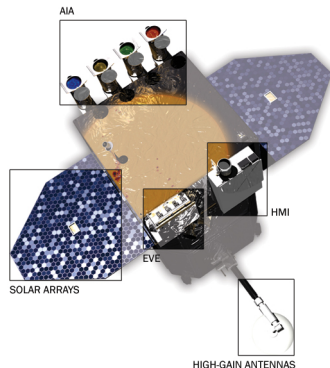
DAVE-DAVE4VM Software Status

- IDL Codes: DAVE/DAVE4VM released with ancillary routines and turnkey code to produce examples/figures from article (Schuck, *ApJ*, **683**, 1134-1152, 2008)
 - Oldest: Archived with ApJ
www.iop.org/EJ/abstract/0004-637X/683/2/1134
 - More recent archived at:
 - NRL wwwppd.nrl.navy.mil/whatsnew/dave/index.html
 - CCMC <http://ccmc.gsfc.nasa.gov/lwsrepository>
 - Latest: contact me at NASA/GSFC – peter.schuck@nasa.gov
- HMI Pipeline Codes: in production (Jacob Hageman GSFC/582), Intel Fortran with C-wrappers, and linked with Intel MKL math libraries with drop-in open source replacements (deliver final versions mid-February).

Summary

Planned Improvements in DAVE4VM

- Incorporate Doppler velocities to constrain vertical flows
- Incorporate HMI covariance matrices
- More verification tests on other MHD codes
- Consider spherical geometry
- *SDO/HMI* first light!



Summary

Products Produced by the Program

Six Publications:

P.W. Schuck, Tracking Vector Magnetograms with the Magnetic Induction Equation, *ApJ*, **683**, 1134-1152, 2008 doi: 10.1086/589434

B. T. Welsch, *et al.*, What is the Relationship Between Photospheric Flow Fields and Solar Flares?, *ApJ*, **705**, 821-843, 2009, doi: 10.1088/0004-637X/705/1/821

B. T. Welsch *et al.*, Tests and Comparisons of Velocity-Inversion Techniques, *ApJ*, **670**, 1434-1452, 2007, doi: 10.1086/522422

P. W. Schuck, Tracking Magnetic Footpoints with the Magnetic Induction Equation, *ApJ*, **646**, 1358-1391, 2006, doi: 10.1086/505015

P. W. Schuck, Local Correlation Tracking and the Magnetic Induction Equation, *ApJ*, **632**, L53-L56, 2005, doi: 10.1086/497633

Foreign and Domestic Patents

Filed: January 25, 2010. (United States Provisional Patent Application No. 61/146,808 filed on January 23, 2009)

References I

- Abbett, W. P., Fisher, G. H., & Fan, Y. 2000, *ApJ*, 540, 548
- Amari, T., Luciani, J. F., Aly, J. J., Mikić, Z., & Linker, J. 2003, *ApJ*, 585, 1073
- Antiochos, S. K. 1998, *ApJ*, 502, L181+
- Antiochos, S. K., DeVore, C. R., & Klimchuk, J. A. 1999, *ApJ*, 510, 485
- Borrero, J. M., Tomczyk, S., Norton, A., Darnell, T., Schou, J., Scherrer, P., Bush, R., & Liu, Y. 2007, *Sol. Phys.*, 240, 177
- Démoulin, P., & Berger, M. A. 2003, *Sol. Phys.*, 215, 203
- DeVore, C. R., & Antiochos, S. K. 2008, *ApJ*, 680, 740
- Fan, Y., Zweibel, E. G., Linton, M. G., & Fisher, G. H. 1999, *ApJ*, 521, 460
- Foullon, C., Crosby, N., & Heynderickx, D. 2005, *Space Weather*, 3, 7004

References II

- Horne, R. B. 2003, in Space Weather Workshop: Looking Towards a European Space Weather Programme, European Space Agency, ESTEC (Nordwijk, The Netherlands: European Space Agency), 139–144
- Howard, R. F., Harvey, J. W., & Forgach, S. 1990, *Sol. Phys.*, 130, 295
- Keller, C. U., Harvey, J. W., & Henney, C. J. 2008, in *Astronomical Society of the Pacific Conference Series*, Vol. 384, 14th Cambridge Workshop on Cool Stars, Stellar Systems, and the Sun, ed. G. van Belle, 166–+
- Kusano, K., Maeshiro, T., Yokoyama, T., & Sakurai, T. 2002, *ApJ*, 577, 501
- Leka, K. D., & Barnes, G. 2007, *ApJ*, 656, 1173
- Leka, K. D., Barnes, G., Crouch, A. D., Metcalf, T. R., Gary, G. A., Jing, J., & Liu, Y. 2009, *Sol. Phys.*, 260, 83

References III

- Lynch, B. J., Antiochos, S. K., DeVore, C. R., Luhmann, J. G., & Zurbuchen, T. H. 2008, *ApJ*, 683, 1192
- Lynch, B. J., Antiochos, S. K., MacNeice, P. J., Zurbuchen, T. H., & Fisk, L. A. 2004, *ApJ*, 617, 589
- Magara, T., Antiochos, S. K., DeVore, C. R., & Linton, M. G. 2005, in *ESA Special Publication, Vol. 596, Chromospheric and Coronal Magnetic Fields*, ed. D. E. Innes, A. Lagg, & S. A. Solanki
- Manchester, IV, W., Gombosi, T., DeZeeuw, D., & Fan, Y. 2004, *ApJ*, 610, 588
- Manchester, W. I. 2001, *ApJ*, 547, 503
- Metcalf, T. R., Leka, K. D., Barnes, G., Lites, B. W., Georgoulis, M. K., Pevtsov, A. A., Balasubramaniam, K. S., Gary, G. A., Jing, J., Li, J., Liu, Y., Wang, H. N., Abramenko, V., Yurchyshyn, V., & Moon, Y. 2006, *Sol. Phys.*, 237, 267

References IV

- Norton, A. A., Graham, J. P., Ulrich, R. K., Schou, J., Tomczyk, S., Liu, Y., Lites, B. W., López Ariste, A., Bush, R. I., Socas-Navarro, H., & Scherrer, P. H. 2006, *Sol. Phys.*, 239, 69
- Schuck, P. W. 2005, *ApJ*, 632, 53
- . 2006, *ApJ*, 646, 1358
- . 2008, *ApJ*, 683, 1134, <http://arxiv.org/abs/0803.3472>
- Vernazza, J. E., Avrett, E. H., & Loeser, R. 1981, *ApJS*, 45, 635
- Welsch, B. T., Li, Y., Schuck, P. W., & Fisher, G. H. 2009, *ApJ*, 705, 821