

THE SUBSURFACE RADIAL GRADIENT OF SOLAR ANGULAR VELOCITY FROM MDI f -MODE OBSERVATIONS

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(Received 6 September 2001; accepted 12 October 2001)

Abstract. We report quantitative analysis of the radial gradient of solar angular velocity at depths down to about 15 Mm below the solar surface for latitudes up to 75° using the Michelson Doppler Imager (MDI) observations of surface gravity waves (f modes) from the Solar and Heliospheric Observatory (SOHO). A negative outward gradient of around -400 nHz/ R_\odot , equivalent to a logarithmic gradient of the rotation frequency with respect to radius which is very close to -1 , is found to be remarkably constant between the equator and 30° latitude. Above 30° it decreases in absolute magnitude to a very small value at around 50° . At higher latitudes the gradient may reverse its sign: if so, this reversal takes place in a thin layer extending only 5 Mm beneath the visible surface, as evidenced by the most superficial modes (with degrees $l > 250$). The signature of the torsional oscillations is seen in this layer, but no other significant temporal variations of the gradient and value of the rotation rate there are found.

1. Introduction

The velocity field of the rotational flow in the Sun's near-surface layers may play a significant role in small-scale dynamo action in that region and in the dynamics of supergranular convection. Surface observations over decades and even centuries have shown that the latitudinal variation of the surface rotation is rather smooth, being rather well described by a three-term (i.e., second-order) polynomial in μ^2 where $\mu = \cos \theta$ and θ is the colatitude. Recent analyses of high-resolution data, in particular those utilizing solar f -mode observations by the Michelson Doppler Imager (MDI) on board the Solar and Heliospheric Observatory (SOHO), have highlighted important departures from such a description of the rotation of the near-surface layers. The polar subsurface layers (i.e., $\theta < 20^\circ$ and depths down to 28 Mm below the surface) have been shown to be approximately 10 nHz slower than expected from a simple three-term extrapolation from lower latitudes (Birch and Kosovichev, 1998; Schou *et al.*, 1998; Schou, 1999) and Kosovichev and Schou (1997) have shown that, at a depth of 2 to 9 Mm beneath the surface, there exist zonal bands of alternate faster and slower rotation rate of $\sim \pm 5$ m s $^{-1}$ superimposed on the general trend described by the second order polynomial. This latter feature, inferred from the first observations of MDI in 1996, was found to be similar to the surface 'torsional oscillations' (Howard and Labonte, 1980) and



also observed in 1995 in Doppler measurements using the first GONG observations (Hathaway *et al.*, 1996.) More recently still, analysis of both p and f modes from the GONG network and MDI instrument have further led to the conclusion that these banded structures extend at least down to 60 Mm below the surface (Howe *et al.*, 2000).

The observed f modes, being confined to the outer layers of the Sun, provide a relatively clean and straightforward measure of conditions there. But those results above that were obtained just from the f modes assumed at least implicitly that the angular velocity is not varying significantly with depth within the layer sensed by those modes. It is, however, well known that another important property of the subsurface layers is that they present a radial gradient of angular velocity. This was first suggested by the fact that different indicators such as Doppler shifts of photospheric Fraunhofer lines, various magnetic field features of different ages and sizes (sunspots, faculae, network elements, H α filaments) or the supergranular network, present different rotation rates (see the review of Howard, 1984; Schroeter, 1985; Snodgrass, 1992). This has been interpreted by assuming that the different magnetic features are anchored at different depths (e.g., Foukal, 1972; Collin *et al.* 1995), their different rotation rate being therefore interpreted as an indication of the existence of radial gradients of angular velocity in the subsurface layers. More specifically, noticing that the supergranular network rotation rate (~ 473 nHz) was found to be $\sim 4\%$ faster than the upper photospheric plasma rate obtained from spectroscopic methods and also $\sim 2\%$ faster than various magnetic indicators thought to be rooted under the supergranulation layer, Snodgrass and Ulrich (1990) inferred that a maximum of angular velocity should exist somewhere between $0.95 R_{\odot}$ and the surface.

From the theoretical point of view, it has been suggested that the angular momentum per unit mass $\Omega r^2 \sin^2 \theta$ could be conserved in the supergranular flow (Foukal and Jokipii, 1975; Foukal, 1977; Gilmand and Foukal, 1979). From $\partial\Omega/\Omega = -2\partial r/r$, at fixed latitude, this simple argument leads effectively to a negative gradient below the surface, and the 4% difference in rotation rates would be explained if the supergranulation network velocity observed at the surface were reflecting the rotation rate at a depth of $2\% R_{\odot} \simeq 15$ Mm, which turns out to correspond to the depth expected for the supergranular convection (Foukal, 1977; Duvall, 1980) (but see also Beck and Schou, 2000, for a more recent estimate). In order to reproduce the observed patterns of solar activity such as the equatorward migration of sunspots, early dynamo models based on a positive surface α -effect indicated also that the angular velocity must decrease outwards, i.e., $\partial\Omega/\partial r < 0$ (e.g., Leighton, 1969; Roberts and Stix, 1972). One of the first goals of helioseismology was therefore to test the assumptions about the negative gradient of angular velocity below the surface suspected from different surface observations. This was indeed first attempted by Deubner, Ulrich, and Rhodes (1979): although they did not resolve individual modes, they were able, from ridge-fitting separately the eastward- and westward-propagating near-equatorial waves in the

(k, w) diagram, to detect such a negative gradient close to the surface. Subsequent helioseismic work using resolved mode frequencies has shifted much theoretical focus to the base of the convection zone by showing that the radial gradient of angular velocity in the bulk of the convection zone is weak and that a strong radial shear, the so-called tachocline, occurs at its base. The gradient $\partial\Omega/\partial r$ is positive in the tachocline at sunspot (i.e., low) latitudes (Brown *et al.*, 1989). This has led various dynamo theories to locate the dynamo action below the convection zone, with a negative α -effect operating there (e.g., Gilman, Morrow, and Deluca, 1989; Parker, 1993) though some recent work has revisited the idea of a positive surface α -effect but invoking the action of a meridional circulation, equatorward below the convection zone and poleward at the surface, to produce the observed equatorward migration of sunspots by advective transport of flux (Dikpati and Charbonneau, 1999; Küker, Rüdiger, and Schultz, 2001). The lack until recently of precise determinations of high-degree mode parameters made it difficult to obtain very localized inferences about the subsurface layers. But, because all the observed modes have large amplitude close to the surface, inverters again got hints about the existence of a radial shear close to the surface (especially using methods such as regularized least-squares which readily extrapolate into regions where the data provide no localized information) though without being able to quantify precisely its extent and amplitude (e.g., Thompson *et al.*, 1996; Corbard *et al.*, 1997).

We show in this work that f -mode observations allow us to make quantitative inferences about the surface radial shear. These should be taken into account when modeling near-surface dynamo action or the dynamics of the supergranulation layer.

2. Observations

The data used here are 23 independent times series of 72 days obtained from the so-called MDI medium- l program. These cover the period from 1 May 1996 to 4 April 2001 with interruptions during the summer 1998 (23 June to 23 October) and between 4 December 1998 and 4 February 1999 due to SOHO spacecraft problems. More details on the production of these time series from the observations can be found in Schou (1999).

A given f -mode multiplet in the spectra comprises $2l+1$ frequencies ν_{lm} , where l and m are the degree and azimuthal order of the spherical harmonic $Y_l^m(\theta, \phi)$ describing the angular dependence of the modes. The so-called a coefficients for the multiplet are defined by the polynomial expansion:

$$\nu_{lm} = \nu_{l0} + \sum_{j=1}^{2l} a_j^l \mathcal{P}_j^{(l)}(m) \quad m = \pm 1, \pm 2 \dots \pm l, \quad (1)$$

where \mathcal{P} are orthogonal polynomials normalized such that $\mathcal{P}_j^{(l)}(l) = l$ (Schou, Christensen-Dalsgaard, and Thompson, 1994). All f modes considered here have

degrees between $l = 117$ and $l = 300$ but the total number of multiplets observed is between 112 and 143, depending on the 72-day interval considered. For each observed mode, the central frequencies ν_{l0} and the first 36 a coefficients have been estimated using the method described in Schou (1992). Odd-indexed a coefficients, which describe the dependence of the frequencies that is an odd function of m , arise from the north-south symmetric part of the solar rotation. Even-indexed coefficients arise from latitudinal structural variation, centrifugal distortion and magnetic fields.

3. Data Analysis

Following Ritzwoller and Lively (1991), we identify the north-south symmetric part of the angular velocity $\Omega(r, \mu)$ with the odd-degree, zonal part of the toroidal component of a general stationary and laminar velocity field and write

$$\Omega(r, \mu) = \sum_{j=0}^{\infty} \Omega_{2j+1}(r) \bar{T}_{2j}^1(\mu), \quad (2)$$

where r is fractional radius and $\bar{T}_{2j}^1 \equiv T_{2j}^1(\mu)/T_{2j}^1(0)$ are Gegenbauer polynomials (see Appendix) normalized such that the equatorial rate is given by the straight sum of the $\Omega_{2j+1}(r)$.

Assuming slow rotation, we can use a linear perturbation theory to predict the effect of rotation on the oscillation modes (e.g., Hansen, Cox, and Van-Horn, 1977). Moreover, with the polynomials \mathcal{P} and expansion Equation (2) as chosen, there is a one-to-one relation between odd a coefficients and the components $\Omega_{2j+1}(r)$ (Ritzwoller and Lively, 1991), thereby reducing the full 2D problem to a set of 1D integral equations often referred to as the 1.5D problem. In the particular case of the f modes, we obtain

$$2\pi a_{2j+1}^l = u_{2j+1}^l \int_0^1 K_h^l(r) \Omega_{2j+1}(r) dr, \quad (3)$$

where the expression for the kernels $K_h^l(r)$ and u_{2j+1}^l are derived in the Appendix.

The 36 a coefficients extracted from observation do not provide information about the terms above $j = 17$ in the summation in Equation (2) and that corresponds to a limitation in the latitudinal resolution we can reach. Defining

$$b_{2j+1}^l \equiv \frac{2\pi a_{2j+1}^l}{u_{2j+1}^l}, \quad (4)$$

from Equations (2)–(4) we obtain

$$\sum_{j=0}^{17} b_{2j+1}^l \bar{T}_{2j}^1(\mu_0) \approx \int_0^1 K_h^l(r) \bar{\Omega}(r, \mu_0) dr, \quad (5)$$

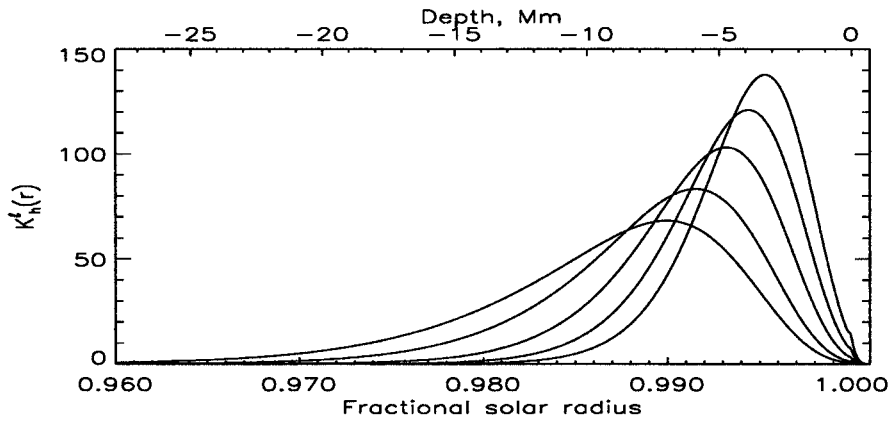


Figure 1. f -modes rotational kernels $K_h^l(r)$ for $l = 117, 150, 200, 250, 300$ from left to right.

where $\bar{\Omega}(r, \mu)$ refers to the part of the rotation profile corresponding to the sum Equation (2) truncated at $j = 17$. We can also show (Pijpers, 1997) that the above linear combination of b coefficients is such that

$$\sum_{j=0}^{17} b_{2j+1}^l \bar{T}_{2j}^1(\mu_0) = \int_0^1 \int_0^1 K_h^l(r) \kappa(\mu_0, \mu) \Omega(r, \mu) dr d\mu, \quad (6)$$

where $\kappa(\mu_0, \mu)$ are the so-called latitudinal averaging kernels which show what latitudinal average of the true rotation rate is made at each latitude. Figure 2(a) shows that these kernels have their main peak centered at μ_0 but present an oscillatory behavior which may lead to systematic errors if some small-scale features (corresponding to terms with $j > 17$) exist in the true rotation rate. In order to avoid this, one may try to find instead the combination of b coefficients that leads to kernels that are optimally localized around a given latitude. This can be achieved following for instance the method of Backus and Gilbert (1968), but we notice here that a similar result can be obtained simply by introducing, in the sum of Equation (6), a correcting factor $e^{-j(j+3/2)/l_0}$ where $l_0 \equiv 117$ corresponds to the lowest degree of the observed f modes (see also Equation (15)). Doing this, the latitudinal averaging kernels are found better peaked (Figure 2(b)) and the formal errors associated with the linear combination of the b coefficients is lowered. Following the definition of Corbard *et al.* (2001), the latitudinal resolution obtained is about 10° at all latitudes.

The kernels $K_h^l(r)$ associated with each f mode have a simple shape with only one maximum located at slightly different radial positions depending on the degree l (Figure 1). If we define $r_0^l \equiv \int_0^1 K_h^l(r) r dr$, the radial location of the center of gravity of these kernels, and assume a linear behavior of the rotation rate at each latitude in the radial domain where the f modes considered have appreciable amplitude, i.e.,

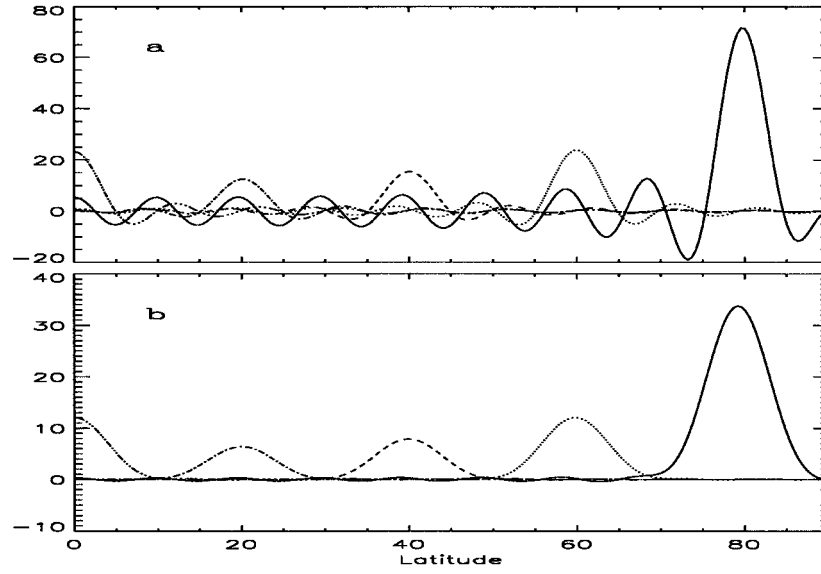


Figure 2. Latitudinal averaging kernels at 0, 20, 40, 60, 80° of latitude (double-dot-dash, dot-dash, dash, dot, and full lines, respectively) corresponding to the combination (a) Equation (8), (b) Equation (9).

$$\Omega(r, \mu_0) = \alpha(\mu_0) - \beta(\mu_0)(r - 1) \quad (7)$$

in $r > 0.97$, say, we simply obtain

$$\sum_{j=0}^{17} b_{2j+1}^l \bar{T}_{2j}^1(\mu_0) \approx \bar{\Omega}(r_0^l, \mu_0), \quad (8)$$

where the meaning of $\bar{\Omega}$ is the same as in Equation (5). Alternatively, a slightly modified choice of weights yields

$$\sum_{j=0}^{17} b_{2j+1}^l \bar{T}_{2j}^1(\mu_0) e^{-[j(j+3/2)]/117} \approx \langle \Omega(r_0^l, \mu) \rangle_{\mu_0} \approx \Omega(r_0^l, \bar{\mu}_0), \quad (9)$$

where the brackets denote the weighted average around μ_0 , the weighting function being the kernels of Figure 2(b). The second approximate equality in Equation (9) would be exact if the rotation profile were a linear function of μ^2 in the domain covered by the averaging kernels (i.e., $\pm 10^\circ$), with $\bar{\mu}_0^2 \equiv \int_0^1 \kappa(\mu_0, \mu) \mu^2 d\mu$; the approximation is less good, however, at high solar latitudes.

The parameters α and β can then be estimated at each latitude from a linear least-squares fit, yielding not only an estimate of the value of the rotation rate at, e.g., the surface, but also an estimate of the average gradient $\partial\Omega/\partial r$ in the region sampled by the f modes. Finally we note that the dependence of Ω as a function of radius in the near-surface layers may sometimes conveniently be described by

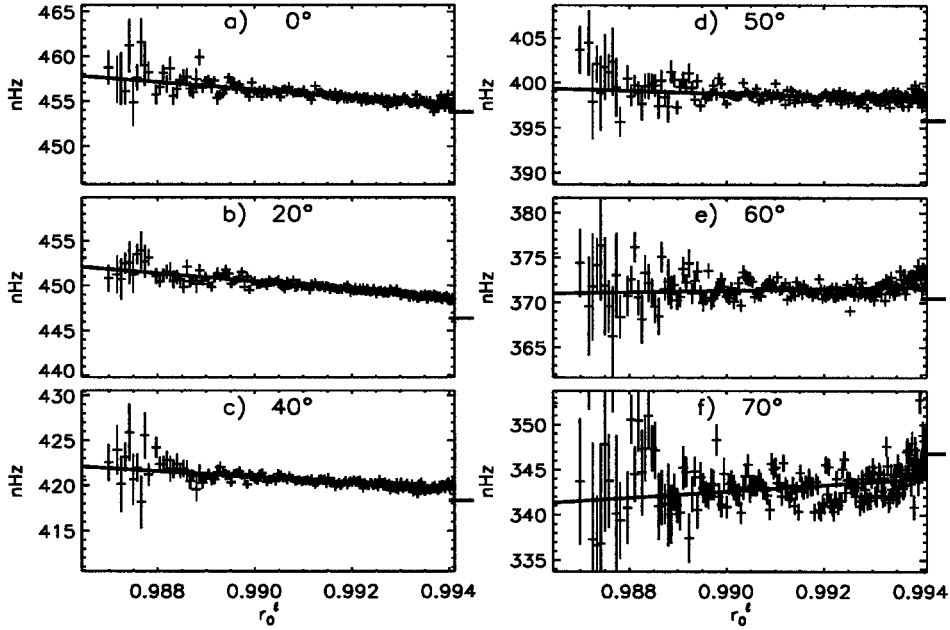


Figure 3. Time average of $\bar{\Omega}(r_0^l, \mu_0)/2\pi$ (Equation (8)) for values of μ_0 corresponding to the latitudes indicated in each panel. The result of the linear fits (Equation (7)) are shown by the *straight lines*. The error bars are the standard deviation associated with the weighted temporal mean. The mark on the right of each panel indicate the surface plasma rate obtained by Snodgrass, Howard, and Webster (1984). Note that the surface spectroscopic value indicated on panel f is essentially an extrapolation from observations at lower latitudes.

a power of r : we note that this description is immediately derivable from our α and β , since for small values of $1 - r$ the right-hand side of Equation (7) is well approximated by $\alpha(\mu_0)r^{-\alpha(\mu_0)/\beta(\mu_0)}$.

4. Results

By combining the frequency splittings within each f -mode multiplet in the manner given by Equation (8), for different choices of target latitude, we obtain measures of the near-surface rotation which are reasonably well localized in latitude and which correspond to different weightings in the depth direction. The latitudinal sensitivity is illustrated in Figure 2 and the depth sensitivity in Figure 1. Figure 3 shows the results of combining the data using Equation (8), averaged in time over all the datasets under study. In depth, the points are plotted at the center of gravity ($r = r_0^l$) of the corresponding kernels (cf., Figure 1). It is evident from these results that, at low latitudes, the weighted rotation increases with depth. If at each latitude separately we fit these results to a rotation profile that is linear in depth, we obtain the linear fits overplotted in Figure 3. These provide an average rotational gradient

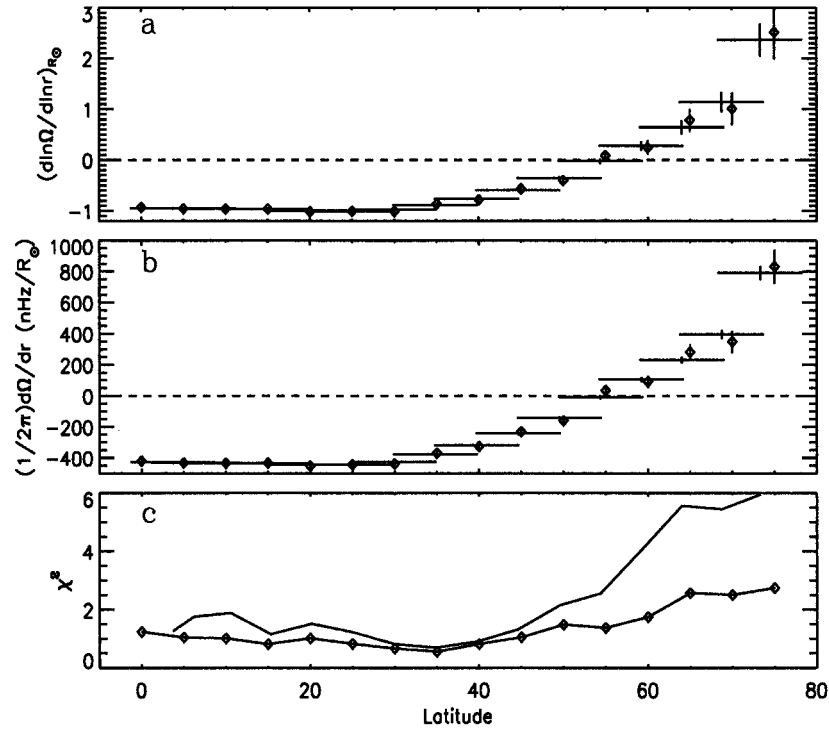


Figure 4. (a) Logarithmic derivative of angular velocity as a function of latitude. This corresponds to the ratio β/α of Equation (7). (b) Radial gradient of angular velocity β as a function of latitude. (c) Normalized χ^2 value of the linear fit. The *diamond symbols* are for the results obtained using Equation (8) while the other points are obtained using Equation (9). The *horizontal error bars* indicate the angular resolution as deduced from Figure 2(b). The *vertical error bars* are formal errors deduced from the linear fit. The *dashed horizontal lines* correspond to no radial gradient of angular velocity.

$\beta(\mu_0)$ in the outer 15 Mm or so of the solar interior, and an extrapolated surface rotation rate $\alpha(\mu_0)$. The gradient, as a function of latitude, is presented in Figure 4, both in terms of its dimensional value and in terms of the logarithmic derivative $\partial \ln \Omega / \partial \ln r$. It may be seen that for latitudes below 50° the gradient of rotation with depth is negative; at about 50° it is close to zero; and for higher latitudes the average rotational gradient becomes positive. We note that the radial gradient is remarkably constant at latitudes up to 30° , and the value of the logarithmic derivative at these latitudes is close to -1 . We return to this in Section 5.

Another way to visualize the changing gradient with latitude is that in Figure 5, where we show the rotation rate extrapolated both to the surface ($r = 1$) and to $r = 0.97$. The deeper rotation is faster than the surface rotation at low- and mid-latitudes, but slower at high latitudes. At low- and mid-latitudes the extrapolated surface rate agrees well with the spectroscopic surface measurements, given the approximately 1.5% spread in recent such determinations (see the review by Beck,

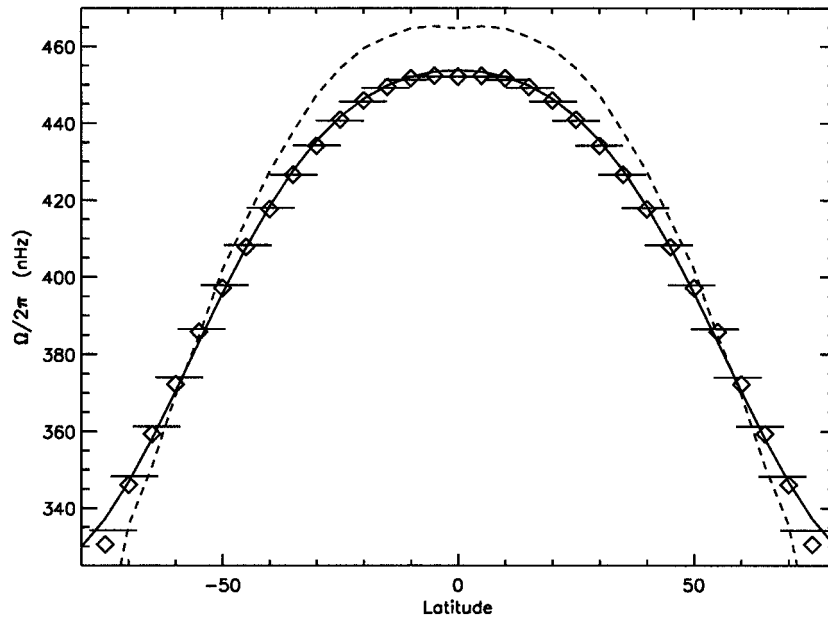


Figure 5. The full line gives the photospheric plasma rotation rate inferred by Snodgrass, Howard, and Webster (1984); the diamond symbols and horizontal bars corresponds to α , the intercept of the linear fit respectively in the case of Equations (8) and (9) and the dashed line corresponds to an extrapolation of the rotation rate at $0.97 R_{\odot}$ using Equation (7) in the case of Equation (8).

2000). For comparison, we have made a fit to our inferred surface rate below 60° latitude and present our fitting coefficients with the spectroscopic coefficients of Snodgrass, Howard, and Webster (1984) in Table I. Similarly to what has been found previously, our inferred rotation rate above 70° is markedly slower than what would be expected from a 3-term fit at low- and mid-latitudes: we return to this issue of the so-called ‘slow pole’ later.

Figure 4(c) shows the chi-squared for the least-squares fits at each latitude. The large chi-squared values at higher latitudes are striking. The difference between the chi-squared values when using Equations (8) and (9) is also very noticeable: this arises largely because the error bars on the fitted points are reduced by the exponential factor in Equation (9), which results in an increased chi-squared. Thus the interpretation of the absolute value of the chi-squared may be a little uncertain, but the trend with latitude for the two cases is similar. The larger values of chi-squared at higher latitudes is consistent with the greater deviation from a linear fit in the high-latitude panels of Figure 3. The systematic deviation of the near-surface points contributes most to the chi-squared: these correspond to the high-degree modes and so motivates taking a closer look at those data. (The scatter of the deepest points is large but less significant because of the large error bars on those points.)

TABLE I

Comparison between the surface plasma rate and our results from f -mode analysis.

Method	$\Omega_1/2\pi$ (nHz)	$\Omega_3/2\pi$ (nHz)	$\Omega_5/2\pi$ (nHz)
Snodgrass, Howard, and Webster (1984) ¹	436.4	21.0	-3.6
f modes (l -averaged) ²			
($117 \leq l \leq 300$) $\langle r_0^l \rangle = 0.991$ ³	438.8	21.0	-3.9
($160 \leq l \leq 250$) $\langle r_0^l \rangle = 0.991$	438.9	21.2	-4.0
f modes (surface extrapolation) ⁴			
($117 \leq l \leq 300$)	435.8	20.2	-3.2
($160 \leq l \leq 250$)	435.7	20.5	-3.6

¹Spectroscopic measurements made at the Mount Wilson 150-foot Tower between 1967 and 1982.

²Average of the first 3 b coefficients (cf., Equations (4) and (8)).

³Center of gravity of the corresponding l averaged radial kernels.

⁴Obtained by fitting the intercept $\alpha(\mu)$ to the expansion equation (2) for latitudes below 60° .

We have therefore repeated our analysis but excluding those modes of degree $l > 250$ and $l < 160$. The resulting gradient and chi-squared are shown in Figure 6. Compared with the previous result (Figure 4) the gradient is similar for latitudes lower than 50° . Now it is evident from Figure 3 that, at high latitudes, excluding the high-degree modes will tend to make the fitted gradient less positive. Indeed, we find that the gradient without the $l > 250$ data remains slightly negative up to about 75° . Also, the values of chi-squared have been more than halved at high latitude, compared with our previous linear fit to all the f -mode data (Figure 6(b)). The inferred low- and mid-latitude surface rate is barely affected (compare the last two lines of Table I).

It is interesting also to compare the linear fit to the $l < 250$ data with a fit of a constant function to the same data: the constant fit is equivalent averaging the f -mode splittings over l (see Table I). It is evident from Figure 6(b) (dotted line) that this provides a very poor fit below about 55° : the data strongly favor the model with a linear depth-dependence there. At high latitudes, the linear fit selects only a very small gradient and so the two chi-squared functions are very similar: the data for $l < 250$ indicate that at high latitudes the gradient is small, in the range of depths spanned by their lower turning points.

If the data for $l > 250$ are indeed reliable, then the discrepancy between the results in Figures 4(a) and 6(a) implies that the model of rotation varying linearly with depth is not appropriate at high latitudes and the extrapolation to the surface at those latitudes will be unreliable. An alternative approach is to attempt to construct

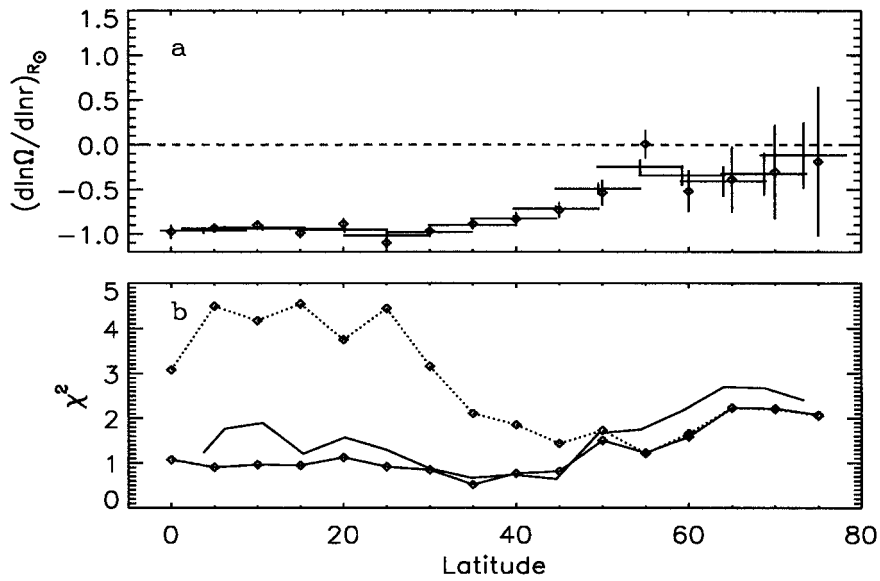


Figure 6. Similar to Figure 4 but using only modes $160 \leq l \leq 250$. The radial gradient of angular velocity remains negative even at latitudes above 55° . The *dotted line* on panel b shows, in the case of Equation (8), the χ^2 values corresponding to a fit by a constant which is equivalent of taking an average over l .

kernels that are localized in depth using the Optimally Localized Averaging (OLA) kernel in depth (cf., Christensen-Dalsgaard, Schou, and Thompson, 1990) in the manner of Backus and Gilbert (1968). Such kernels at two selected depths are shown in Figure 7: they were constructed using all the available f modes. It should be noted that the method succeeds in producing kernels which are reasonably localized and which have their center of gravity outside the range of abscissa values in Figure 3, that is, the method uses the mode sensitivities to extrapolate to greater depths and closer to the surface. In particular, in the latter case one expects that the increasing trend of values for the near-surface points in Figures 3(e) and 3(f) means that the near-surface Backus–Gilbert inversion at those latitudes will have values higher than those seen in Figure 3. This is exactly what is found (Figure 7): the Backus–Gilbert inversion at high latitudes for $r = 0.986$ interestingly falls below the 3-term spectroscopic surface rate, but even more strikingly the corresponding near-surface result at $r = 0.997$ lies above it by 2–4 standard deviations. This is another way of demonstrating that the increasing values of the combined splittings for $l > 250$, if they are reliable, indicate a strongly positive gradient of rotation with radius in the rather superficial subsurface layers at high latitudes.

To look for possible temporal variations of the subsurface shear, we have analyzed each one of the 23 72-day datasets individually in exactly the same manner as we analyzed the time-averaged set (e.g., Figure 3), and derived an intercept value $\alpha(\mu_0; t)$ (corresponding to the surface rate at that location and epoch) and slope

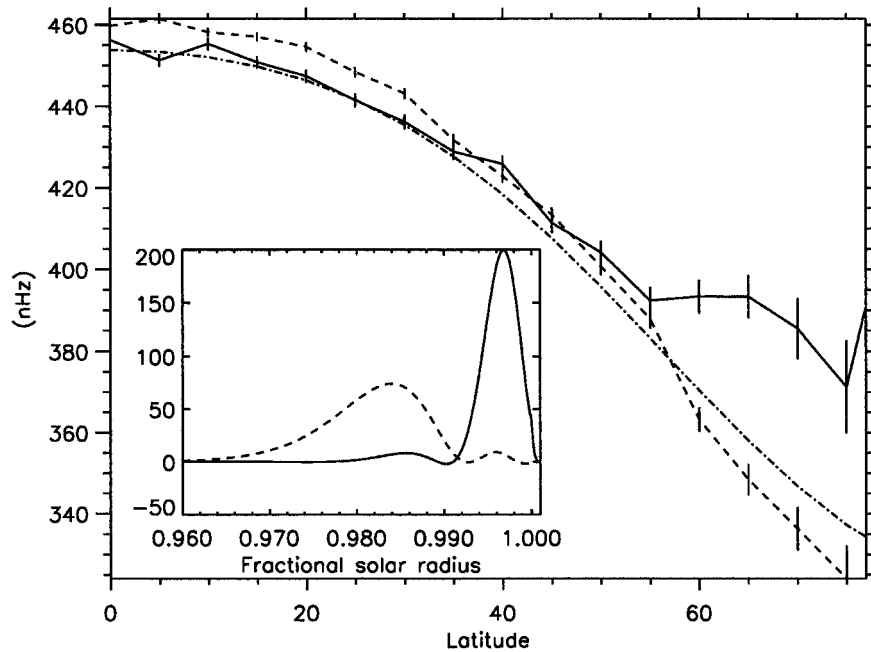


Figure 7. Rotation profiles as a function of latitude corresponding to depth averaged shown in the sub panel. The *dashed* and *full* lines correspond, respectively, to the shallower and deeper kernels which have respectively $0.986 R_{\odot}$ and $0.997 R_{\odot}$ as center of gravity. The *dot-dashed* line corresponds to the Snodgrass, Howard, and Webster (1984) plasma rotation rate. These results are obtained by using all modes from $l = 117$ to $l = 300$.

$\beta(\mu_0; t)$ from a linear fit to the combined splittings for each latitudinal location μ_0 and time t . The resulting estimated surface rates and slopes at three latitudes (equator, 30° , 60°) are shown in Figure 8. The large-scale variations in the surface rate correspond very well to the migrating banded zonal flows (torsional oscillations) measured by Schou (1999) and by Howe *et al.* (2000): the equatorial surface rate starts high because of the tail-end of one migrating band of faster flow, then drops down and rises again towards solar maximum as another band of faster flow reaches the equator: the latter was at 30° at the beginning of the cycle, hence the rate at that location starts high and drops as the band migrates closer to the equator. The 60° rate rises as the high-latitude banded flow reported by, e.g., Schou (1999) strengthens towards solar maximum. The slope shows no significant corresponding variations, implying that the torsional oscillations raise and lower the rotation rate across the whole depth of the layer without changing the shear gradient. There are indications of annual variations in the inferred values of the slope (most strikingly at 30°), which are almost certainly an artifact: such artifacts can conceivably arise from annual variations in SOHO's orbit. Other evidence for one-year artifacts in the f -mode data is presented by Antia *et al.* (2001). These should not affect the time-

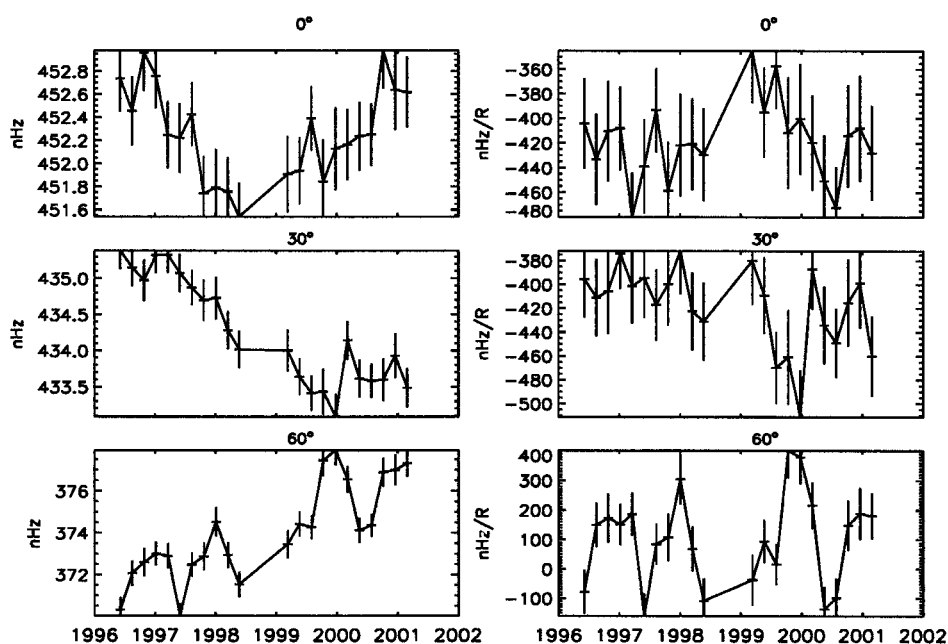


Figure 8. Intercept (left column) and slope (right column) of the linear fit Equation (7) at the equator, 30° and 60° of latitude (from top to bottom).

averaged values, however. There is no noticeable annual variation in our inferred values of the surface rotation rate.

5. Discussion

We have used the depth and latitude variation in the sensitivities of the solar f modes to deduce the rotation profile in the subsurface shear layer of the Sun in the outer 15 Mm of the solar interior. Our work differs from earlier seismic investigations. These were either based on the f modes but implicitly assumed a depth-independent model of the rotation (e.g., Schou, 1999), or used global inversions of p - and f -mode splittings and consequently may suffer from any systematic difference between the p - and f -mode data (e.g., Schou *et al.*, 1998), or used local helioseismic ring analysis (e.g., Basu, Antia, and Tripathy, 1999; Haber *et al.*, 2000), which promises to be a powerful diagnostic of near-surface flows and stratification but the sensitivity and systematics of which are still under investigation (Hindman *et al.*, 2001, in preparation). By using just the splittings of the f modes, which are arguably the most straightforward helioseismic modes to interpret, we believe we are able to obtain not only a simple but also a clean measure of the near-surface shear. As with all inferences about rotation from global splittings, we

note that only that component of rotation which is symmetric about the equator is recovered.

The most robust results concern the low-latitude shear. The average gradient $\partial \ln \Omega / \partial \ln r$ (at constant latitude) in the outer 15 Mm is close to -1 and remarkably constant from the equator to 30° latitude. Between 30° and 55° latitude, the gradient is still negative but makes a steady transition to a small (absolute) value. All our analyses show this. The variation of rotation at these latitudes appears to be well described by a linear function of depth, within the outer 15 Mm.

As discussed in the Introduction, if moving parcels of fluid were to conserve their specific angular momentum as they moved towards or away from the rotation axis, one would find that the rotation rate varied as the inverse square of the distance from the axis of rotation, so at low latitudes one would have that $\partial \ln \Omega / \partial \ln r \approx -2$. In reality other effects such as diffusion will cause exchange of angular momentum between parcels, so we may expect a logarithmic gradient somewhat smaller in magnitude than -2 . A precise measurement of this value in the Sun provides information about the relative effectiveness of competing mechanisms transporting angular momentum. Our finding is that at latitudes below 30° the value of the logarithmic gradient is much closer to -1 than to -2 . In fact, this seems in reasonable agreement with the equatorial value found by DeRosa in numerical simulations of rotating compressible convective fluid in a thin shell representing the Sun between about $0.94R$ and $0.98R$ (DeRosa, 2001; DeRosa, Gilman, and Toomre, 2001). Also these simulations show a tendency for the gradient to decrease in magnitude as one moves from equator to mid-latitudes, albeit at lower latitudes than we find for the Sun. Although these simulations exclude for numerical reasons the near-surface layers that we are probing, the qualitative agreement is nonetheless encouraging.

At latitudes above $\sim 55^\circ$, the depth-averaged gradient over the layer appears to change sign with respect to the low-latitude shear, though this is largely a consequence of the behavior in the very near-surface layers (outer 5 Mm) which in turn is deduced from the splittings of the highest-degree f modes. The gradient in the range of depths 5–15 Mm is small at these high latitudes; and such significant gradient $\partial \Omega / \partial r$ as does exist at high latitude (if any) is in the outer 5 Mm and predominantly positive. We note that, using a ring-analysis technique, Basu, Antia, and Tripathy (1999) deduced a similar behavior at high latitudes, finding a reversal of gradient in a zone above $0.994 R_\odot$.

Concerning the surface rotation rate itself, below 55° our extrapolation of the rotation rate to the surface is in satisfactory agreement with the directly measured spectroscopic surface rotation rate (cf., Table I). Our inferred surface rate should be more accurate than one simply inferred from the averaged f -mode splittings, because we take out the linear gradient with depth which undoubtedly exists at these latitudes: this can make a difference of ~ 5 nHz, even over the fairly small range of depths sampled by the observed f modes.

The seismically inferred surface rate at high latitudes is considerably less secure. It has previously been reported from helioseismic investigations that the high-latitude surface rate is lower what one would expect from a simple three-term extrapolation from lower latitudes (Schou, Christensen-Dalsgaard, and Thompson, 1998; Schou, 1999). Indeed it can be seen from Figure 3(f) that many of the points fall below the extrapolated spectroscopic rate for that latitude, implying that the rotation rate at *some* depth is lower than the spectroscopic surface rate one would infer from the values in Table I. The rather flat plateau of values in those panels strongly suggests that the rotation rate at about 10–15 Mm depth is slower than the extrapolated spectroscopic rate, which is confirmed by our OLA inversion result at those depths. However, the combined splittings at high degree are *increasing* with l and if taken at face value, as is done in our OLA inversion result for $r = 0.997R$, this behavior implies that the very near-surface rotation rate is actually higher than the spectroscopic rate. Thus the matter is still open. Since the quoted spectroscopic rate is principally an extrapolation of surface observations at low- and mid-latitudes, the true rotation rate that would be determined by spectroscopy at high latitudes is uncertain. Direct spectroscopic determinations at high latitude would resolve the question. The very high-degree splittings could contain some systematic errors, and if these affect the low- m data the most (some evidence for such an effect for p modes at lower degrees is offered by the comparison of GONG and MDI splittings by Schou *et al.* (2001)), then the near-surface, high-latitude rotation rates inferred here could be erroneously high. We hope that this possibility will shortly be addressed by independent determinations of these splittings by the GONG experiment using the new higher-resolution GONG+ observations.

6. Conclusion

Finally, to return again to our principal focus which is the shear gradient of the near-surface rotation, we find that at low and mid-latitudes the gradient $\partial\Omega/\partial r$ in the outer 15 Mm or so is close to -1 and is quite independent of latitude below 30° ; between 30° and $\sim 50^\circ$ latitude, it is still negative but makes a transition to small absolute value. At higher latitudes, the gradient in the bulk of the outer 15 Mm is probably small, but if the highest-degree ($l > 250$) data are to be believed there is a region of positive gradient in the outer 5 Mm at high latitudes, similar to what (Basu, Antia, and Tripathy (1999) found from ring analysis. We find no evidence for the gradient to vary with time: the torsional oscillation seems to pass through without changing the shear gradient in the outer 15 Mm.

Interestingly, the most recent circulation-dominated dynamo models (Dikpati and Charbonneau, 1999; Küker, Rüdiger, and Schultz, 2001) are able to reproduce to some extent the equatorward migration patterns without invoking any radial gradient of angular velocity at the surface. Such negative gradient at low latitude should however probably be taken into account because if it is associated with

a positive surface α -effect, it will compete against the surface poleward circulation and contribute to producing the equatorward migration of magnetic patterns observed at the surface of the Sun.

Acknowledgements

We thank Jesper Schou for providing the data and Marc DeRosa, Mausumi Dikpati, François Lignières, and Peter Gilman for useful discussions. Juri Toomre and Steve Tomczyk are thanked for hospitality at JILA and HAO, respectively, where part of this work was carried out. The work was supported by the UK Particle Physics and Astronomy Research Council through the award of grant PPA/A/S/2000/00171.

Appendix. Derivation of f -Mode 1.5D Kernels

The polynomial \mathcal{P} used to describe the frequency splittings can be expressed in terms of the Clebsh-Gordan coefficients $C_{j_1 m_1 j_2 m_2}^{j m}$ (e.g., Edmonds, 1960) by

$$\mathcal{P}_j^{(l)}(m) = \beta_j^l C_{lmj0}^{lm}, \quad \beta_j^l \equiv \frac{l\sqrt{(2l-j)!(2l+j+1)!}}{(2l)!\sqrt{2l+1}}. \quad (10)$$

The Gegenbauer polynomials used in Equation (2) are defined by (e.g., Morse and Feshbach, 1953)

$$T_{2j}^1(\mu) = \sqrt{\frac{4\pi}{4j+3}} \frac{\partial Y_{2j+1}^0(\theta, \phi)}{\partial \mu}. \quad (11)$$

From Ritzwoller and Lively (1991) we can deduce that

$$2\pi a_{2j+1}^l = \frac{v_{2j+1}^l}{T_{2j}^1(0)} \int_0^1 K_j^l(r) \Omega_{2j+1}(r) dr, \quad (12)$$

where $v_{2j+1}^l \equiv L^2 C_{l1(2j+1)0}^{l1} / \beta_{2j+1}^l$, $L^2 \equiv l(l+1)$ and

$$K_j^l(r) = \frac{(\xi_l^2 + (L^2 - 1 - j(2j+3))\eta_l^2 - 2\xi_l\eta_l) \rho r^2}{\int_0^1 (\xi_l^2 + L^2\eta_l^2) \rho r^2 dr}, \quad (13)$$

ξ_l and η_l being respectively the radial and horizontal displacement eigenfunctions which are determined by solving the differential equations describing the motion of a self-gravitating fluid body in a standard solar model (e.g., Unno *et al.*, 1989) and ρ is the density profile given by the model, all these being functions of the fractional solar radius r .

Other expressions of practical interest can be found for v_{2j+1}^l that are recalled here for completeness. Pijpers (1997) established the recurrence relation

$$v_{2j+1}^l = \frac{(j-l)(2j+1)}{j(2l+2j+1)} v_{2j-1}^l \quad (14)$$

and Schou (1999) noticed that to a very good approximation

$$v_{2j+1}^l / T_{2j}^1(0) \approx e^{-j(j+3/2)/l}. \quad (15)$$

The f modes are horizontally propagating surface gravity waves for which the displacement eigenfunctions satisfy the following surface boundary condition under the Cowling approximation (e.g., Berthomieu and Christensen-Dalsgaard, 1991):

$$\eta_l(r) \approx \frac{g_s}{R_\odot w_l^2} \xi_l(r), \quad (16)$$

where $g_s = GM_\odot/R_\odot^2$ is the surface gravitational acceleration. Moreover, the angular frequencies $w_l = 2\pi\nu_{l0}$ of the f modes follow asymptotically (for $l \rightarrow \infty$) the dispersion relation $w_l^2 \approx g_s L/R_\odot$. Therefore we have $\xi_l \approx L\eta_l$ and, from Equation (13), the rotational kernels associated with the f modes can be written as a function of the horizontal displacement only:

$$K_j^l(r) \approx k_j^l K_h^l(r) \begin{cases} K_h^l(r) \equiv \frac{\eta_l(r)^2 \rho(r) r^2}{\int_0^1 \eta_l(r)^2 \rho(r) r^2 dr} \\ k_j^l \equiv 1 - \frac{1}{L} - \frac{1}{2L^2} (1 + j(2j+3)) \end{cases}. \quad (17)$$

Finally, Equation (3) is obtained by taking

$$u_{2j+1}^l \approx k_j^l e^{-j(j+3/2)/l}. \quad (18)$$

We note that Equation (5) is obtained by using the fact that, in the approximation Equation (17) valid for f modes, the rotational kernels depend on j only by a multiplicative factor. Taking instead $K_j^l \approx K_0^l$ for all j as usually done for high degree modes would also allow us to write Equation (5) but the integrated difference $\int (K_j^l - K_0^l) dr$ would reach 2.2% for $l = 117$, $j = 17$ whereas it remains negligible for all l and j in the case of the approximation used here.

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