Global-Mode Helioseismology: Extensions of a Well-Used Method

tim defends his thesis

11 april 2016

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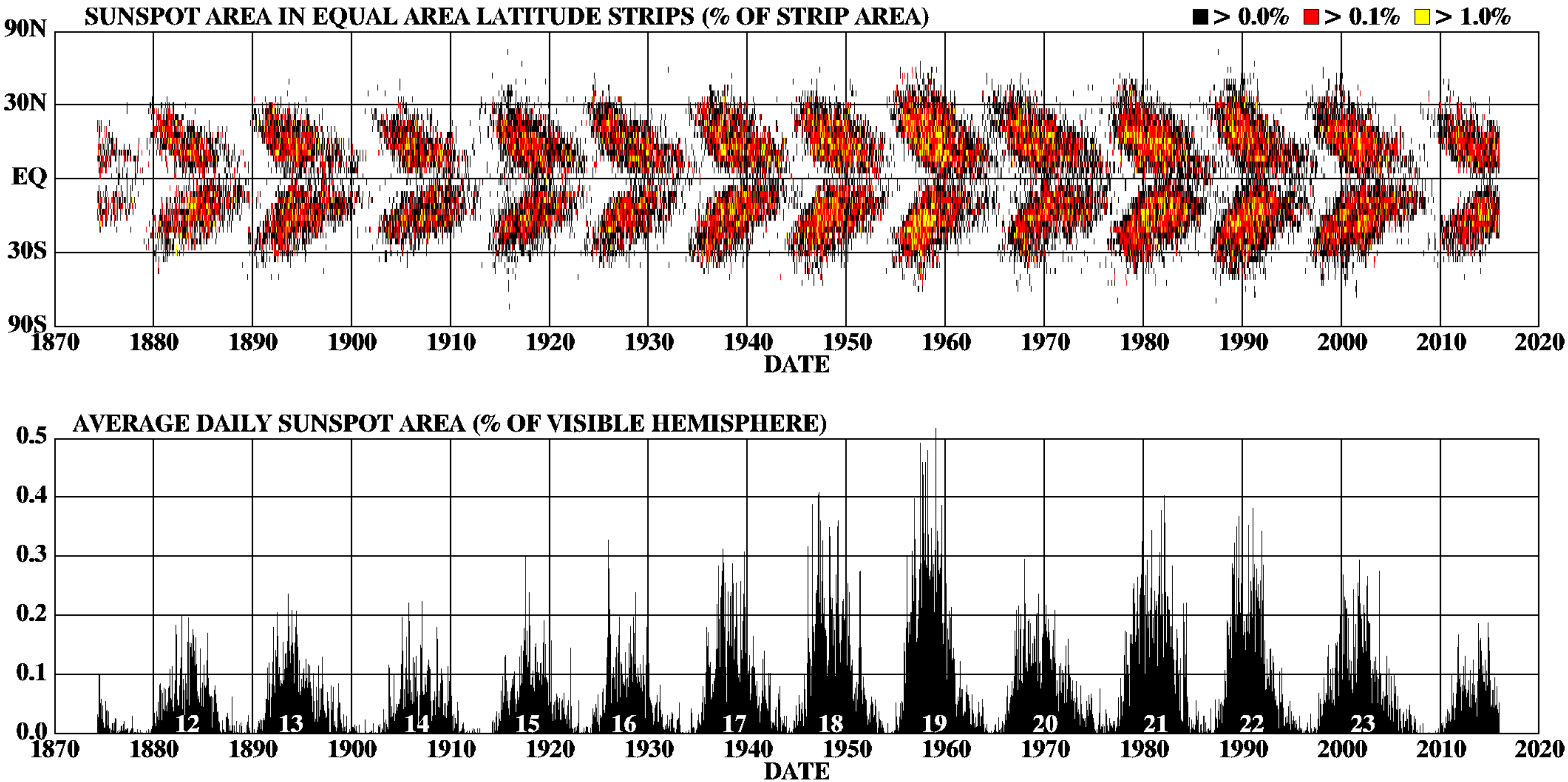
DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS



http://solarscience.msfc.nasa.gov/

HATHAWAY NASA/ARC 2016/01

Where did helioseismology come from?

- Richard Carrington

- George Ellery Hale and Mount Wilson Observatory (MWO)

- Robert Leighton

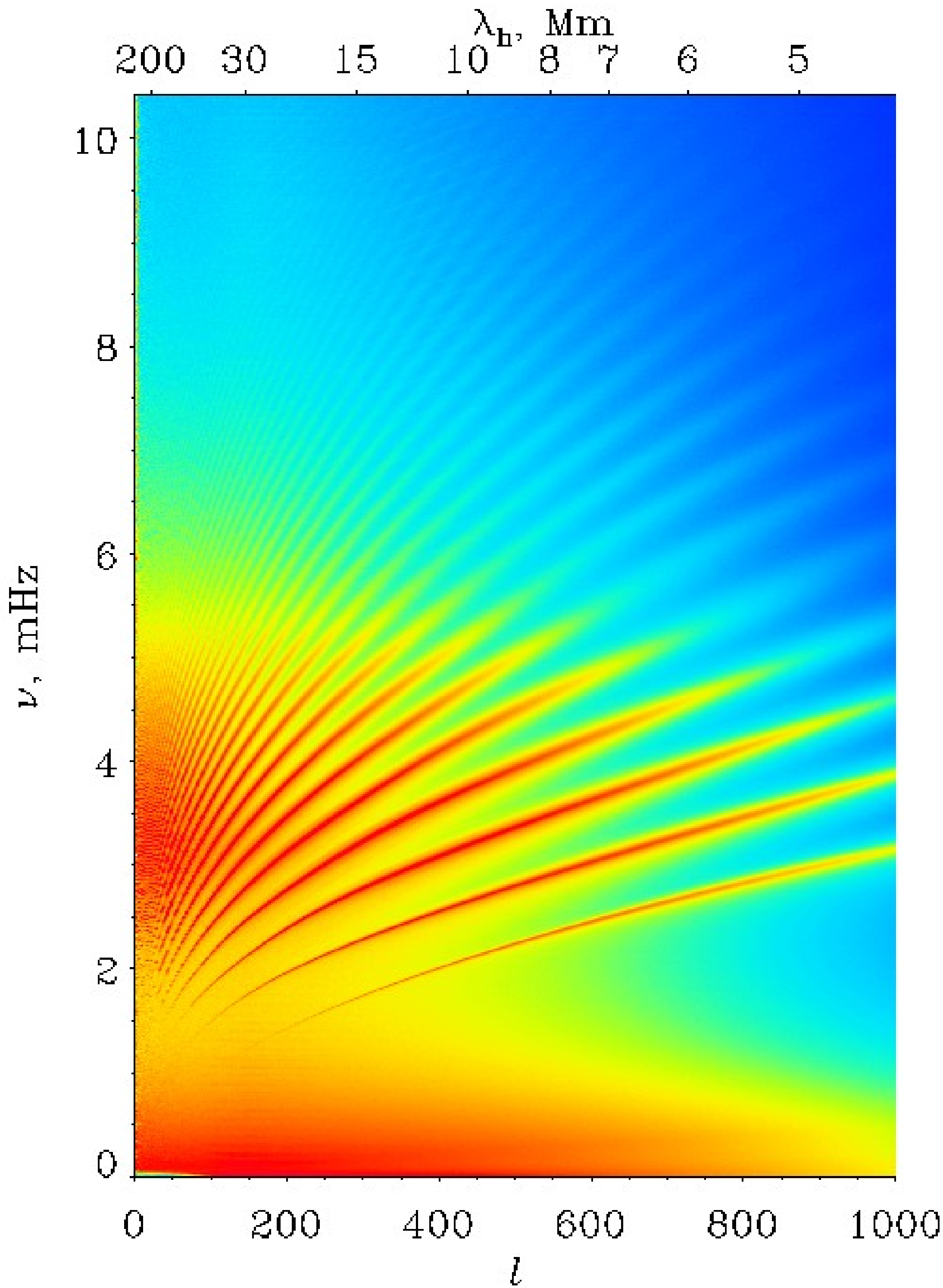
- Roger Ulrich

- Franz Deubner

- Birmingham Solar Oscillation Network (BiSON)

- Tom Duvall and Jack Harvey

- Ed Rhodes and the 60-foot Solar Tower at MWO


λ_h, Mm
200
30
15
10
8
7
6
5
10
8
6
4
2
0
ν, mHz
0
200
400
600
800
1000
l

Modern Helioseismic Instruments

(relevant to this work)

- Mount Wilson Observatory	MWO
- Global Oscillation Network Group	GONG
- Michelson Doppler Imager	MDI
- Helioseismic and Magnetic Imager	HMI

Mathematical Preliminaries

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\,(\rho \mathbf{v}) = 0$$

continuit
y

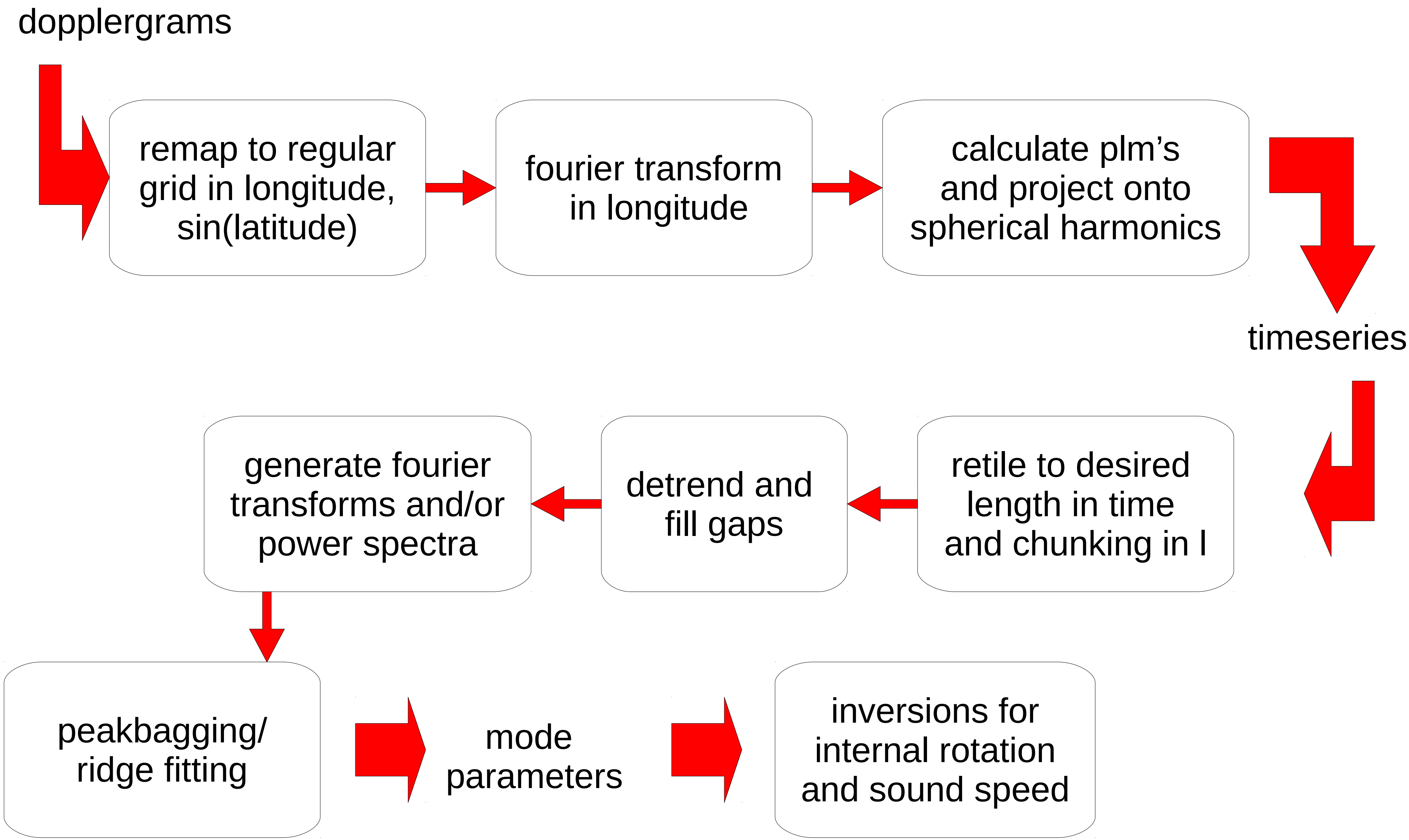
$$\rho\,\frac{\partial\,\mathbf{v}}{\partial t} + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\,\nabla p + \rho\,\mathbf{f}$$

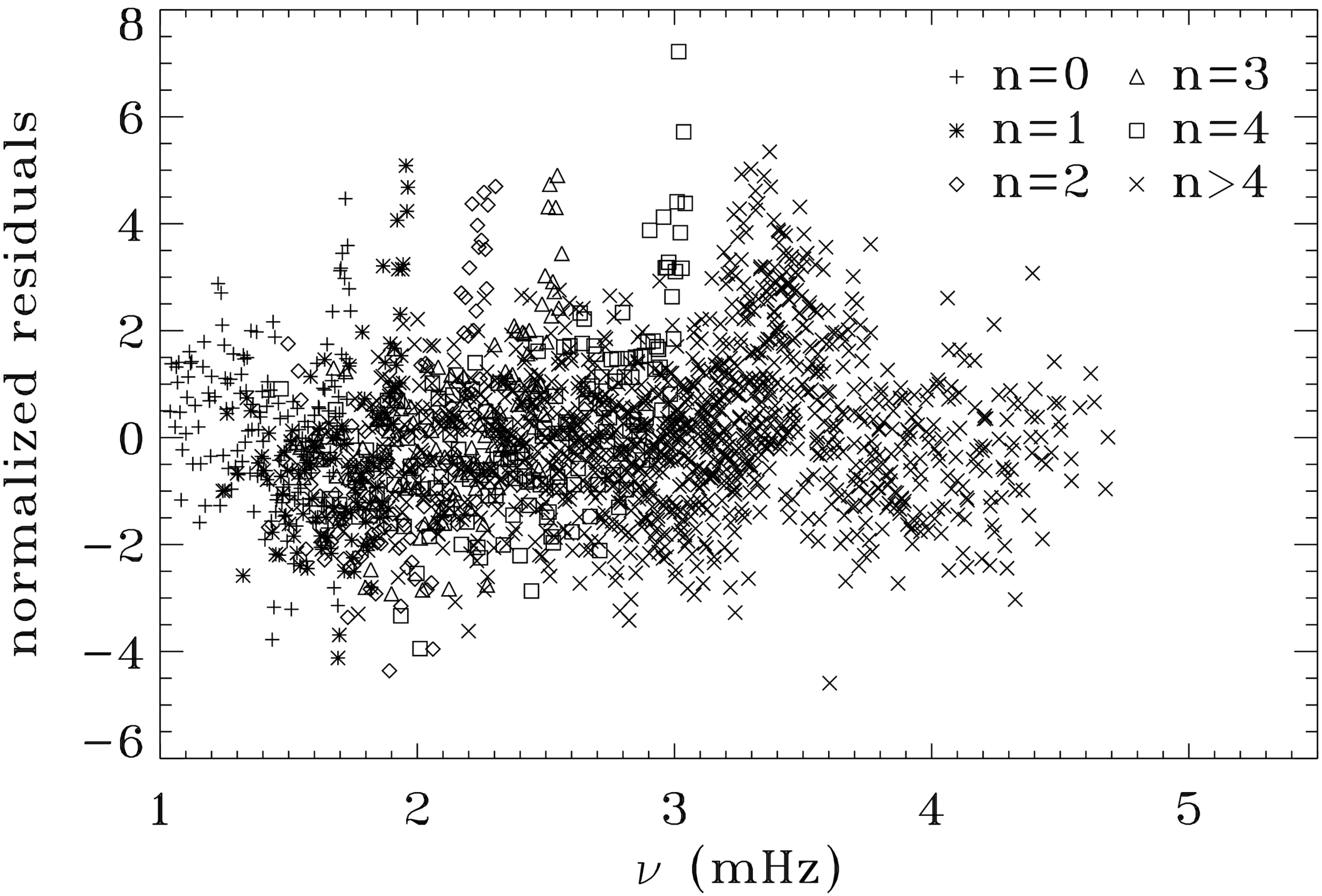
$$\mathbf{g} = -\nabla\Phi \qquad \nabla^2\Phi = 4\pi G\rho$$

equations of
motion

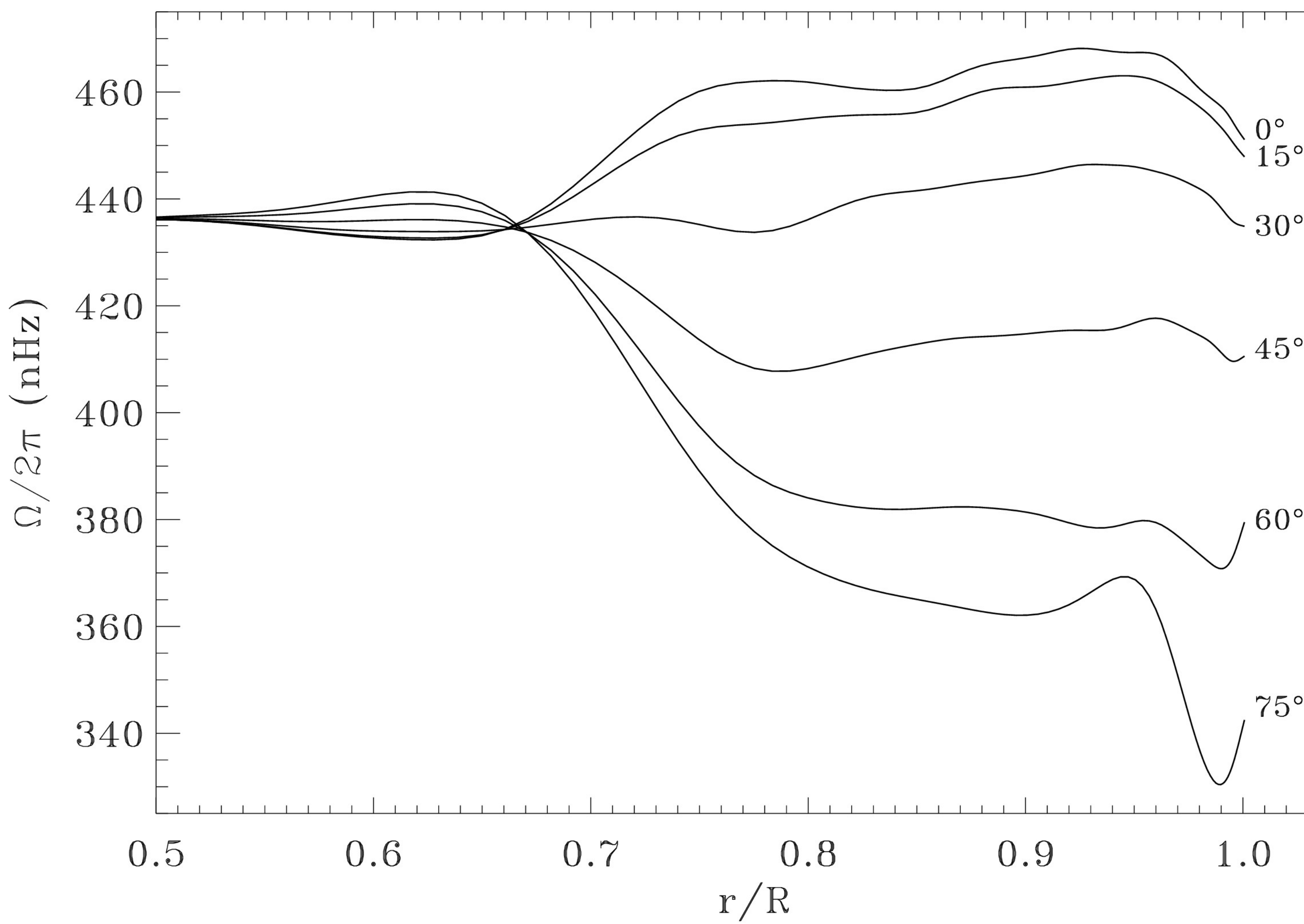
$$\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\Gamma_1 p}{\rho}\,\frac{\mathrm{d}\rho}{\mathrm{d}t}$$

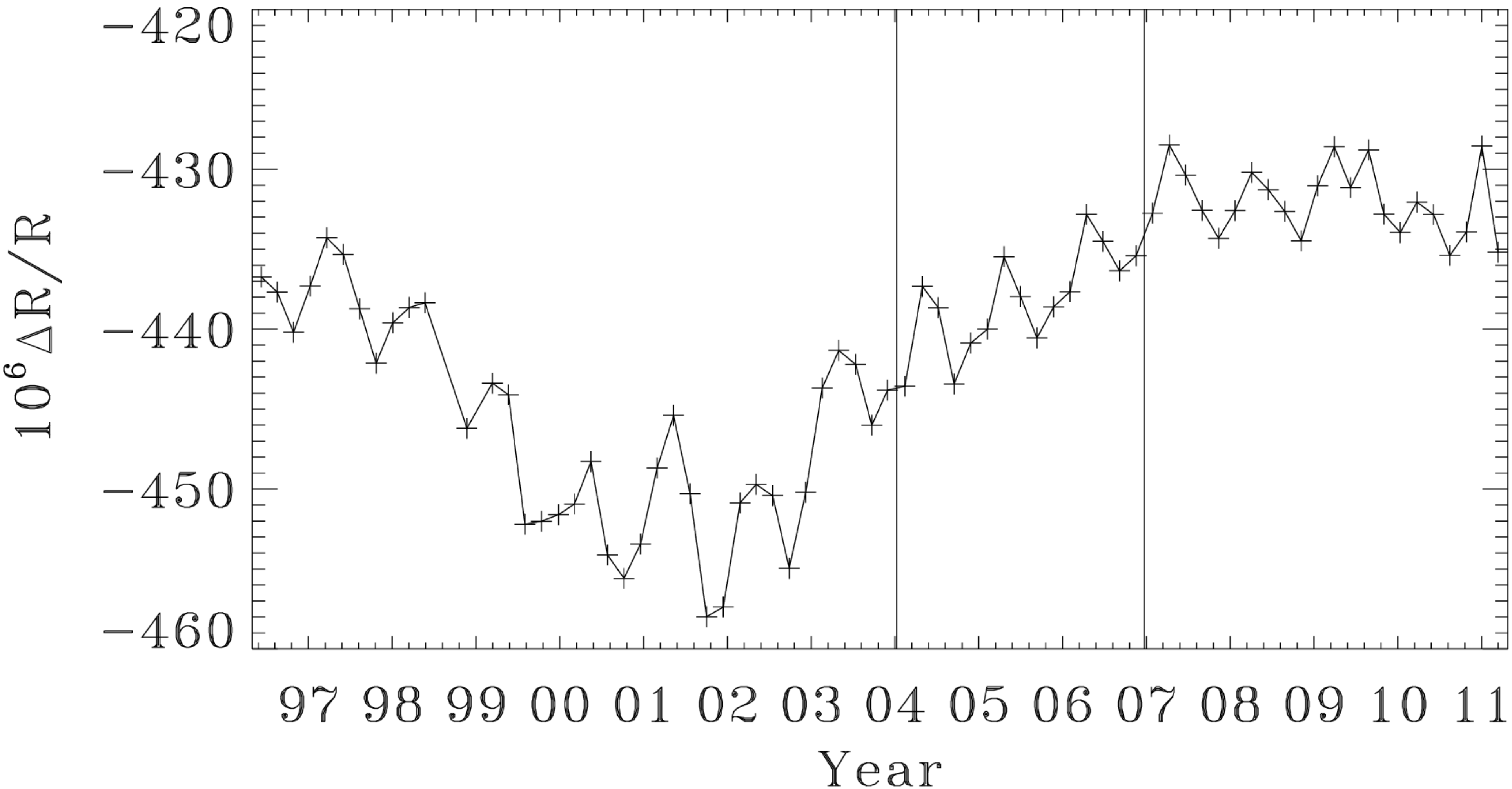
adiabaticity


dopplergrams
remap to regular grid in longitude, sin(latitude)
fourier transform in longitude
calculate plm's and project onto spherical harmonics
timeseries
retile to desired length in time and chunking in l
detrend and fill gaps
generate fourier transforms and/or power spectra
peakbagging/ ridge fitting
mode parameters
inversions for internal rotation and sound speed



n=0
n=1
n=2
n=3
n=4
n>4
normalized residuals
8
6
4
2
0
−2
−4
−6
1
2
3
4
5
ν (mHz)


460
440
420
400
380
360
340
Ω/2π (nHz)
0.5
0.6
0.7
0.8
0.9
1.0
r/R
0°
15°
30°
45°
60°
75°



−420
−430
−440
−450
−460
$10^6 \Delta R/R$
97 98 99 00 01 02 03 04 05 06 07 08 09 10 11
Year

Table 2. Sequence of changes made to the analysis; each analysis includes the changes made in all previous ones.

0	original analysis
1	image scale
2	cubic distortion
3	CCD misalignment
4	inclination error
5	CCD tilt
6	window functions and detrending
7	gapfilling
8	horizontal displacement
9	distortion of eigenfunctions ("Woodard effect")
10	asymmetric line profiles

Image Scale

$$f(t) = b_0 + D[b_1 + b_2(t - t_0) + b_3(t - t_0)^2]$$

fit to $(A_{\mathrm{major}} + A_{\mathrm{minor}})/(2R_0)$

Optical Distortion

$$C_{\mathrm{dist}}(r^2 - R^2)$$

C_{dist} = 7.06×10^{-9}

Angle Corrections

$$B_0' = B_0 + \delta I \sin[2\pi(t_{\mathrm{obs}} - t_{\mathrm{ref}})]$$

$$P_{\mathrm{eff}}' = P_{\mathrm{eff}} + \delta P + \delta I \cos[2\pi(t_{\mathrm{obs}} - t_{\mathrm{ref}})]$$

$\delta P = 0.2°$ $\delta I = 0.1°$

Ellipticity

$$x' = x\cos\beta + y\sin\beta,$$

$$y' = -x\sin\beta + y\cos\beta.$$

$$x'' = x'\left(1 + \frac{y'}{f}\alpha\right) - x'\frac{\alpha^2}{4},$$

$$y'' = y'\left(1 + \frac{\alpha^2}{2} + \frac{y'}{f}\alpha\right) - y'\frac{\alpha^2}{4} - \frac{r_m^2}{f}\alpha,$$

Gapfilling

$$x_t = \sum_{j=1}^{L} \alpha_j x_{t-j} + e_t^{\mathrm{f}}. \quad (1)$$

The coefficients α_j are the model parameters to be determined. The parameter e_t^{f} is most easily interpreted when equation (1) is written in the form

$$e_t^{\mathrm{f}} = x_t - \sum_{j=1}^{L} \alpha_j x_{t-j} = \sum_{j=0}^{L} \gamma_j x_{t-j}. \quad (2)$$

$$\frac{d\epsilon}{d\alpha_K} = 0, \quad (5)$$

where

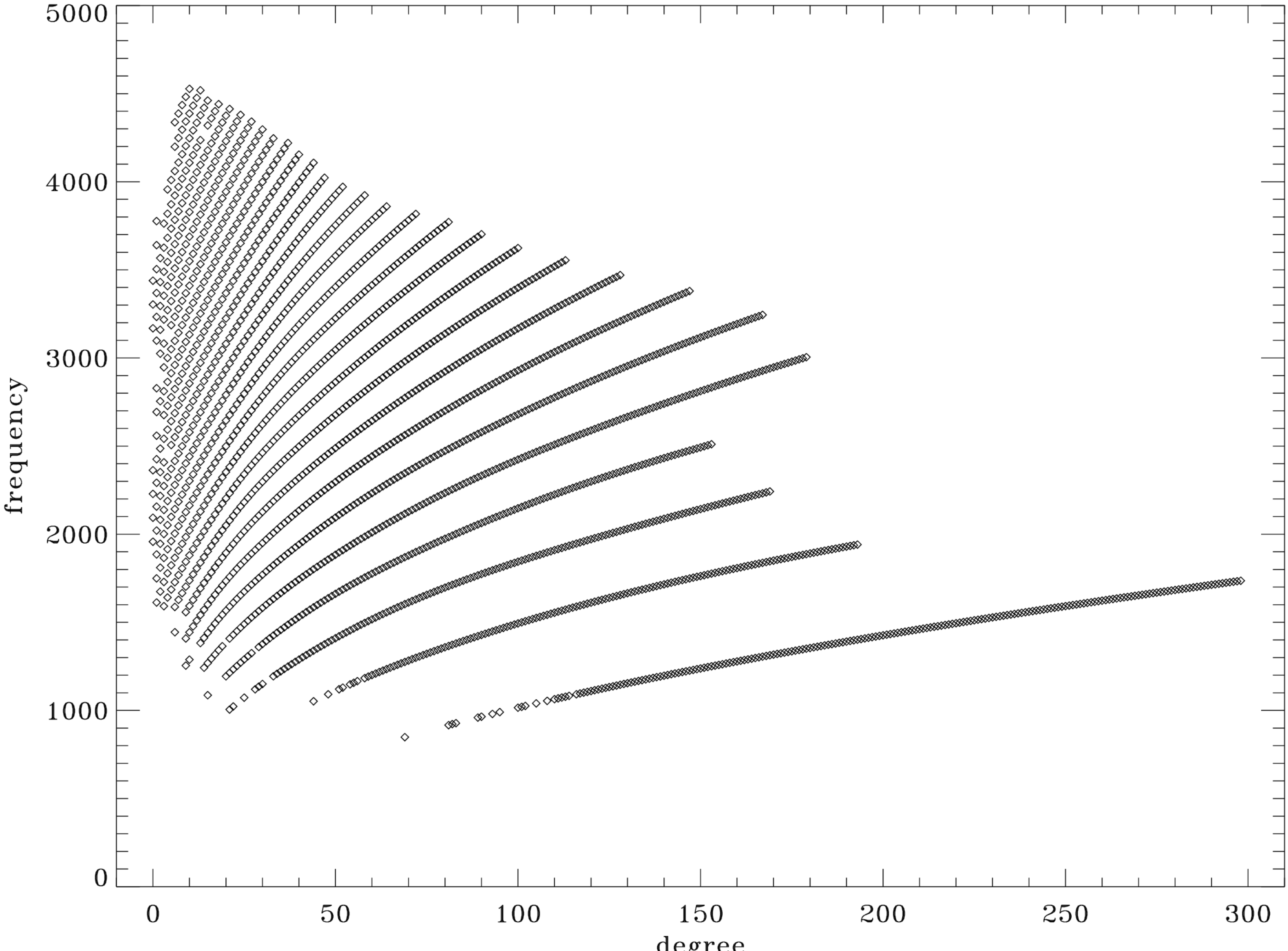
$$\epsilon = \sum_{q=1}^{Q} \sum_{t=M_q+K+1}^{N_q} (e_t^{\mathrm{f}})^2 + \sum_{t=M_q+1}^{N_q-K} (e_t^{\mathrm{b}})^2.$$

$$\frac{\partial \epsilon_q}{\partial x_t} = 0, \qquad t = (N_q + 1, M_{q+1}), \quad (9)$$

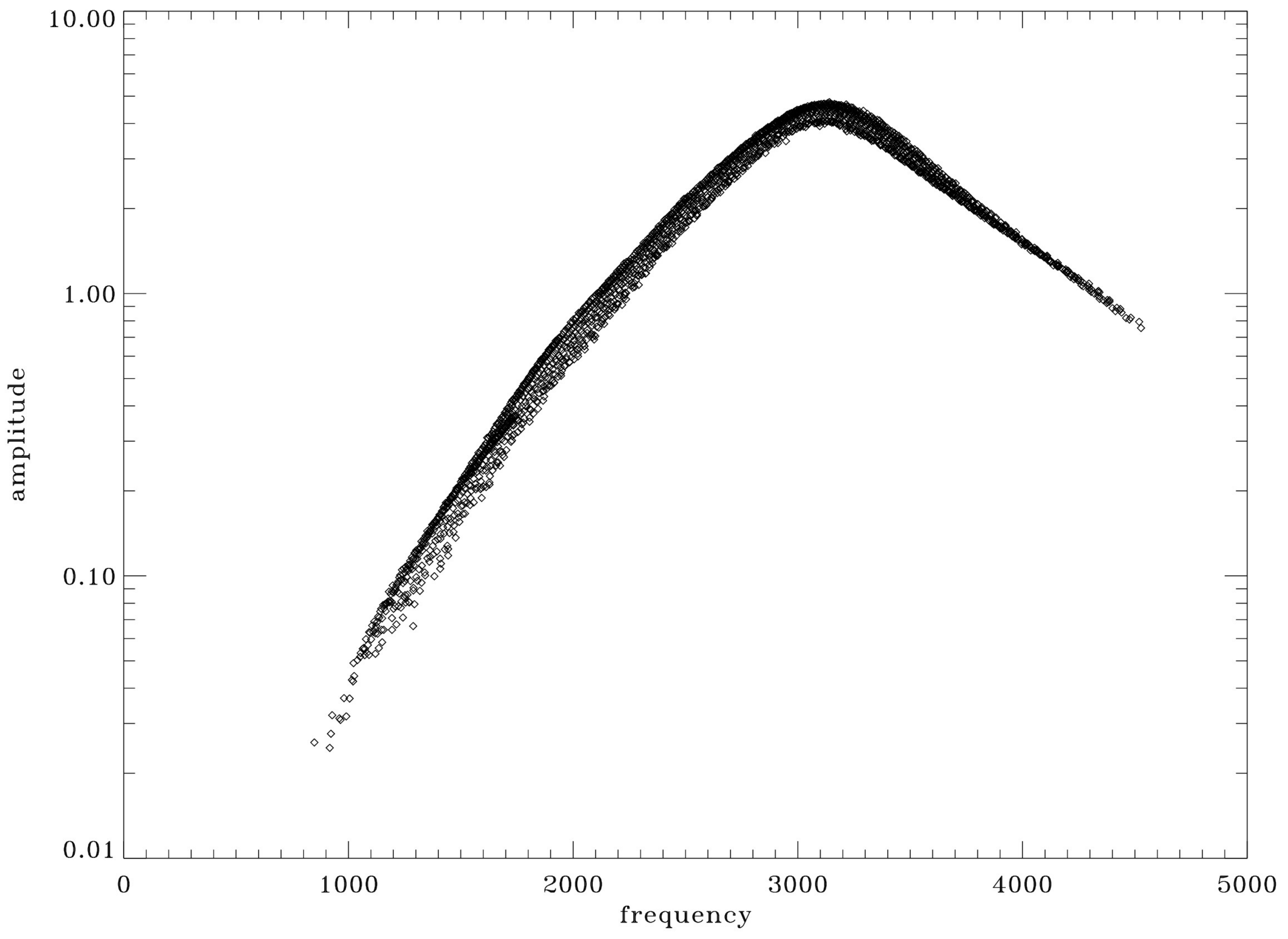
Peakbagging

Asymmetry

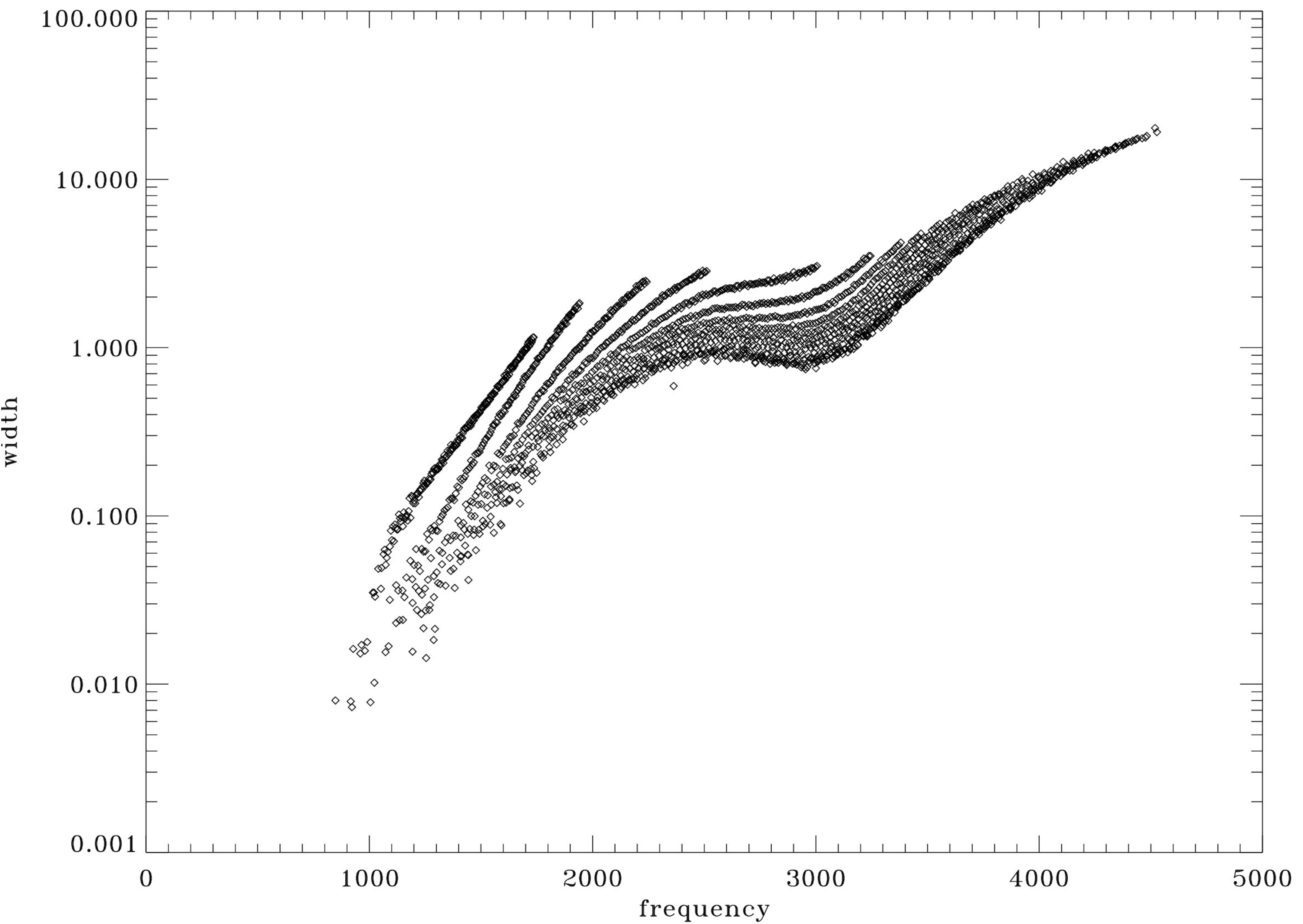
$$A_{\Psi}(X) = -A_{w} \frac{\sin(\beta X - p_{w}) + (i/2)\,\beta\cos(\beta X - p_{w})}{\sin(\beta X) + (i/2)\,\beta\cos(\beta X)}$$



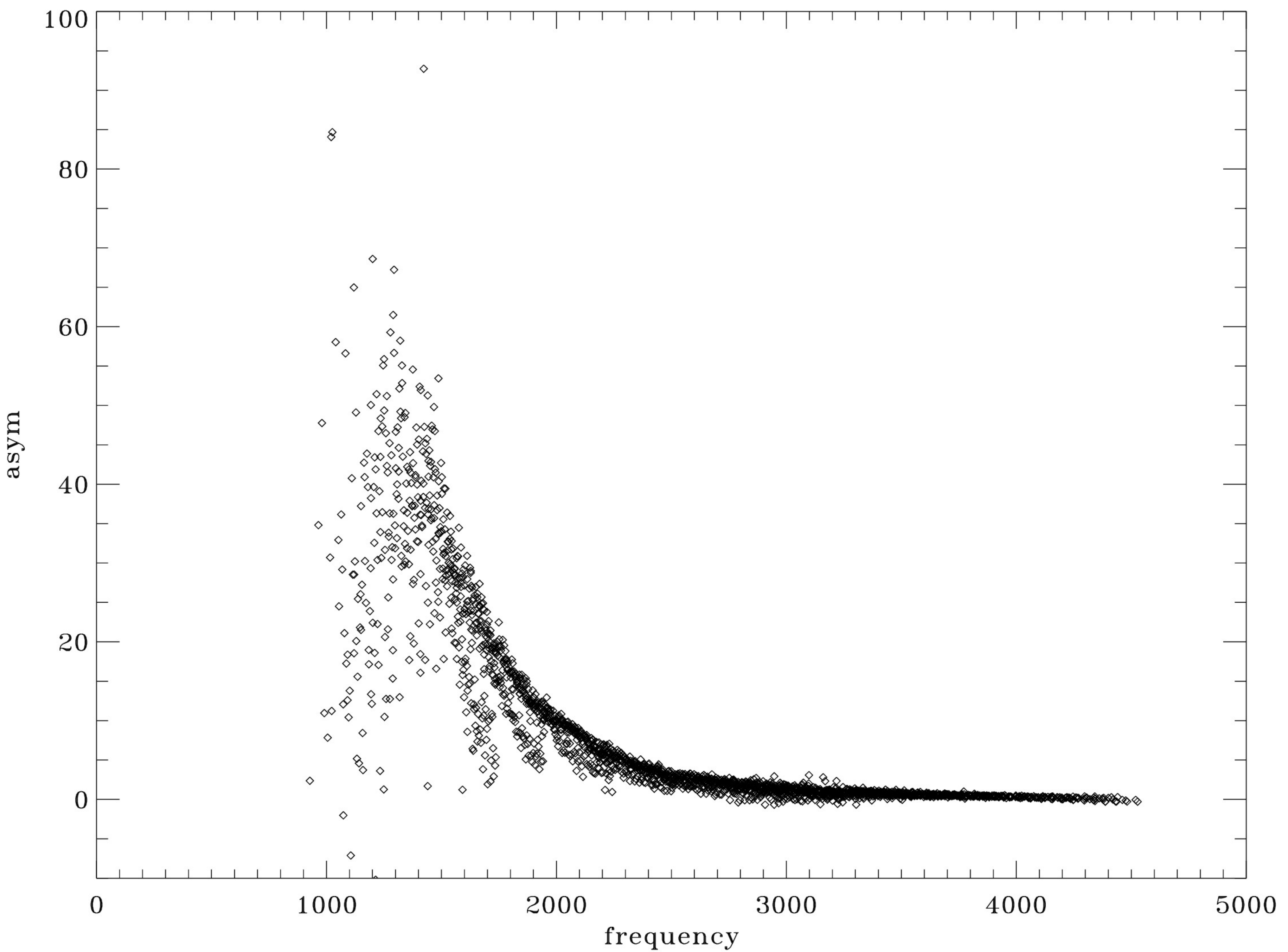
5000
4000
3000
2000
1000
0
frequency
0
50
100
150
200
250
300
degree



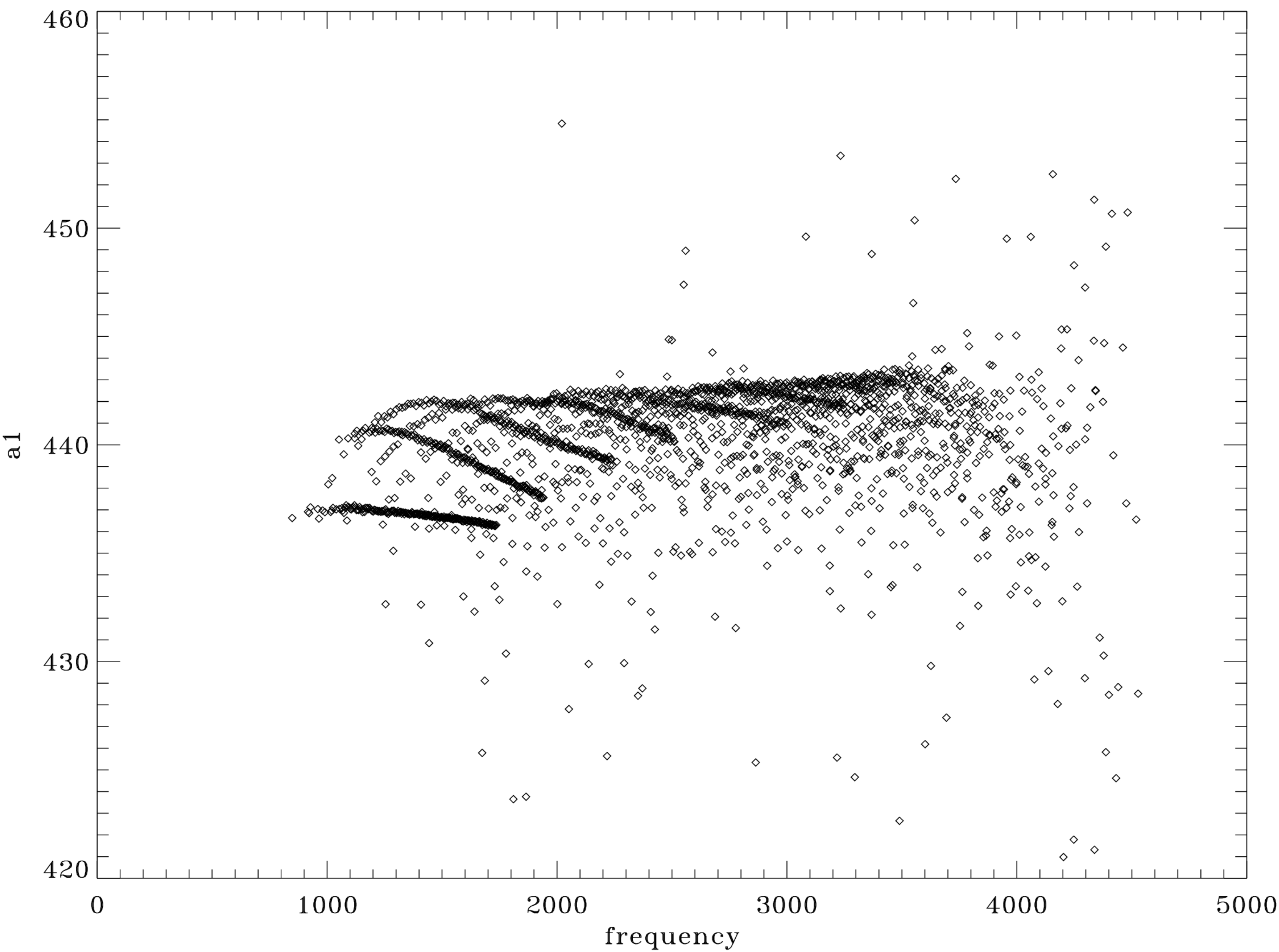
10.00
1.00
0.10
0.01
amplitude
0
1000
2000
3000
4000
5000
frequency



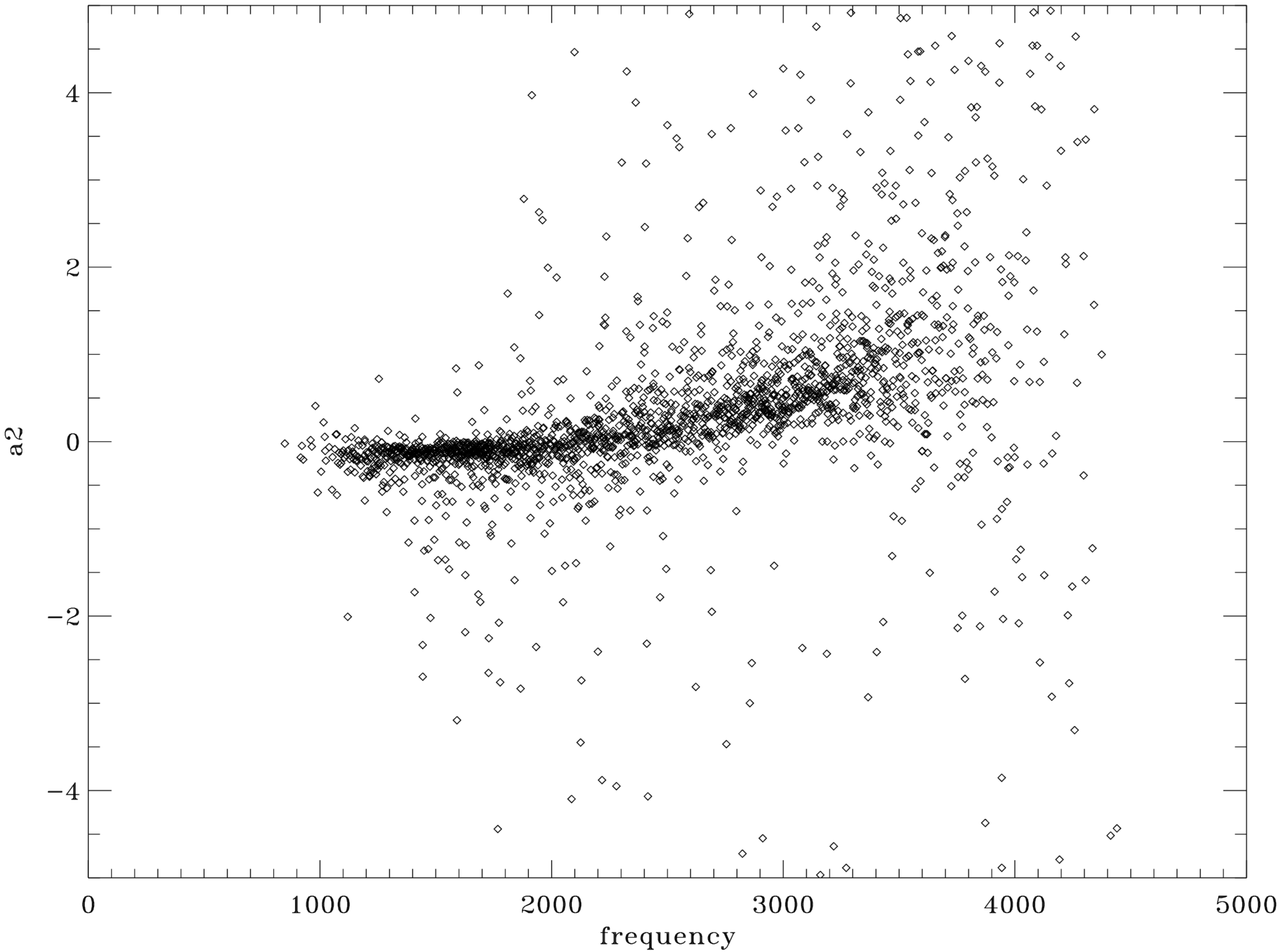
100.000
10.000
1.000
0.100
0.010
0.001
width
0
1000
2000
3000
4000
5000
frequency



100
80
60
40
20
0
asym
0
1000
2000
3000
4000
5000
frequency



460
450
440
430
420
a1
0
1000
2000
3000
4000
5000
frequency



a2
4
2
0
−2
−4
0
1000
2000
3000
4000
5000
frequency

Effect on Mode Parameters

frequency

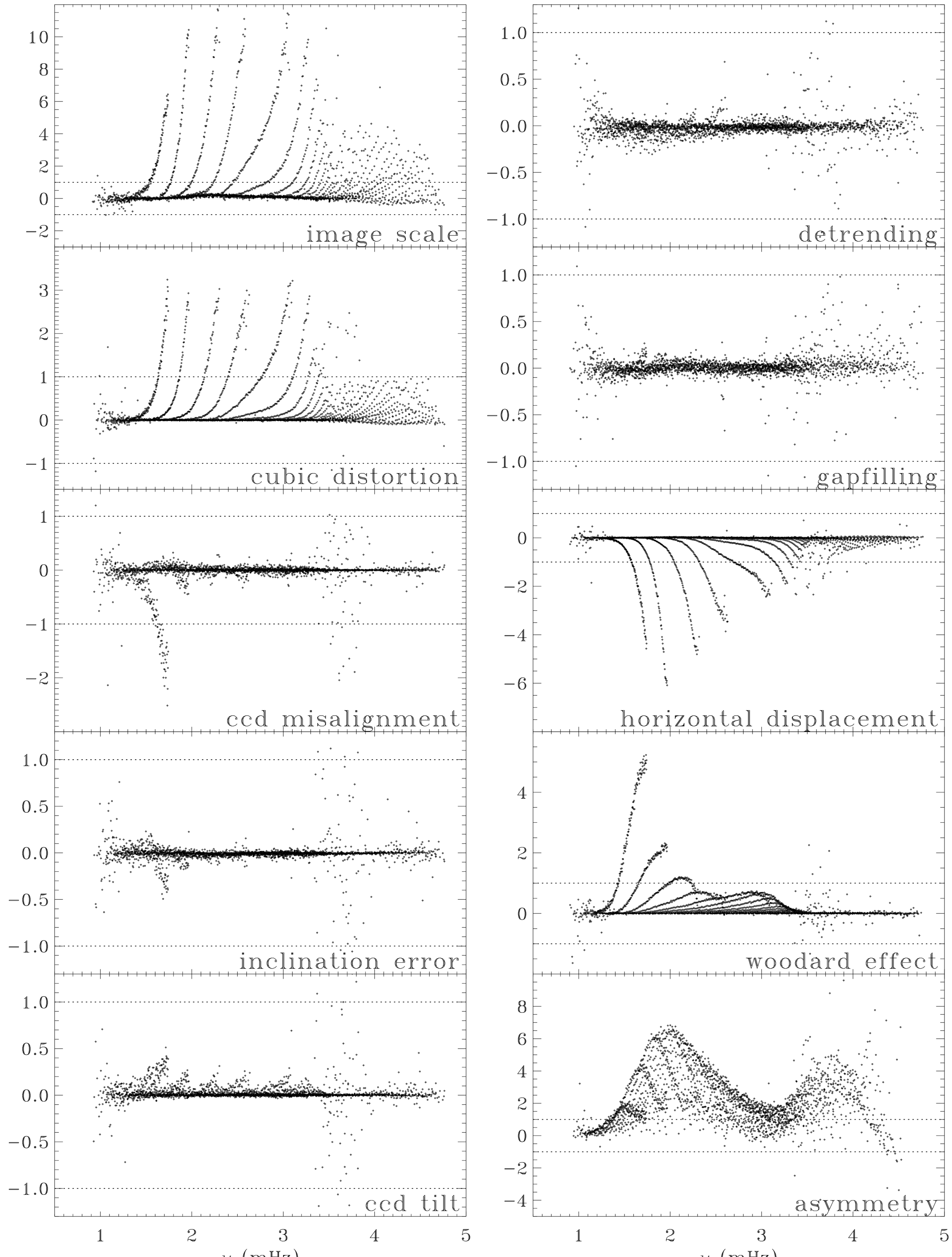
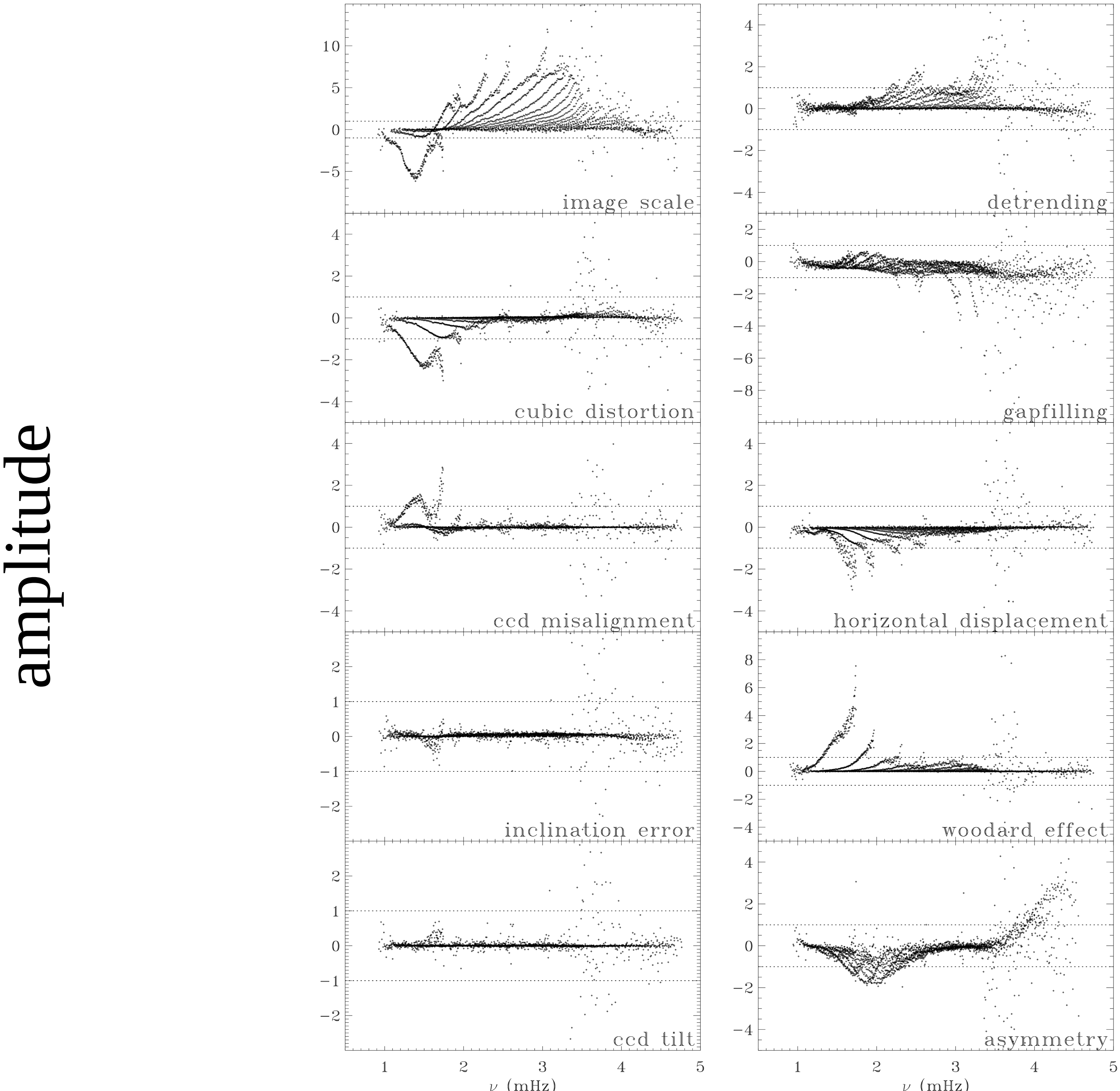


image scale
detrending
cubic distortion
gapfilling
ccd misalignment
horizontal displacement
inclination error
woodard effect
ccd tilt
asymmetry
ν (mHz)
ν (mHz)



amplitude
image scale
detrending
cubic distortion
gapfilling
ccd misalignment
horizontal displacement
inclination error
woodard effect
ccd tilt
asymmetry
ν (mHz)
ν (mHz)

width

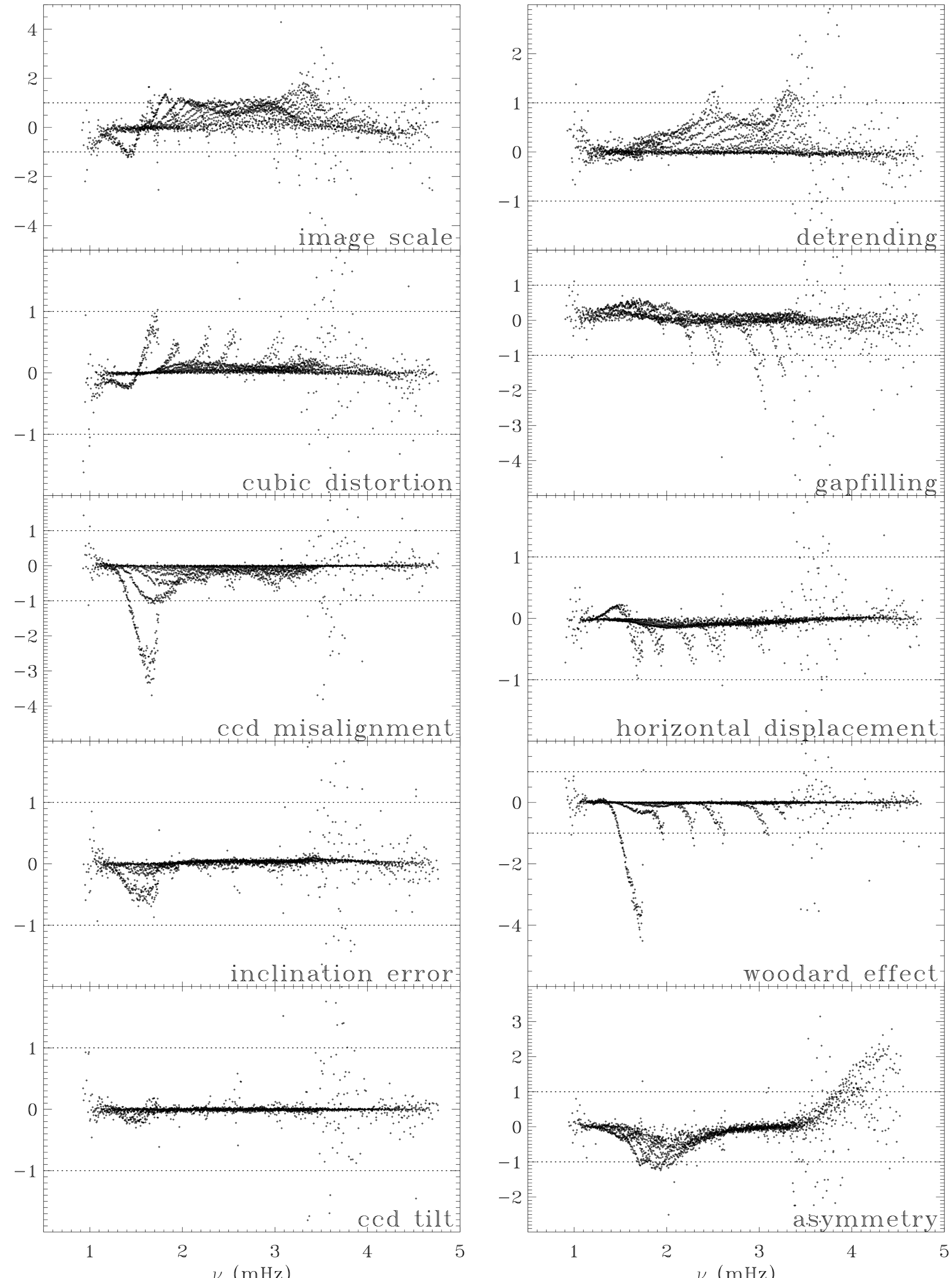


image scale
detrending
cubic distortion
gapfilling
ccd misalignment
horizontal displacement
inclination error
woodard effect
ccd tilt
asymmetry
ν (mHz)
ν (mHz)

background

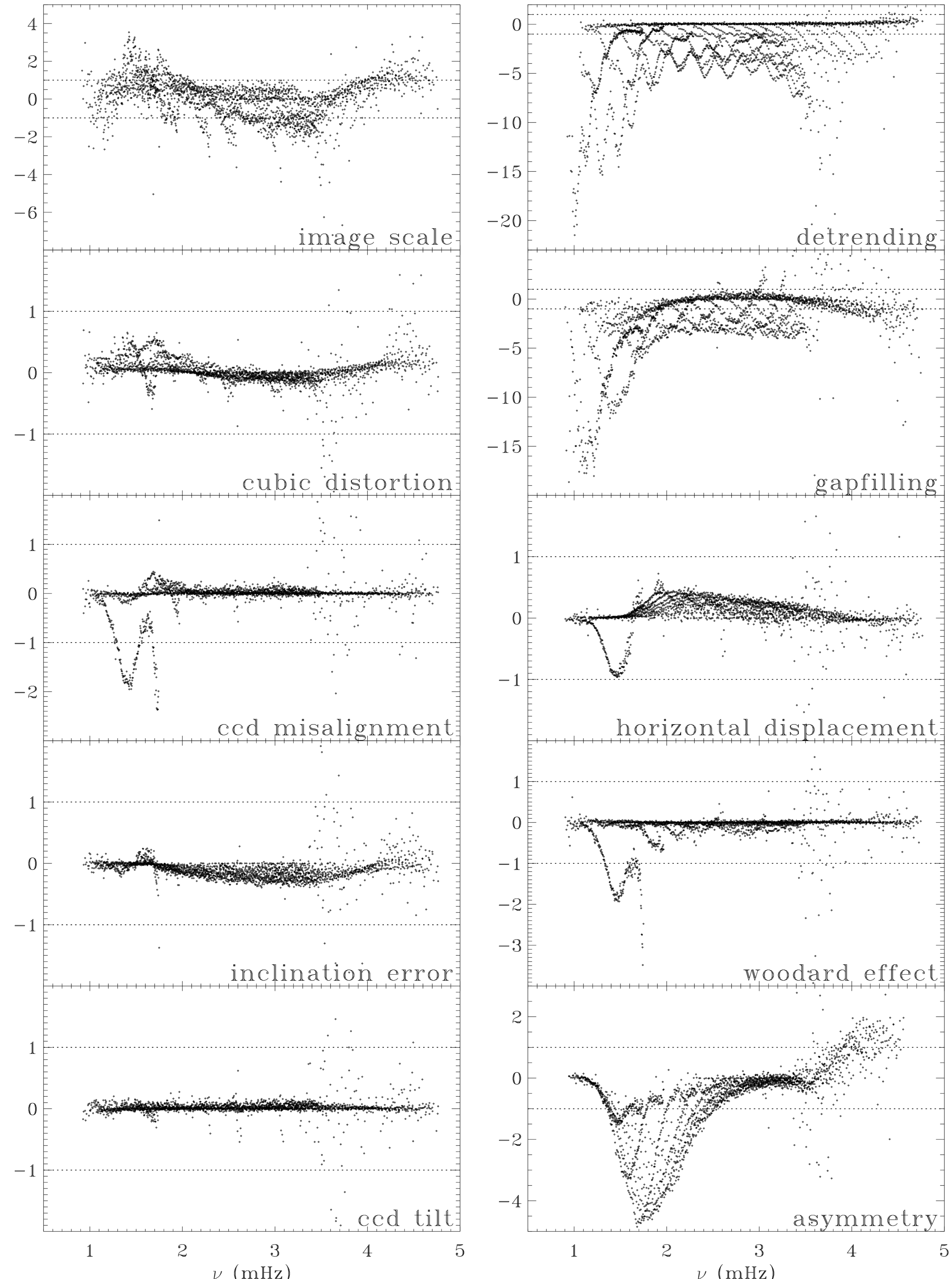


image scale
detrending
cubic distortion
gapfilling
ccd misalignment
horizontal displacement
inclination error
woodard effect
ccd tilt
asymmetry
ν (mHz)
ν (mHz)

a1

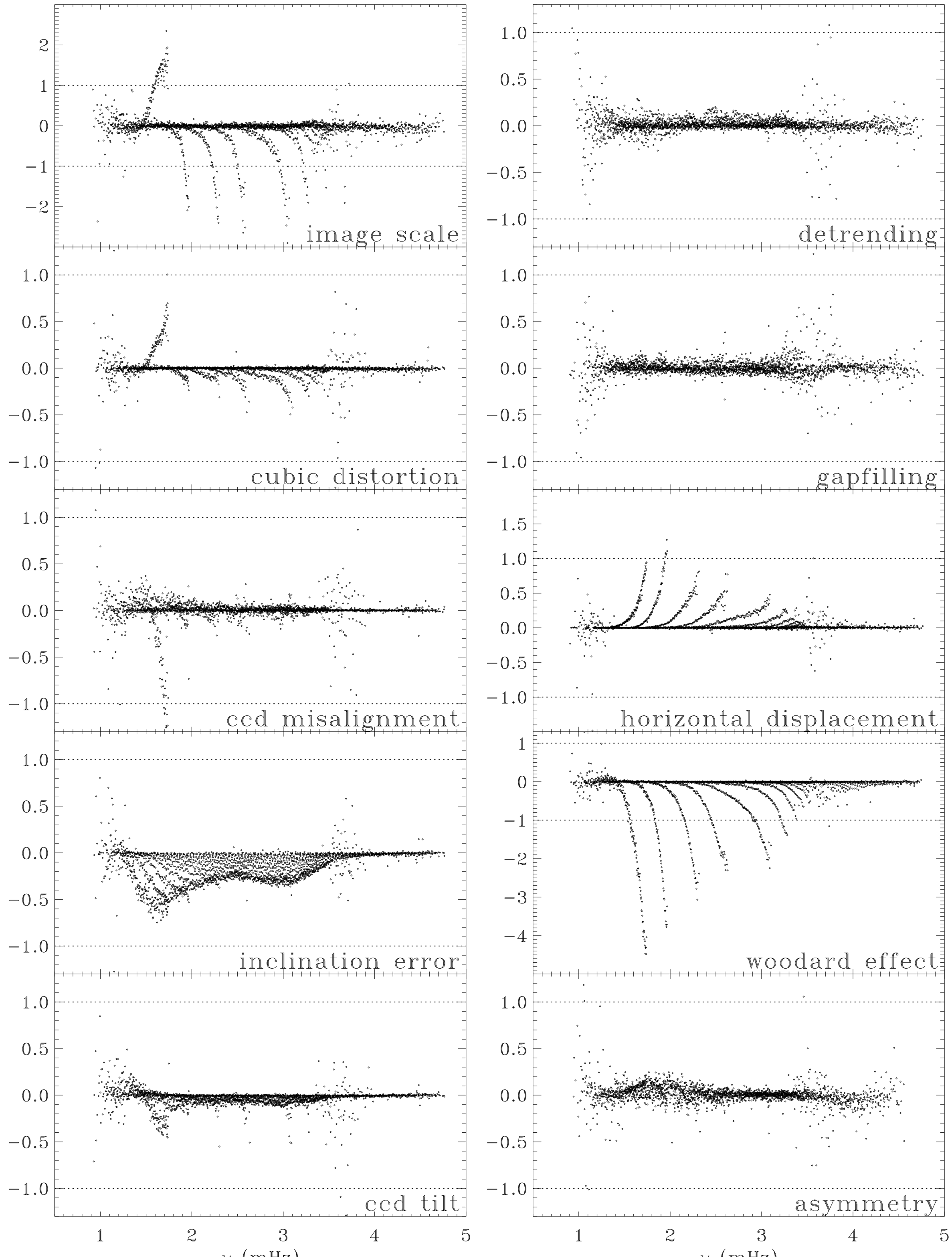
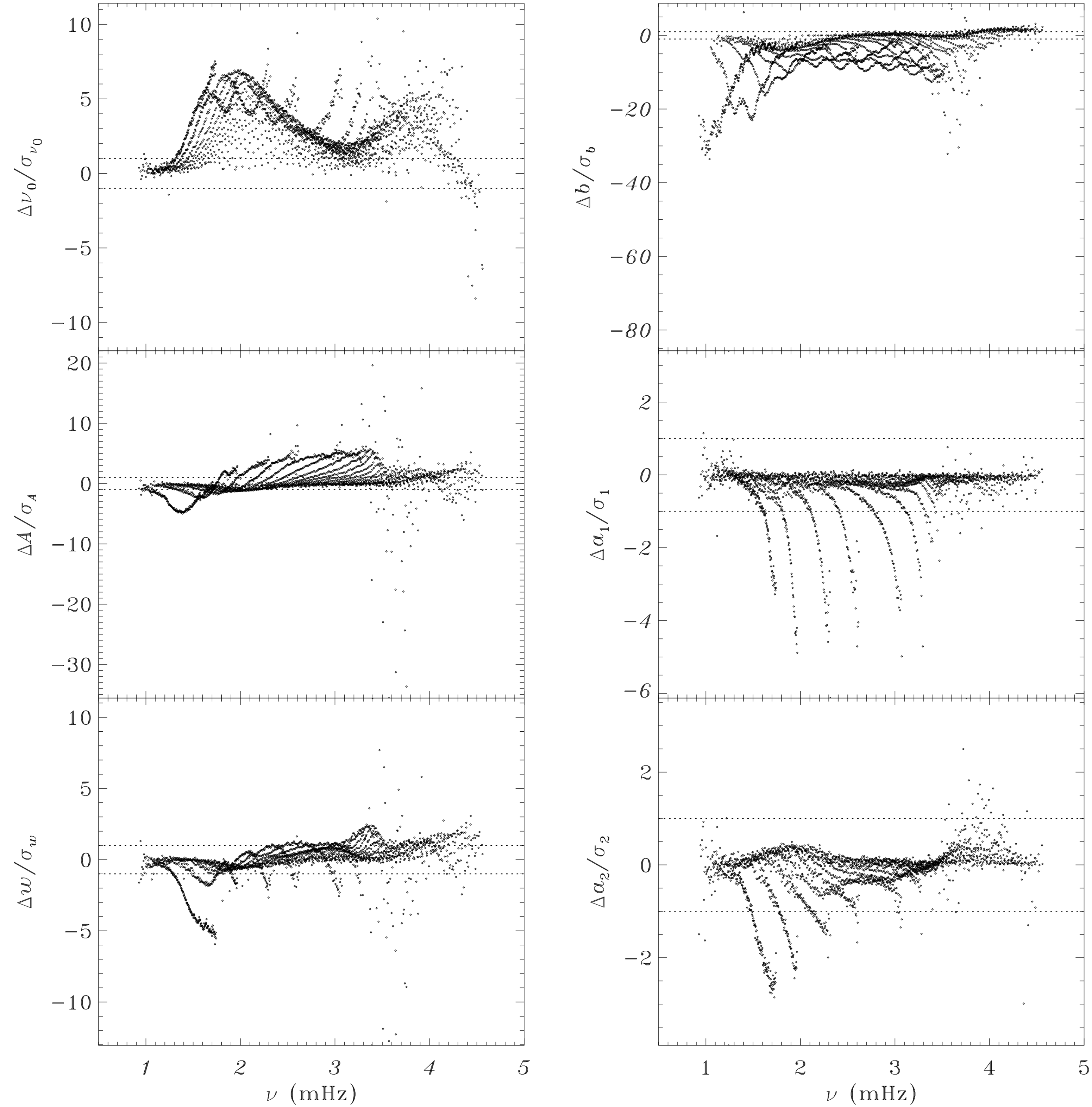
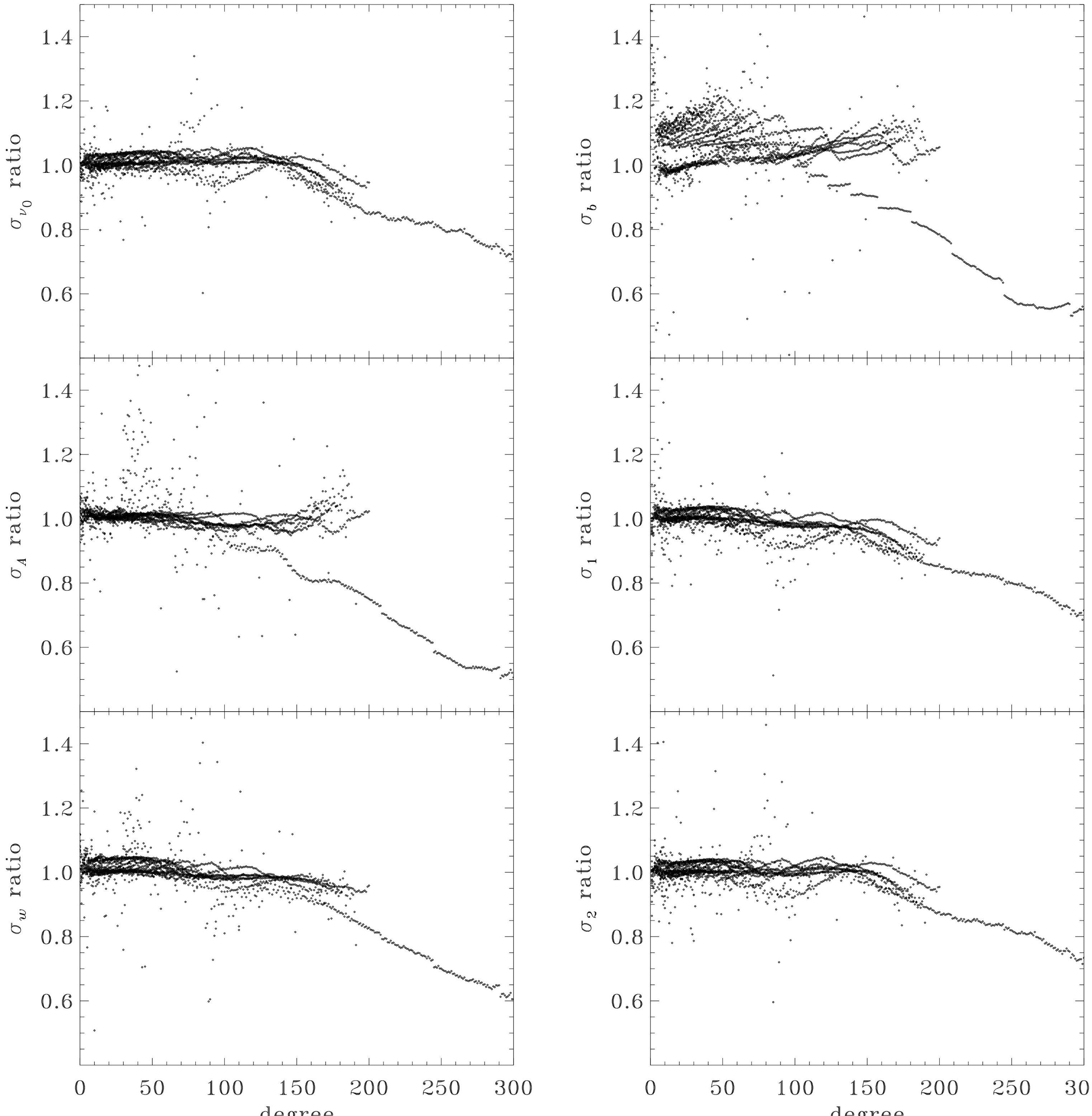


image scale
detrending
cubic distortion
gapfilling
ccd misalignment
horizontal displacement
inclination error
woodard effect
ccd tilt
asymmetry
ν (mHz)
ν (mHz)

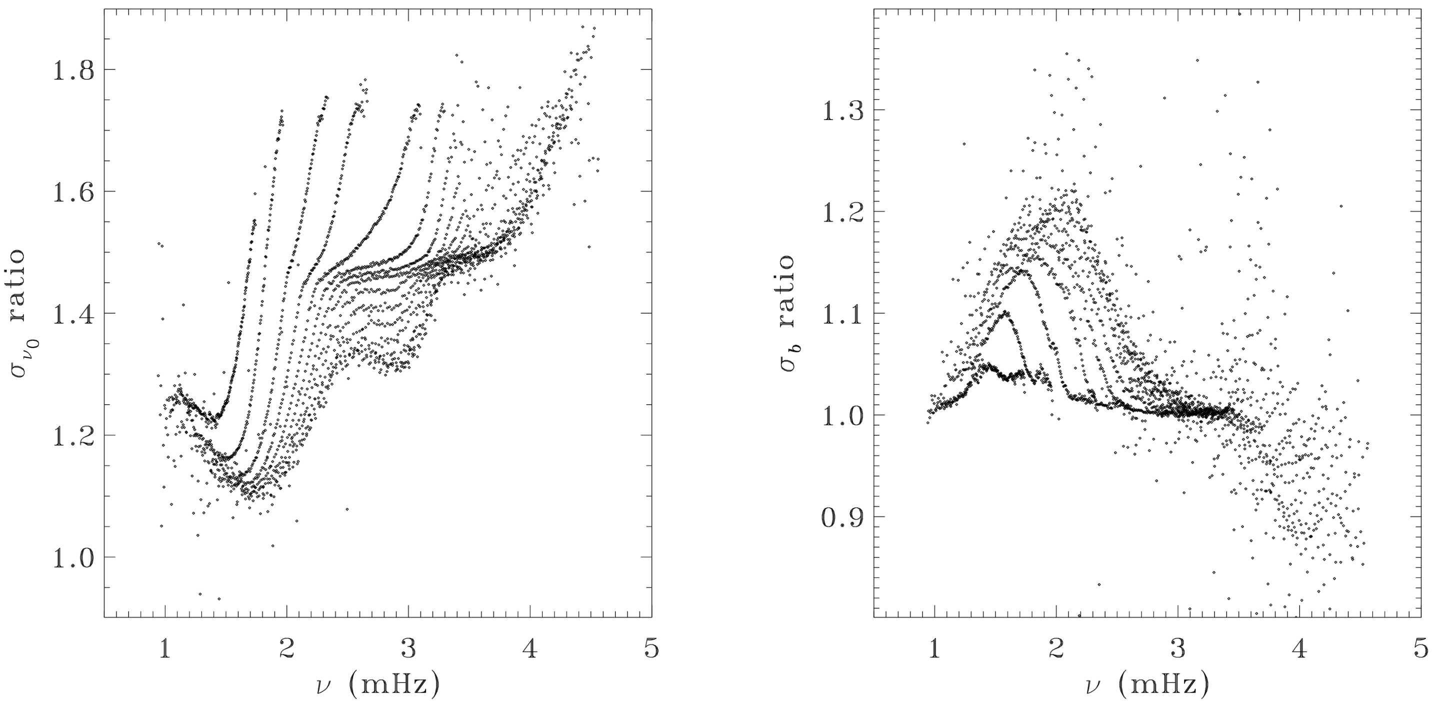
final - initial

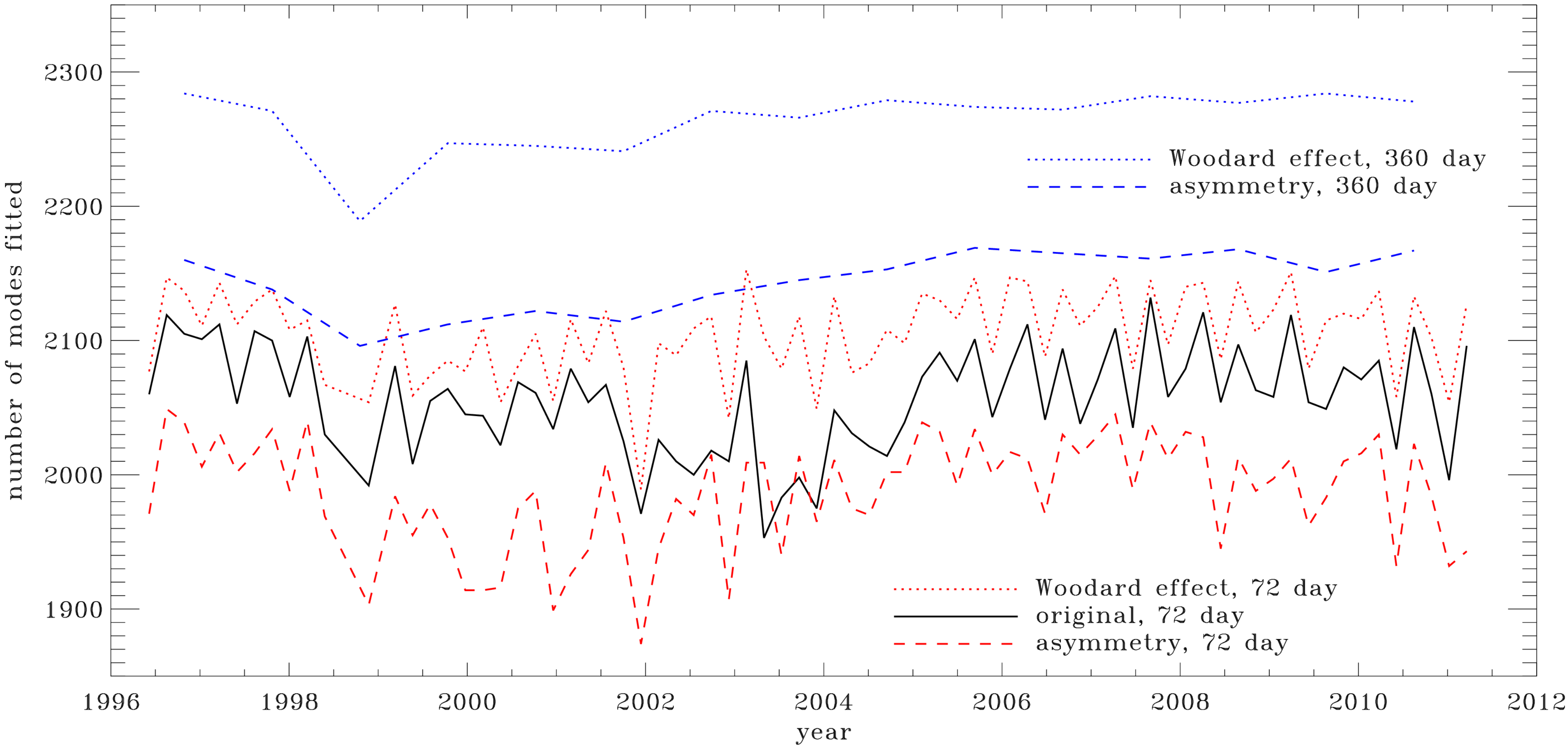


error reduction



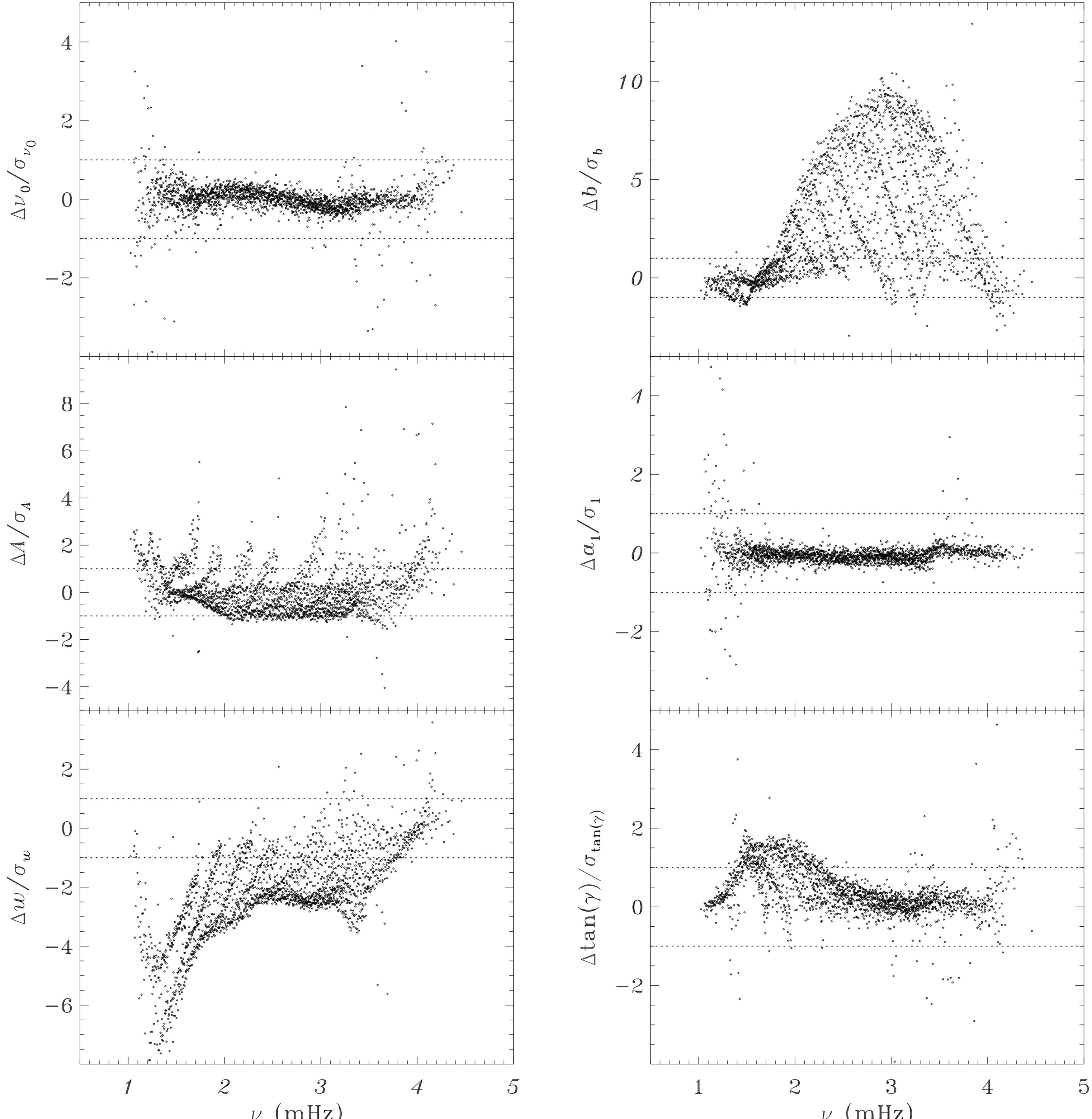
error magnification



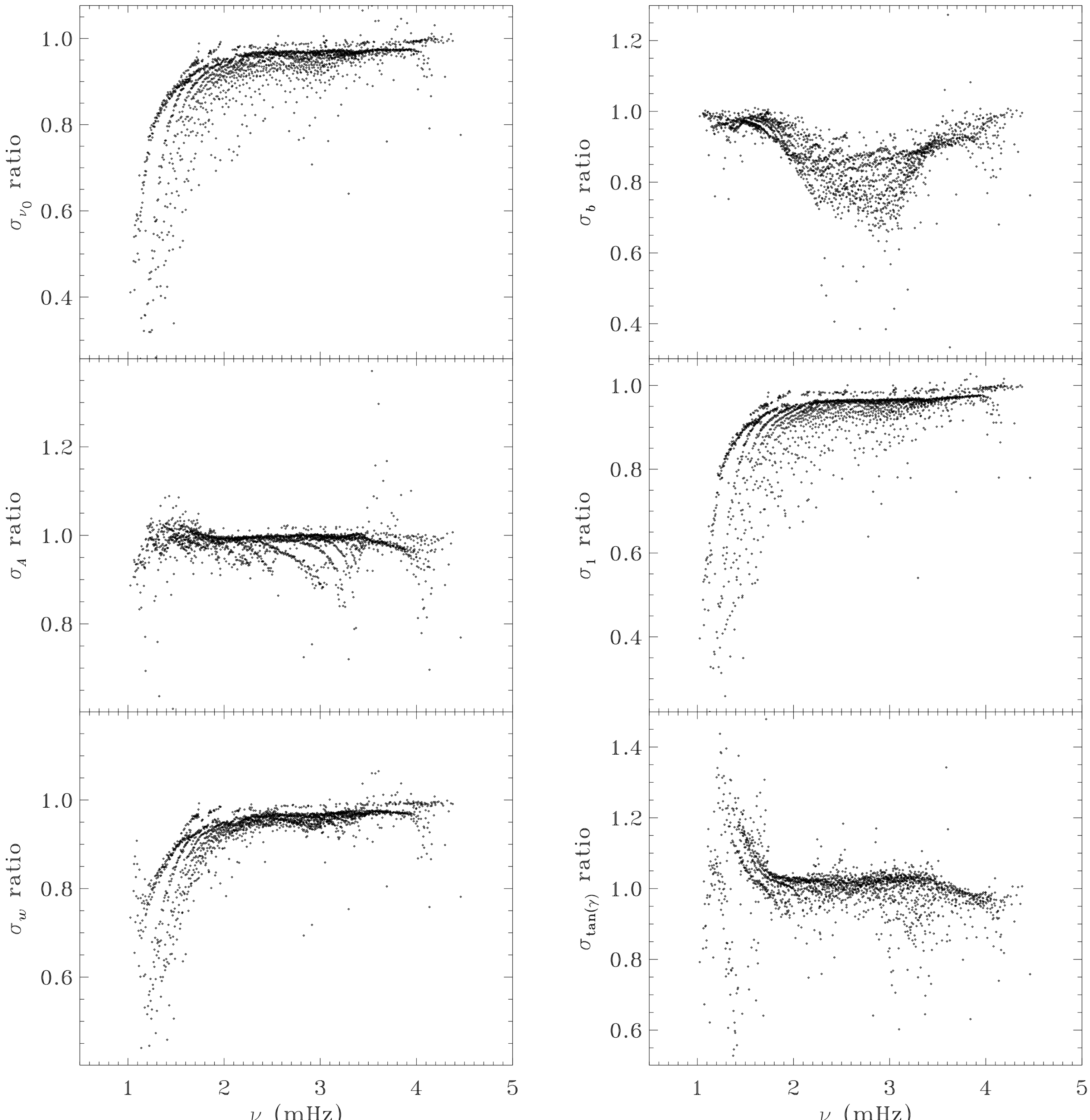


Woodard effect, 360 day
asymmetry, 360 day
Woodard effect, 72 day
original, 72 day
asymmetry, 72 day
number of modes fitted
year
2300
2200
2100
2000
1900
1996
1998
2000
2002
2004
2006
2008
2010
2012

360d fits

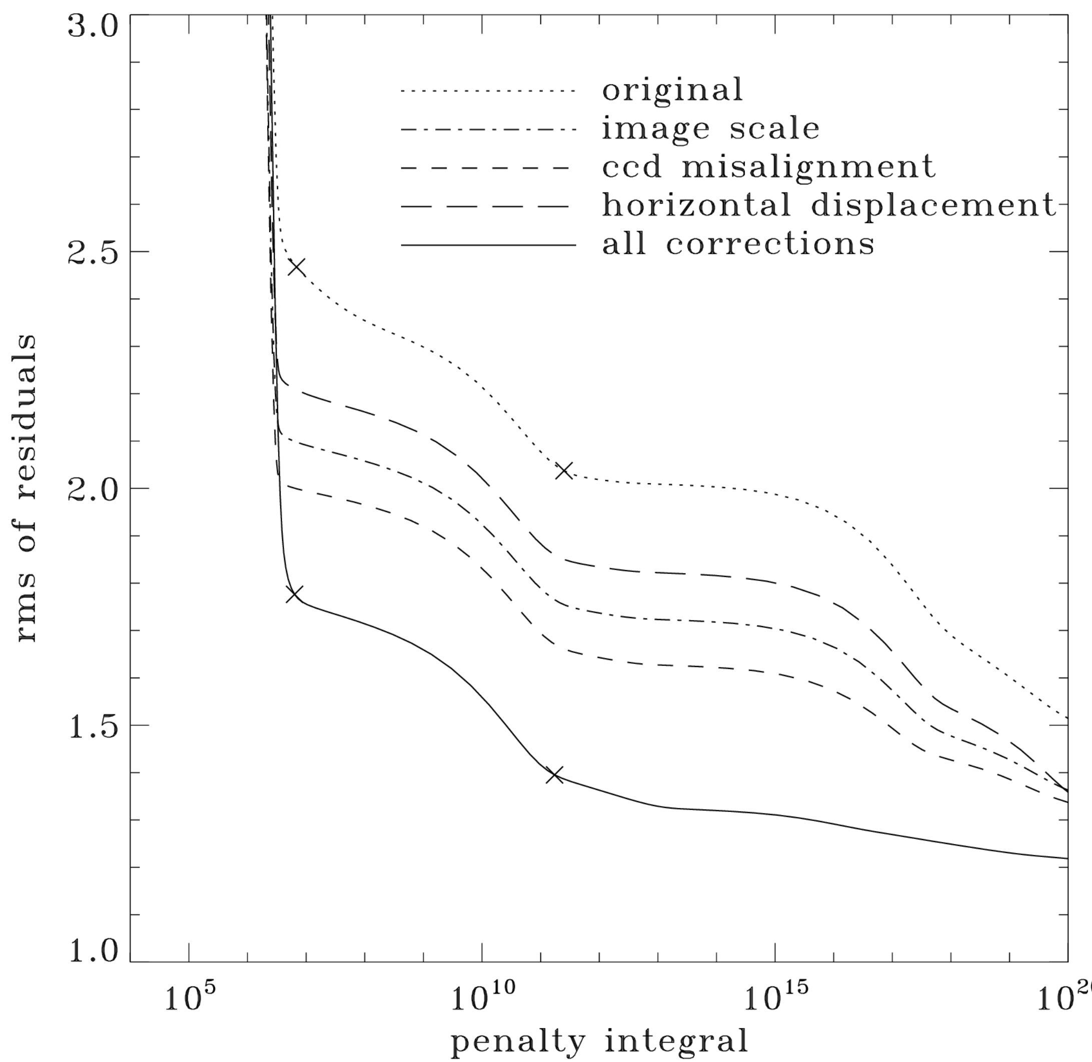


360d error

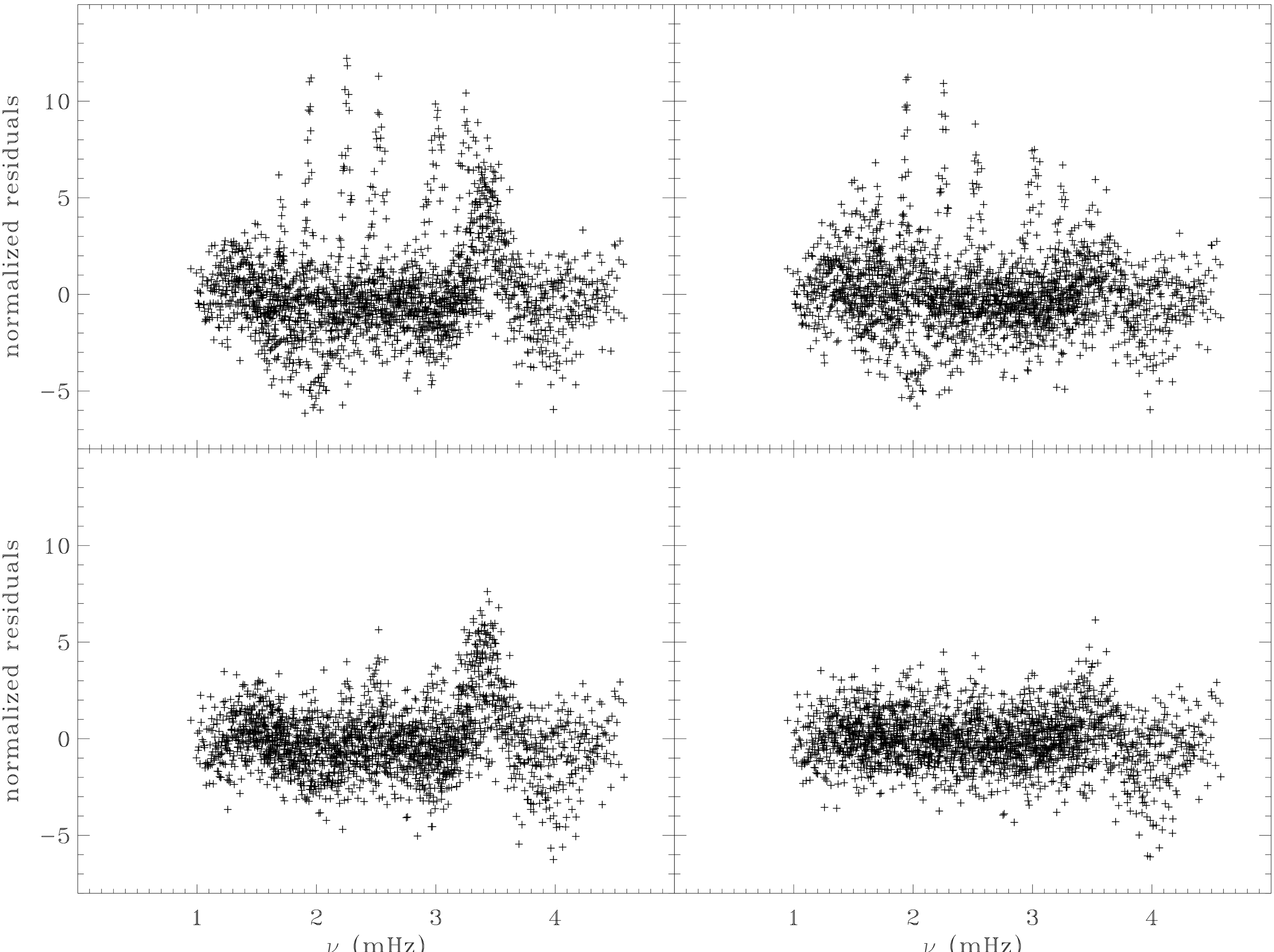


σ_{ν_0} ratio
σ_b ratio
σ_A ratio
σ_1 ratio
σ_w ratio
$\sigma_{\tan(\gamma)}$ ratio
ν (mHz)
ν (mHz)

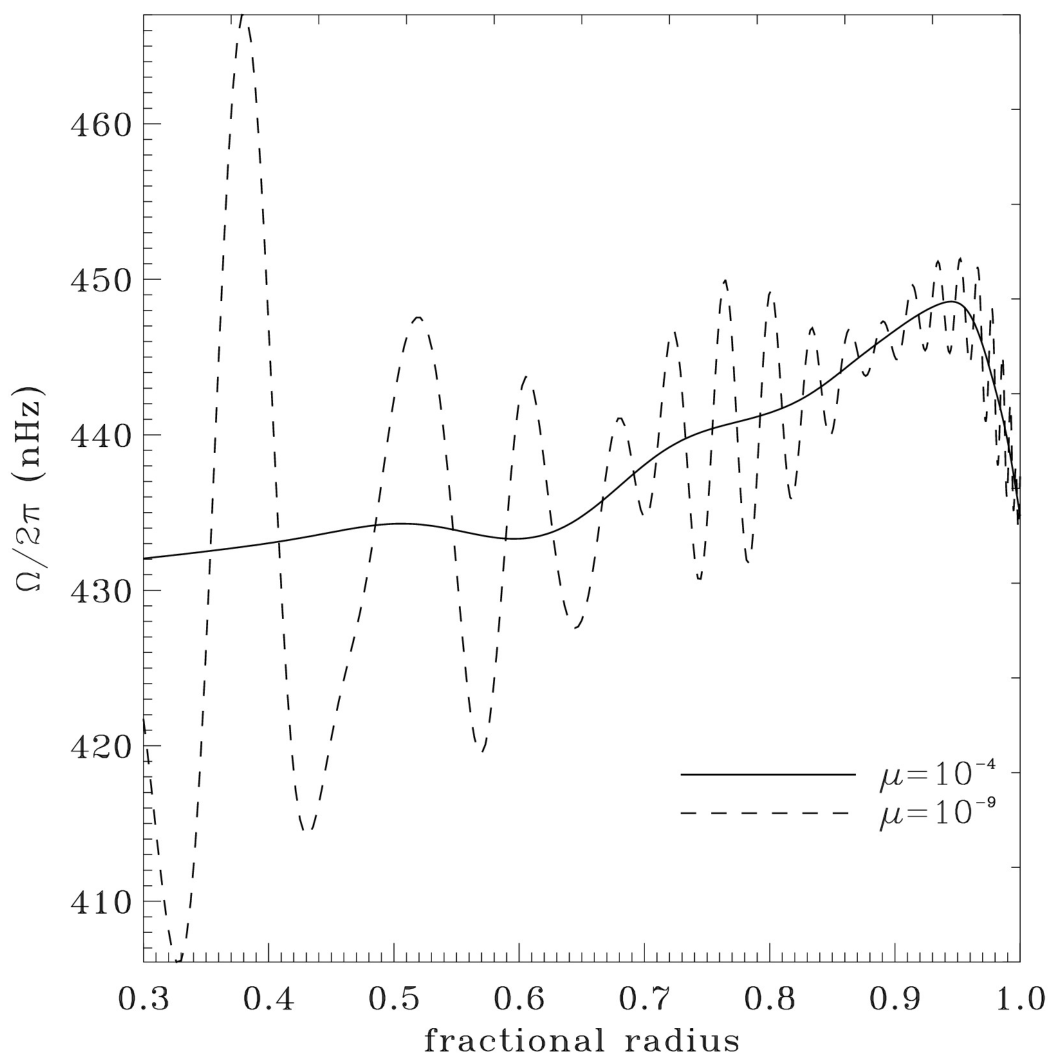
Systematic Errors



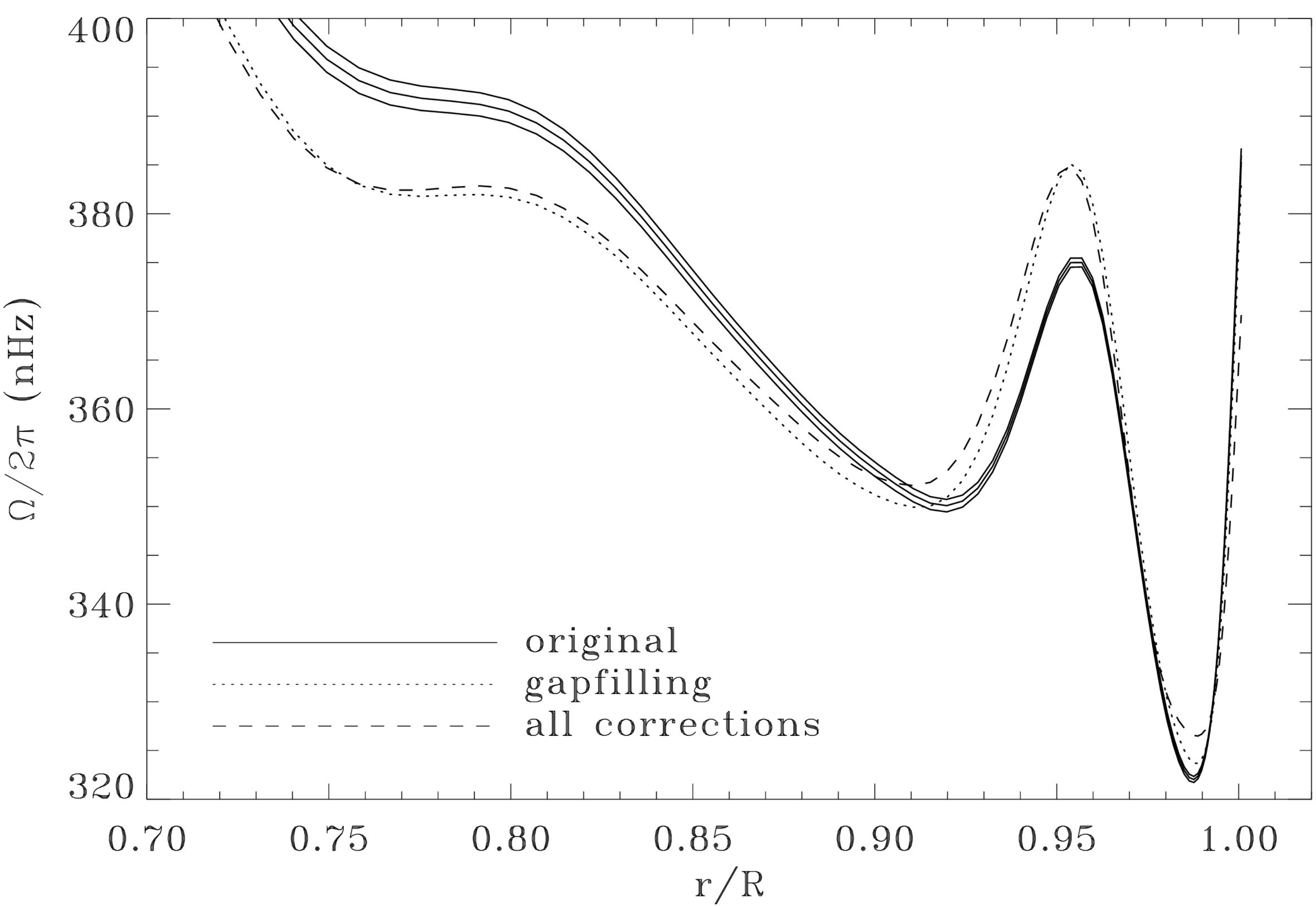
3.0
2.5
2.0
1.5
1.0
10^5
10^{10}
10^{15}
10^{20}
penalty integral
rms of residuals
original
image scale
ccd misalignment
horizontal displacement
all corrections



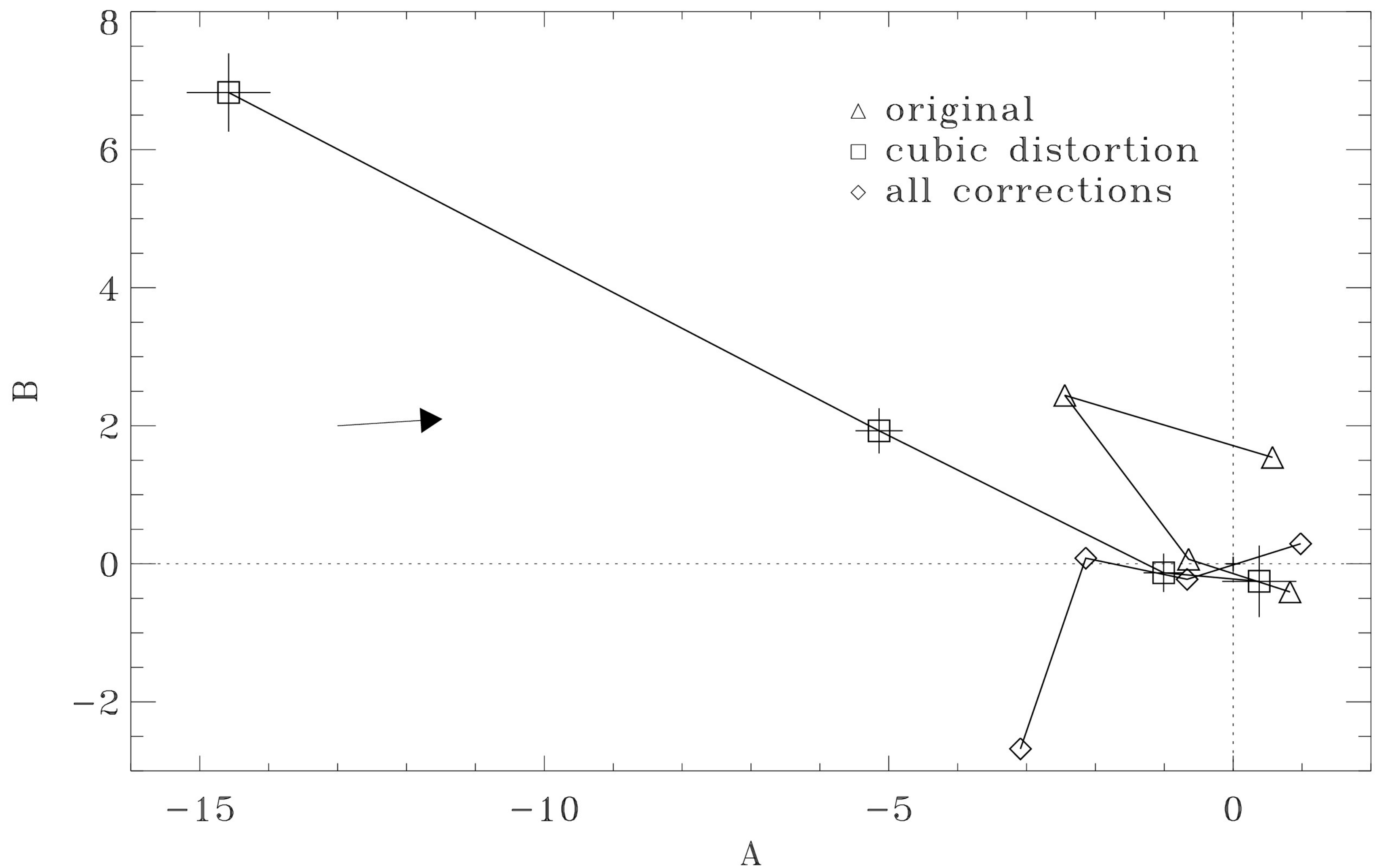
normalized residuals
10
5
0
−5
normalized residuals
10
5
0
−5
1
2
3
4
ν (mHz)
1
2
3
4
ν (mHz)



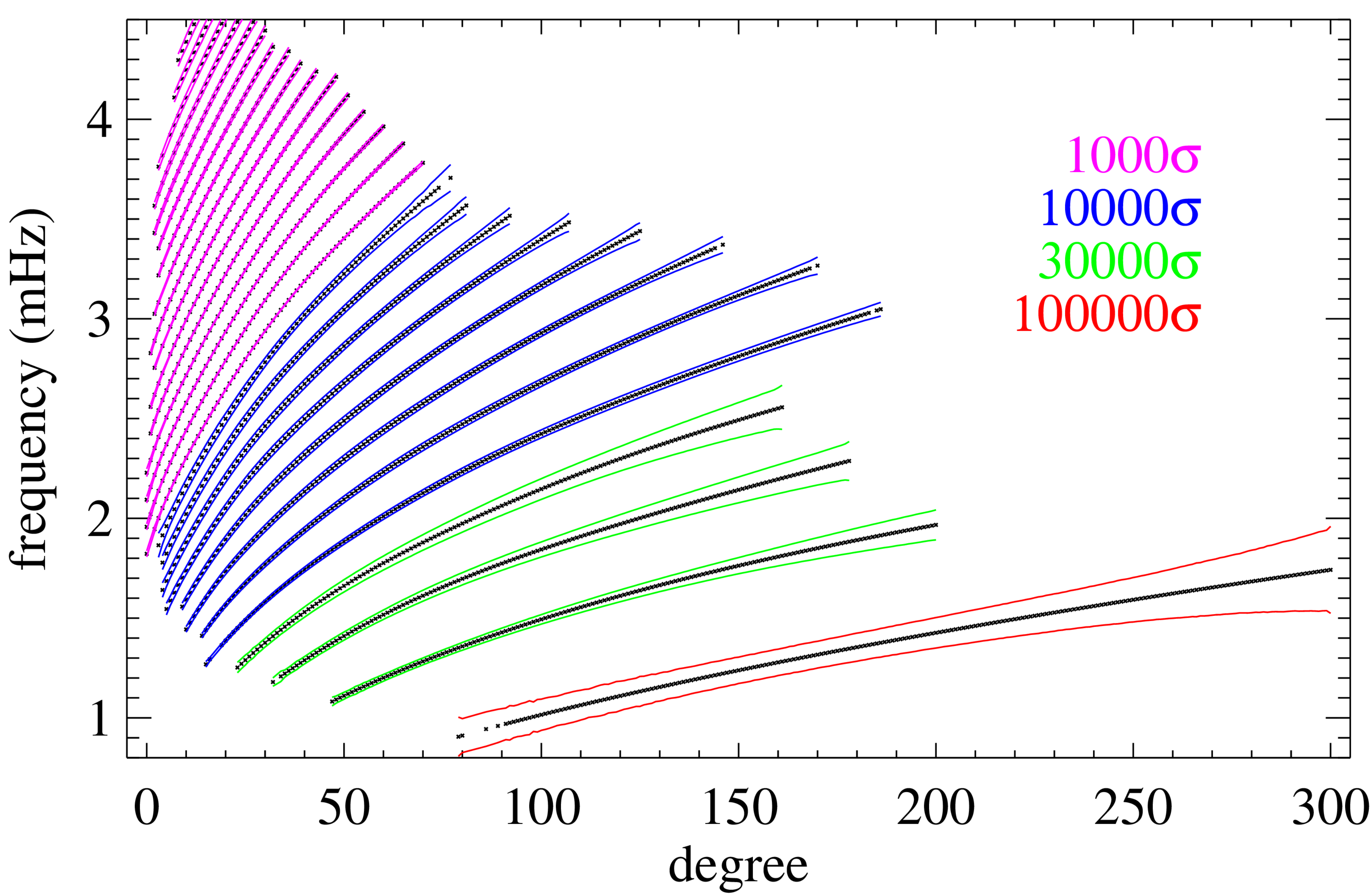
460
450
440
430
420
410
Ω/2π (nHz)
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
fractional radius
μ=10⁻⁴
μ=10⁻⁹



400
380
360
340
320
Ω/2π (nHz)
0.70
0.75
0.80
0.85
0.90
0.95
1.00
r/R
original
gapfilling
all corrections

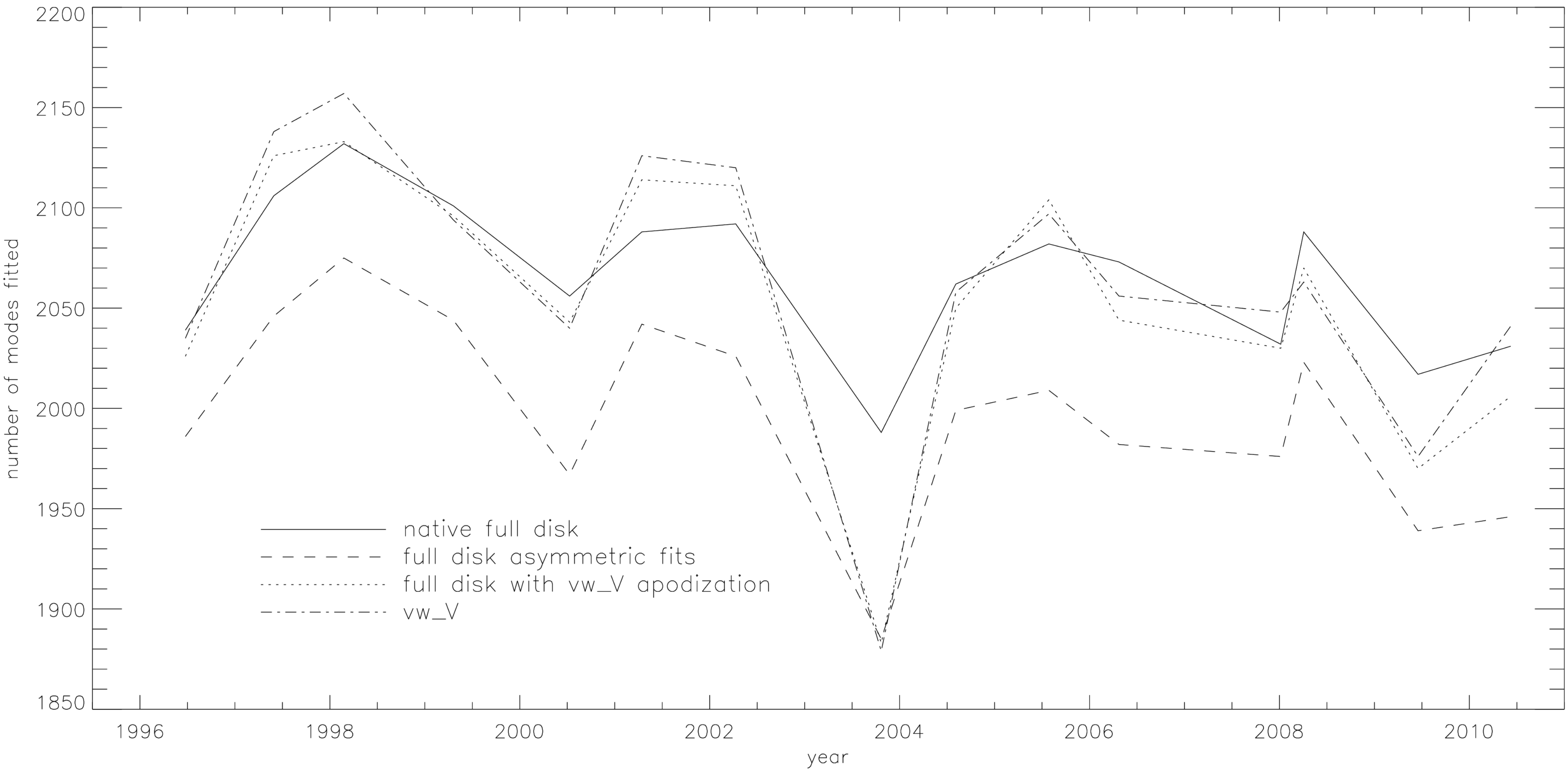


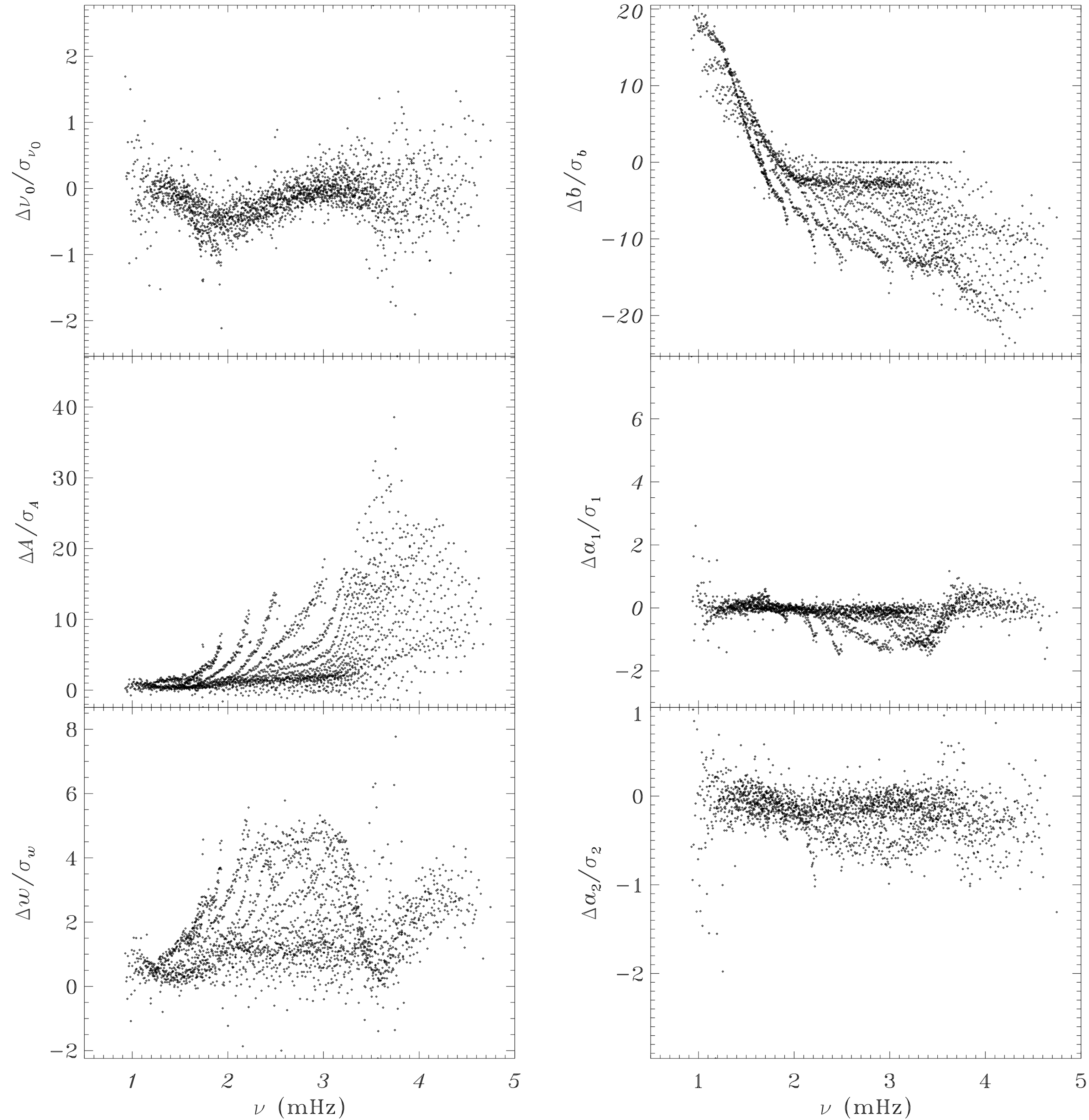
$$f(t) = A\sin(\omega_{\mathrm{yr}}t) + B\cos(\omega_{\mathrm{yr}}t) + Ct + D$$



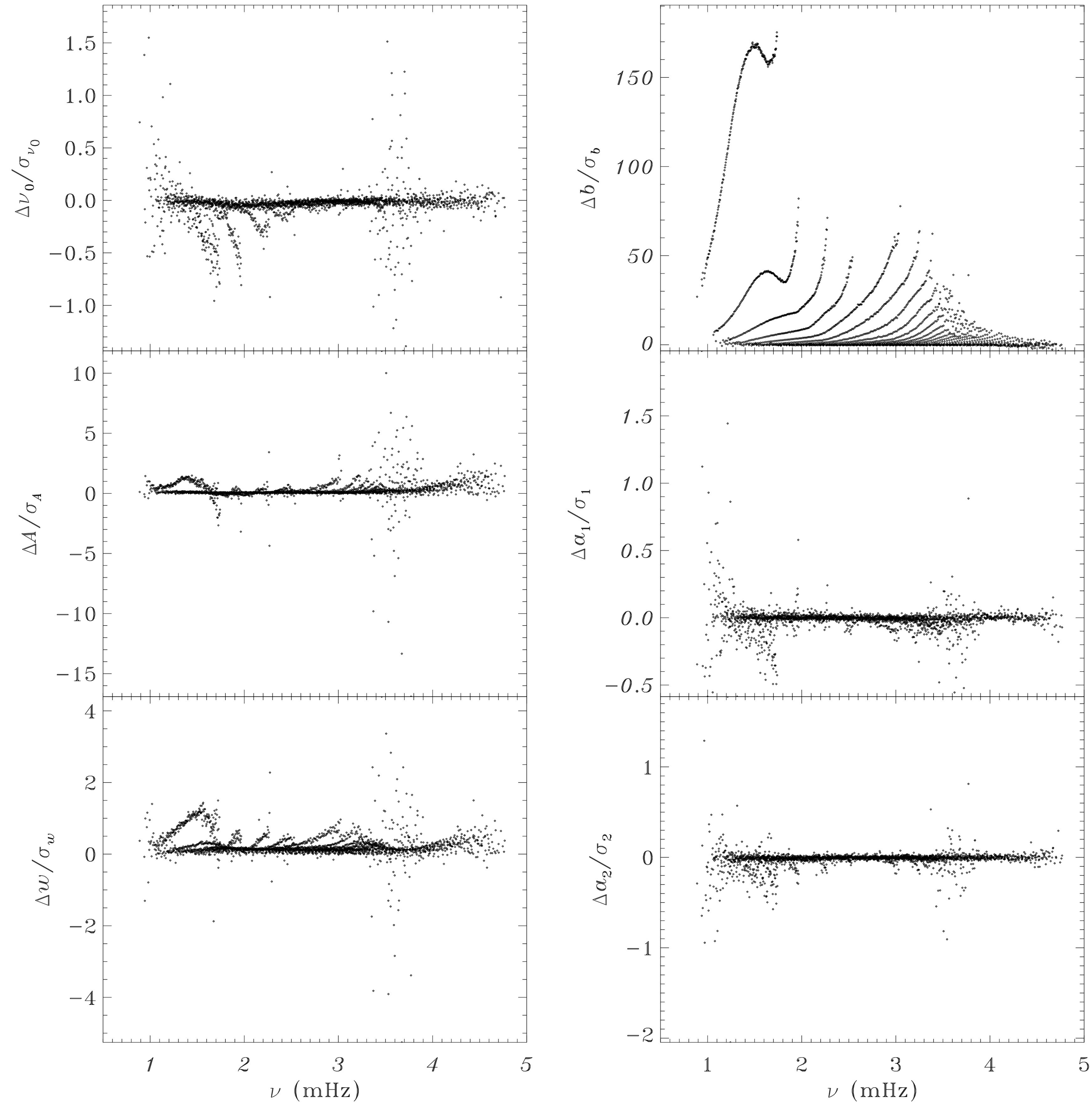
1000σ
10000σ
30000σ
100000σ
frequency (mHz)
1
2
3
4
degree
0
50
100
150
200
250
300

Full Disk Results from MDI

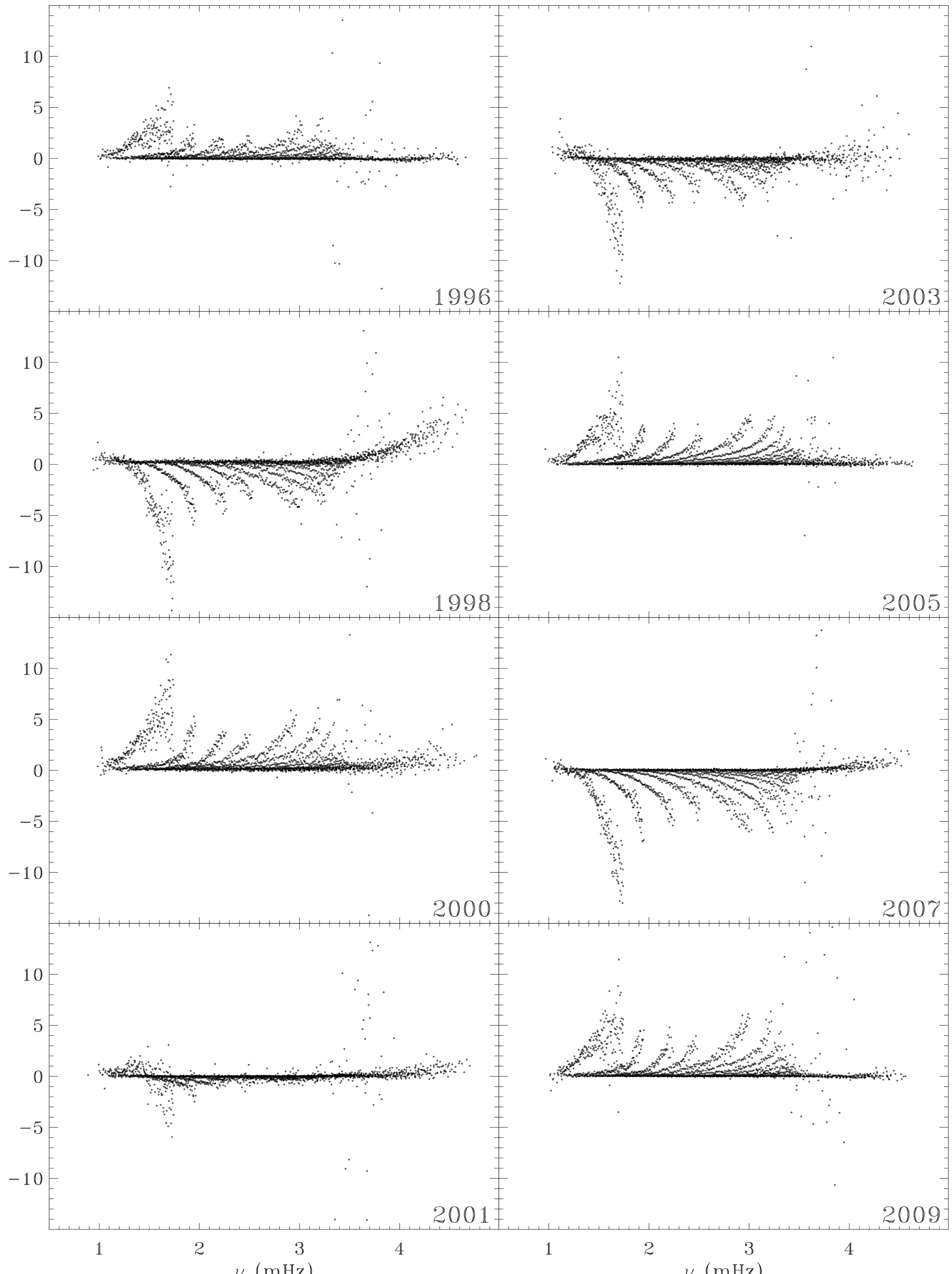




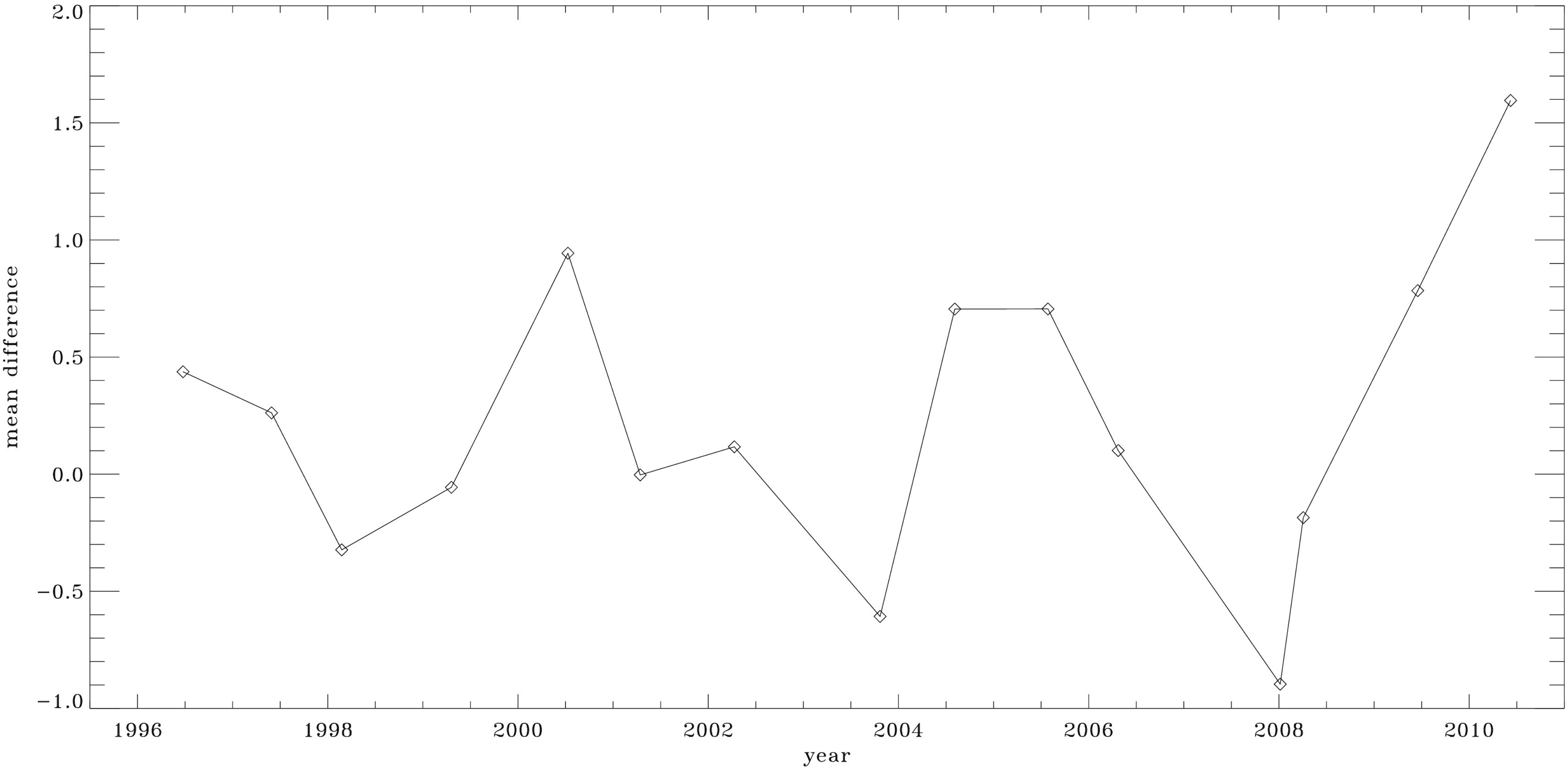
$\Delta\nu_0/\sigma_{\nu_0}$
$\Delta b/\sigma_b$
$\Delta A/\sigma_A$
$\Delta a_1/\sigma_1$
$\Delta w/\sigma_w$
$\Delta a_2/\sigma_2$
ν (mHz)
ν (mHz)



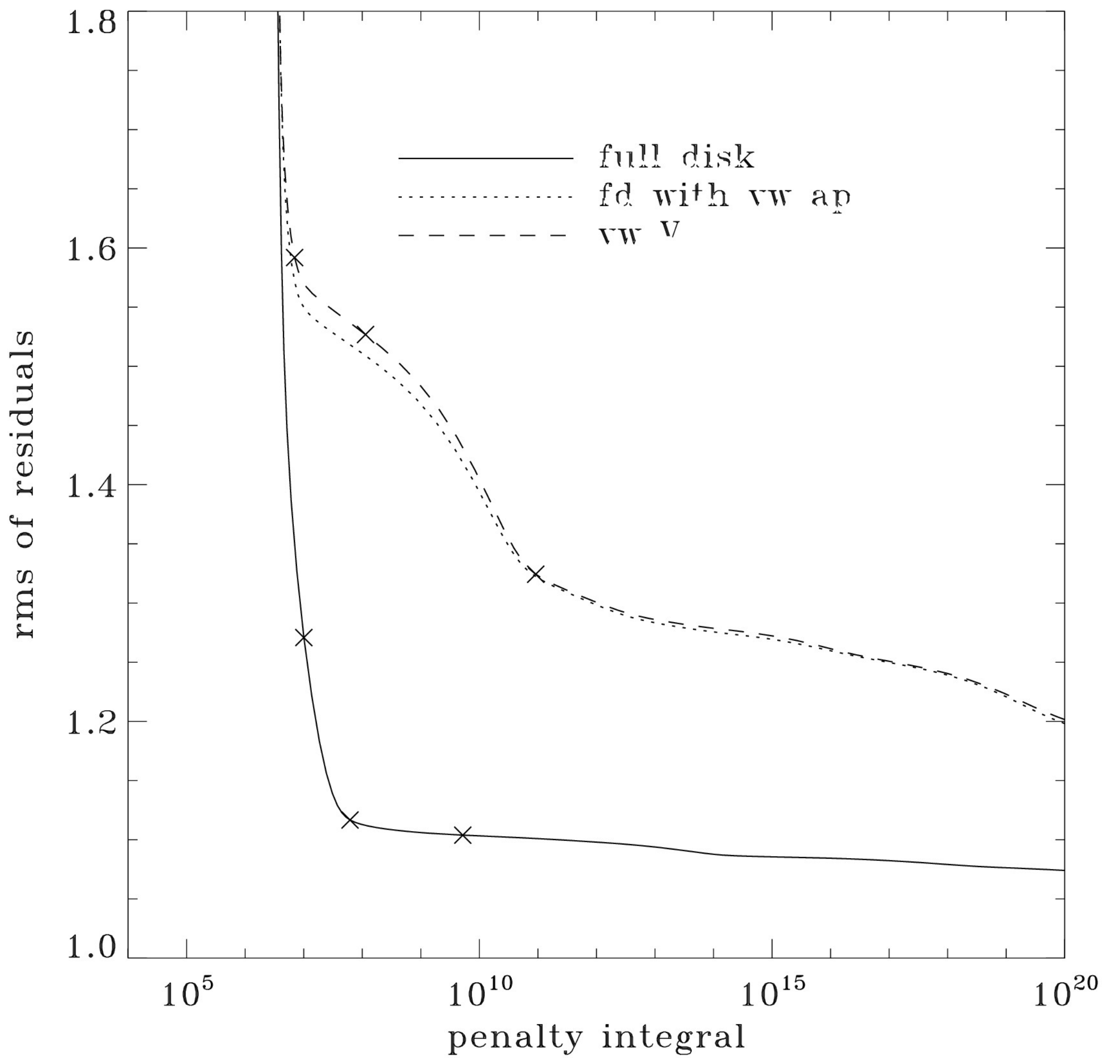
$\Delta\nu_0/\sigma_{\nu_0}$
$\Delta b/\sigma_b$
$\Delta A/\sigma_A$
$\Delta a_1/\sigma_1$
$\Delta w/\sigma_w$
$\Delta a_2/\sigma_2$
ν (mHz)
ν (mHz)



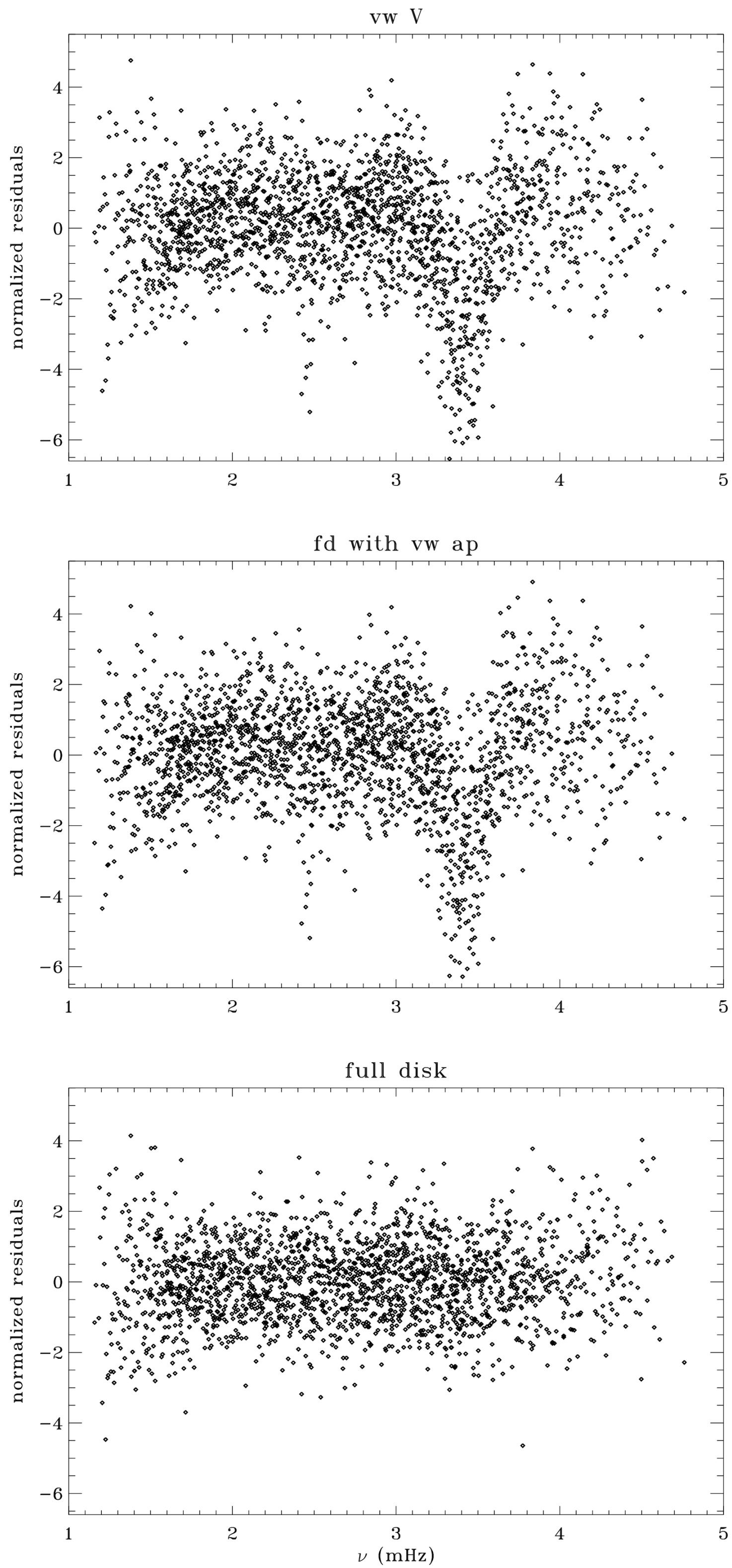
10
5
0
−5
−10
1996
2003
1998
2005
2000
2007
2001
2009
1
2
3
4
ν (mHz)

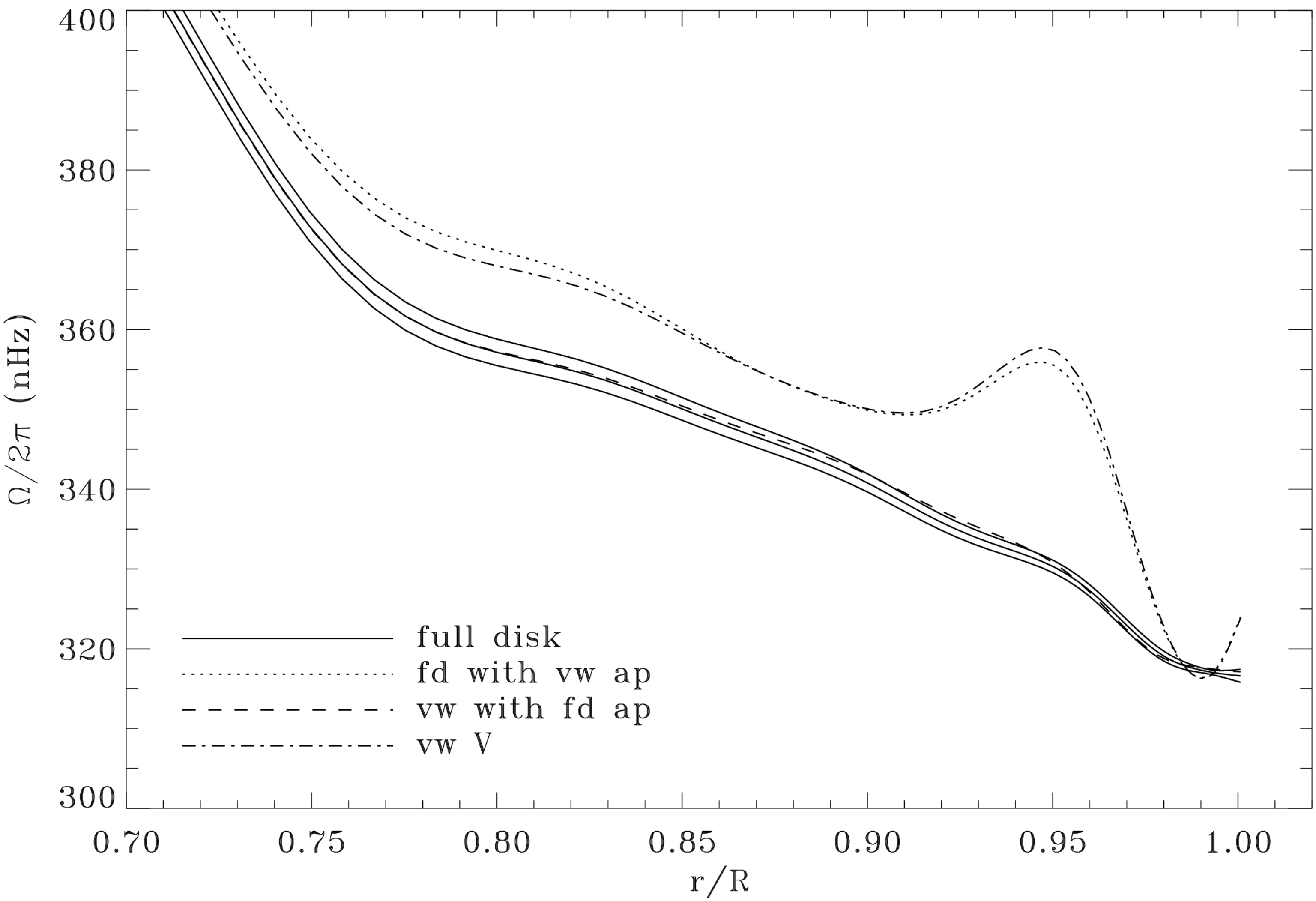


2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
mean difference
1996
1998
2000
2002
2004
2006
2008
2010
year



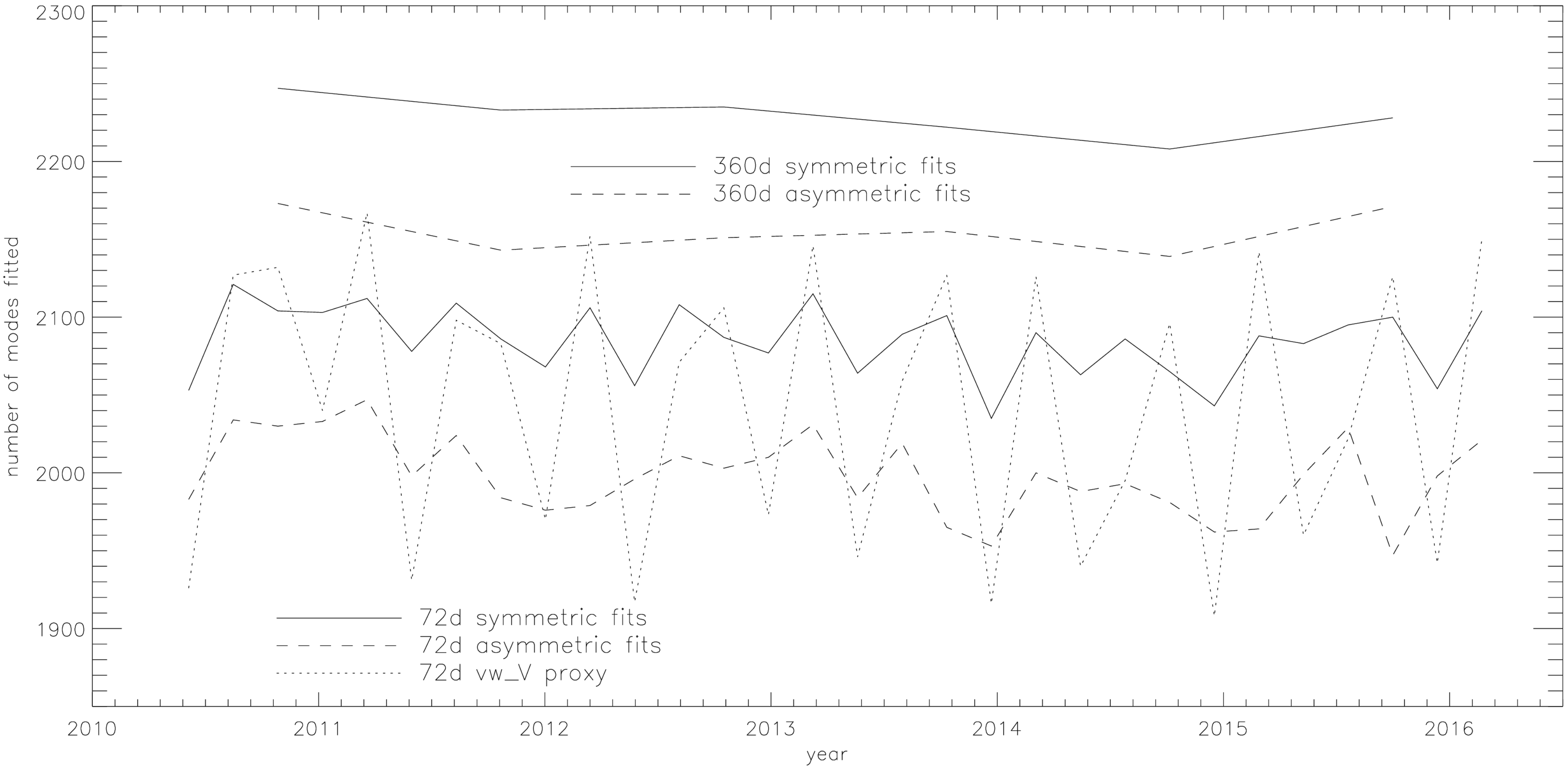
1.8
1.6
1.4
1.2
1.0
rms of residuals
10^5
10^{10}
10^{15}
10^{20}
penalty integral
full disk
fd with vw ap
vw V


vw V
fd with vw ap
full disk
normalized residuals
ν (mHz)

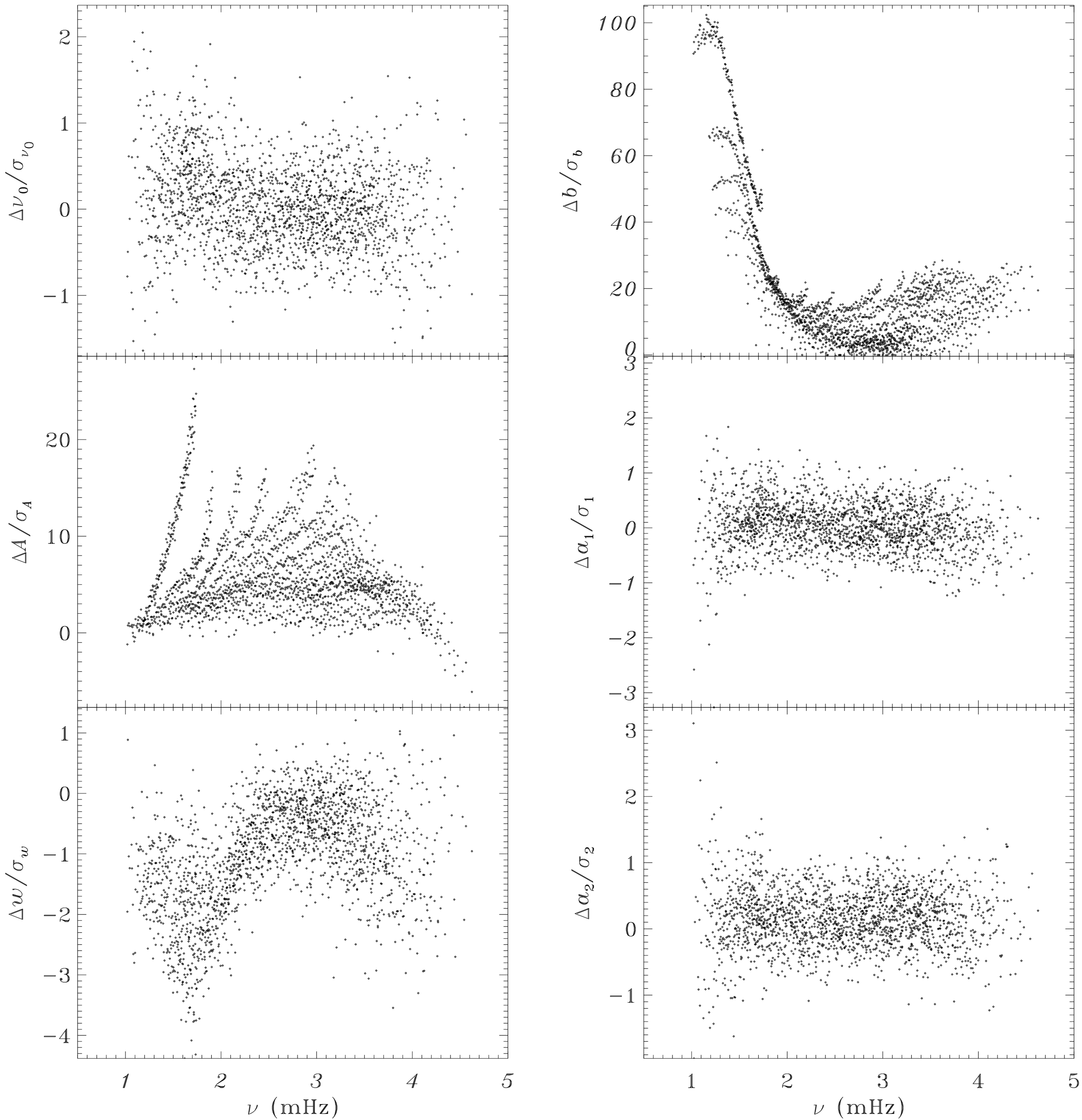


400
380
360
340
320
300
Ω/2π (nHz)
0.70
0.75
0.80
0.85
0.90
0.95
1.00
r/R
full disk
fd with vw ap
vw with fd ap
vw V

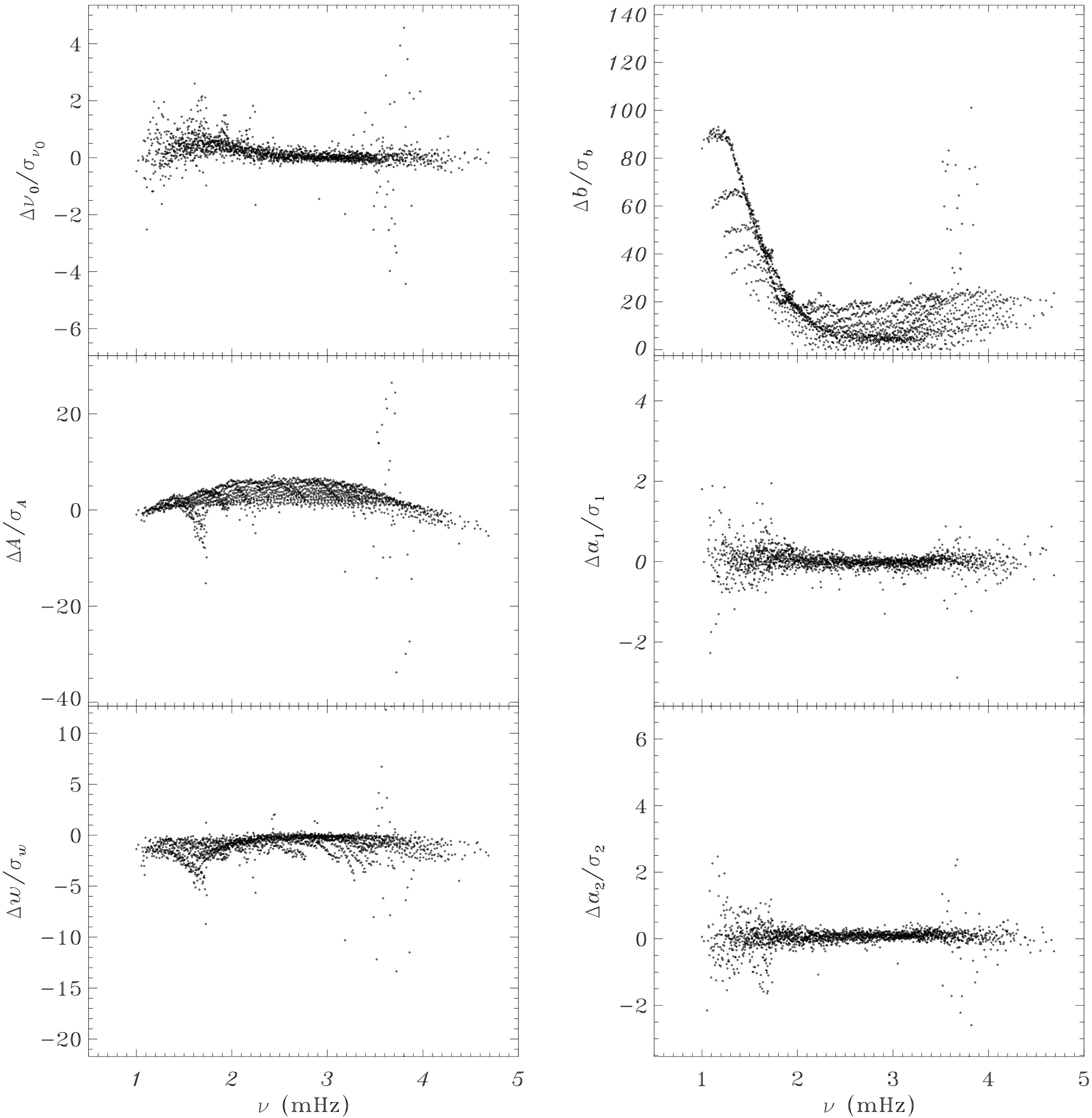
and HMI



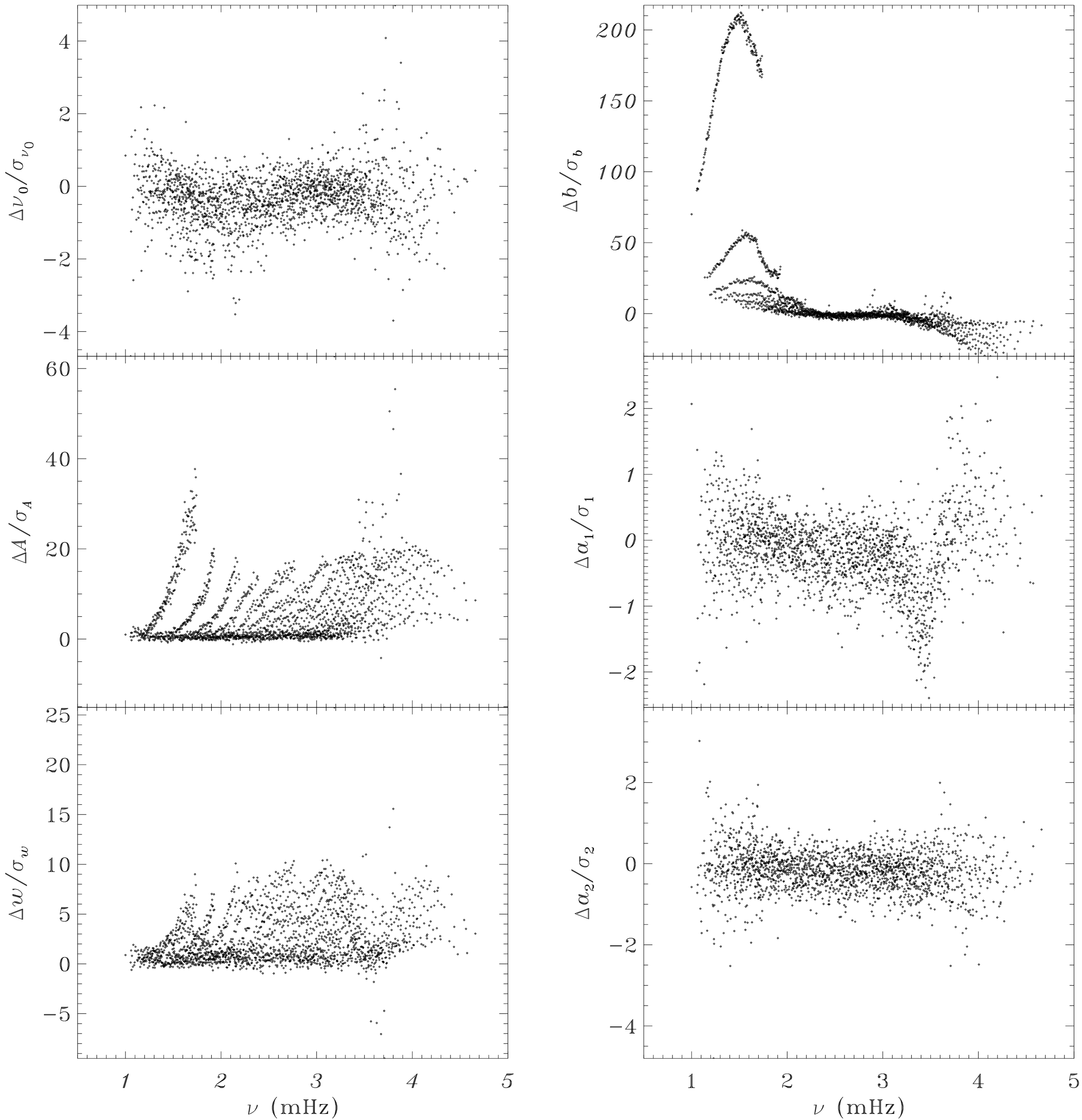
2300
2200
2100
2000
1900
number of modes fitted
360d symmetric fits
360d asymmetric fits
72d symmetric fits
72d asymmetric fits
72d vw_V proxy
2010
2011
2012
2013
2014
2015
2016
year



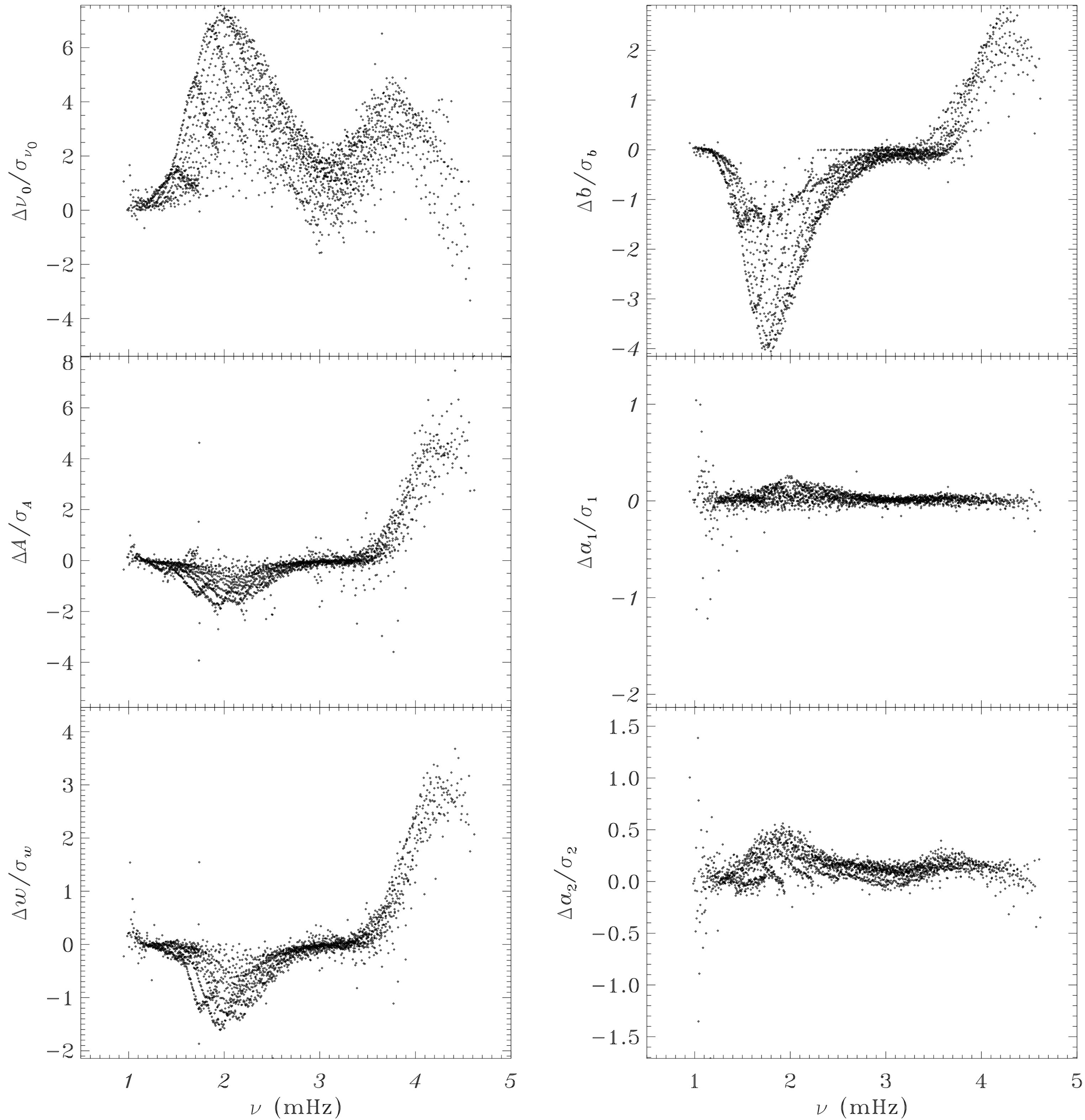
$\Delta\nu_0/\sigma_{\nu_0}$
$\Delta b/\sigma_b$
$\Delta A/\sigma_A$
$\Delta a_1/\sigma_1$
$\Delta w/\sigma_w$
$\Delta a_2/\sigma_2$
ν (mHz)
ν (mHz)



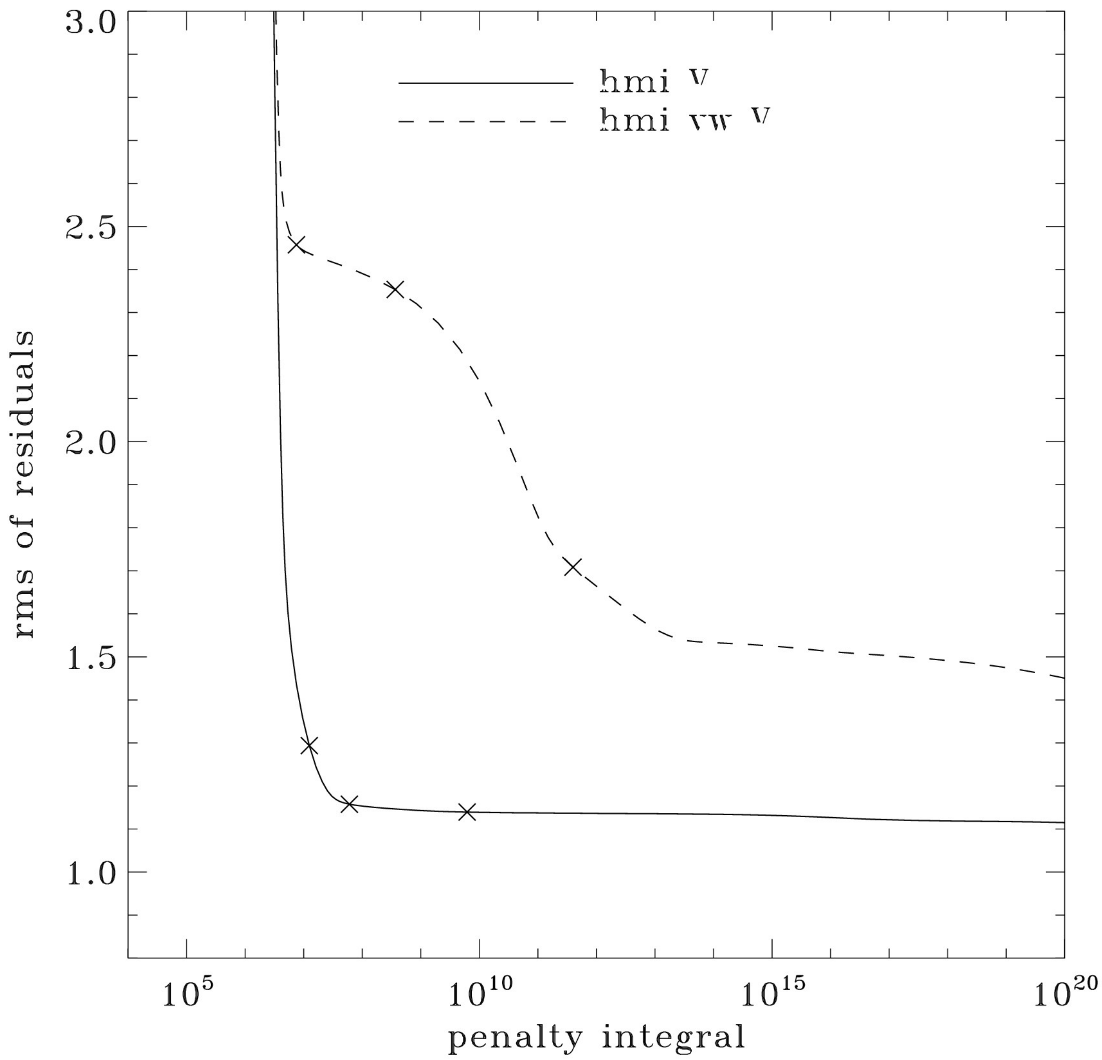
$\Delta\nu_0/\sigma_{\nu_0}$
$\Delta A/\sigma_A$
$\Delta w/\sigma_w$
$\Delta b/\sigma_b$
$\Delta a_1/\sigma_1$
$\Delta a_2/\sigma_2$
ν (mHz)
ν (mHz)



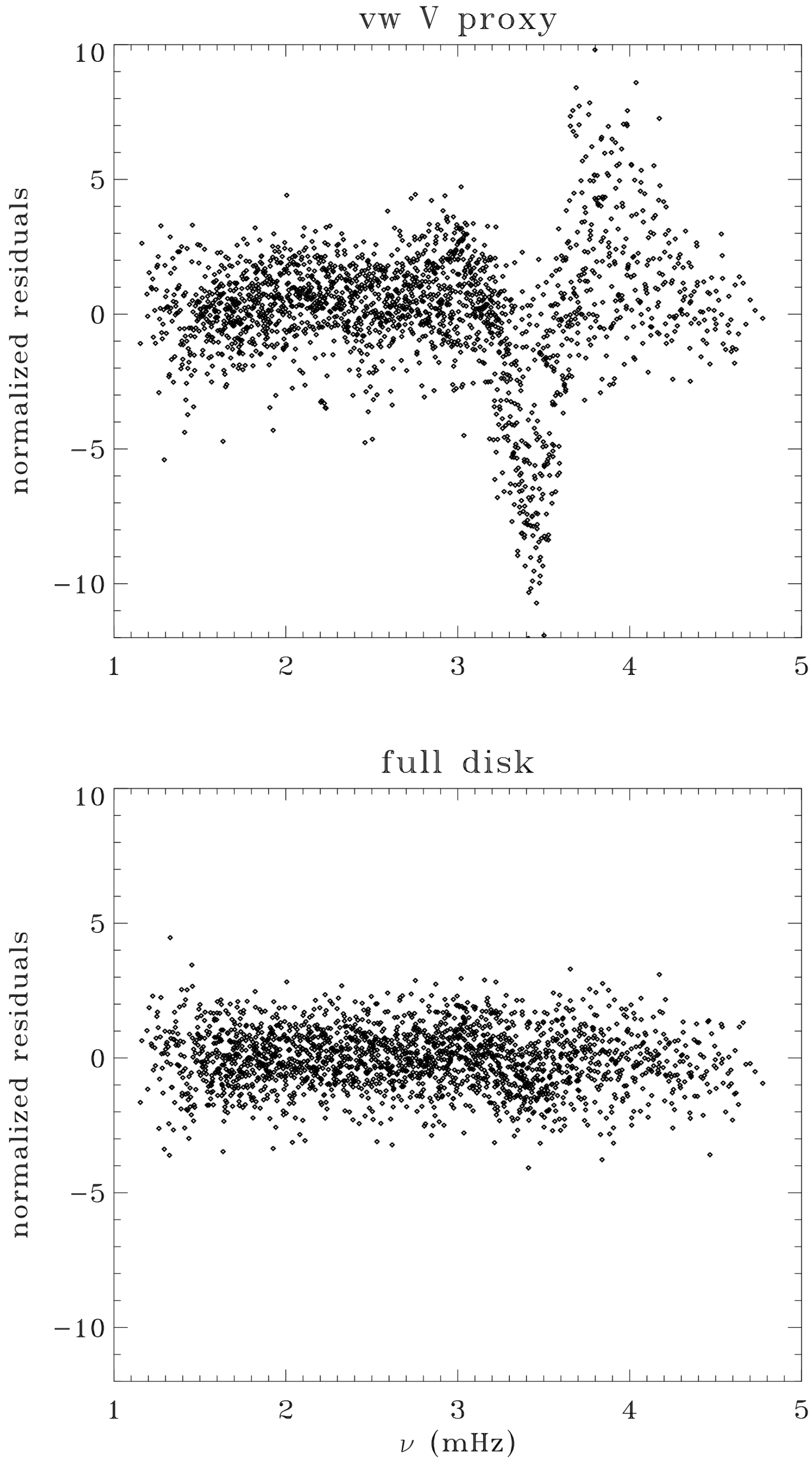
$\Delta\nu_0/\sigma_{\nu_0}$
$\Delta b/\sigma_b$
$\Delta A/\sigma_A$
$\Delta a_1/\sigma_1$
$\Delta w/\sigma_w$
$\Delta a_2/\sigma_2$
ν (mHz)
ν (mHz)



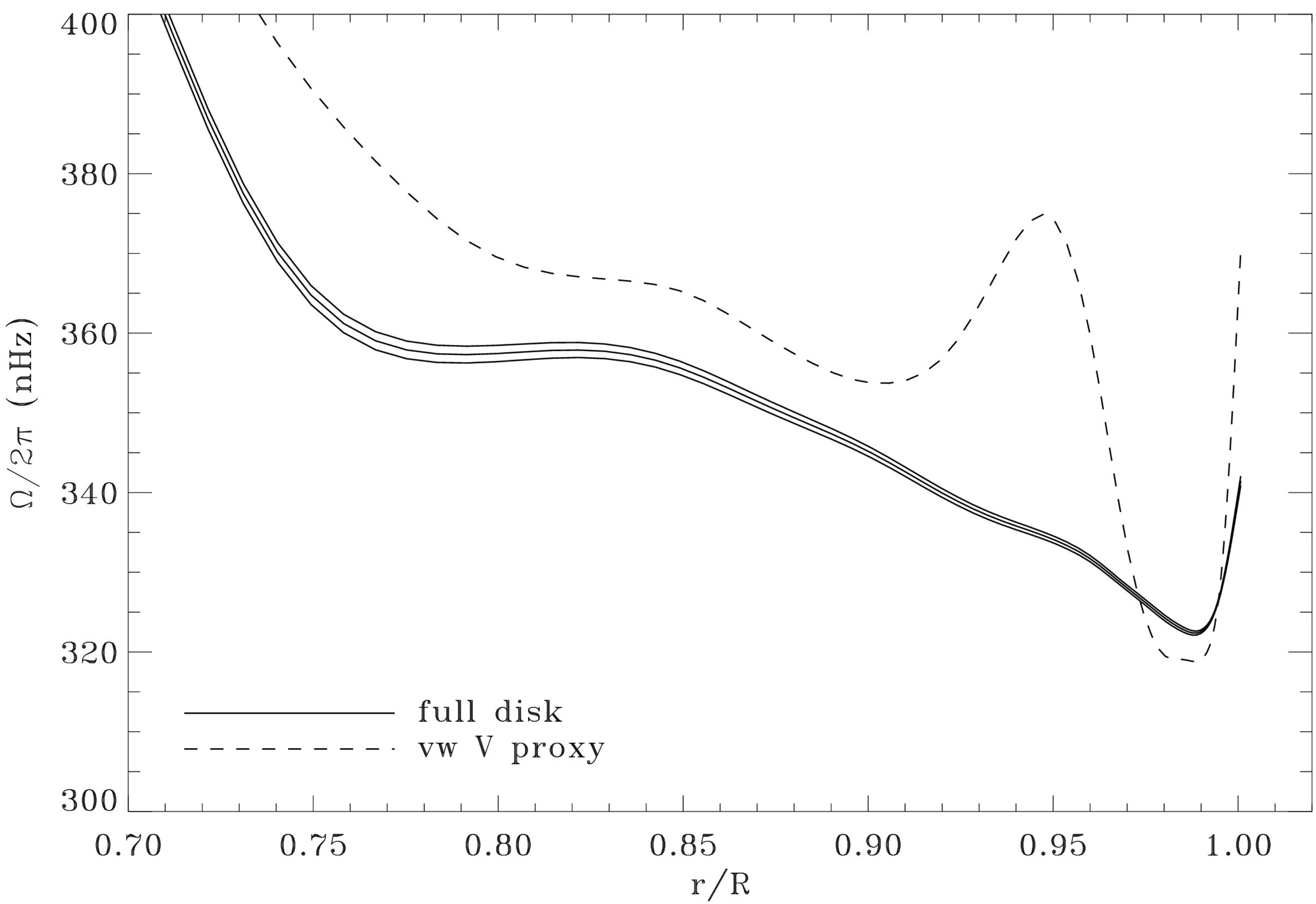
$\Delta\nu_0/\sigma_{\nu_0}$
$\Delta A/\sigma_A$
$\Delta w/\sigma_w$
$\Delta b/\sigma_b$
$\Delta a_1/\sigma_1$
$\Delta a_2/\sigma_2$
ν (mHz)
ν (mHz)



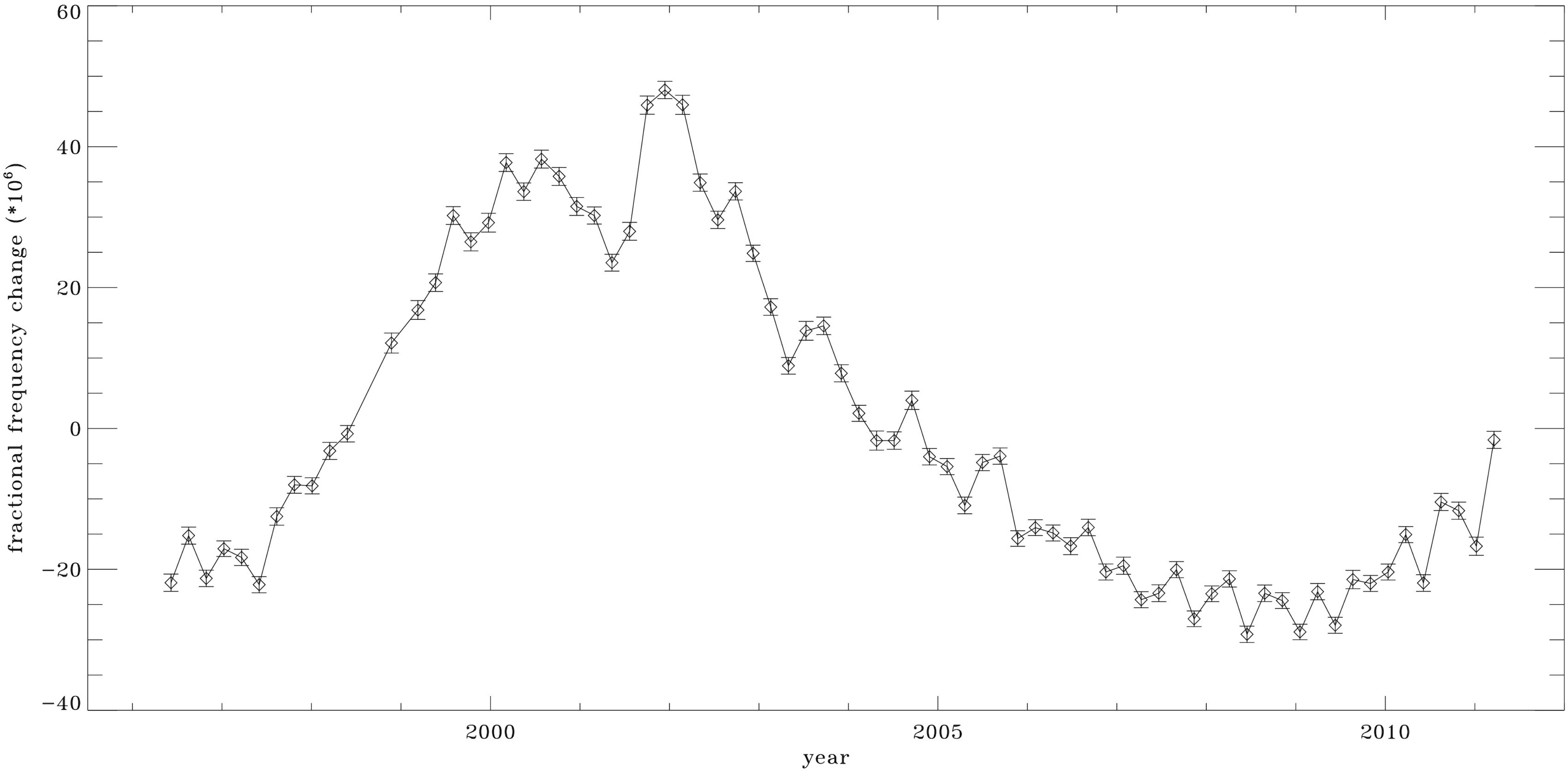
3.0
2.5
2.0
1.5
1.0
rms of residuals
hmi V
hmi vw V
10^5
10^{10}
10^{15}
10^{20}
penalty integral



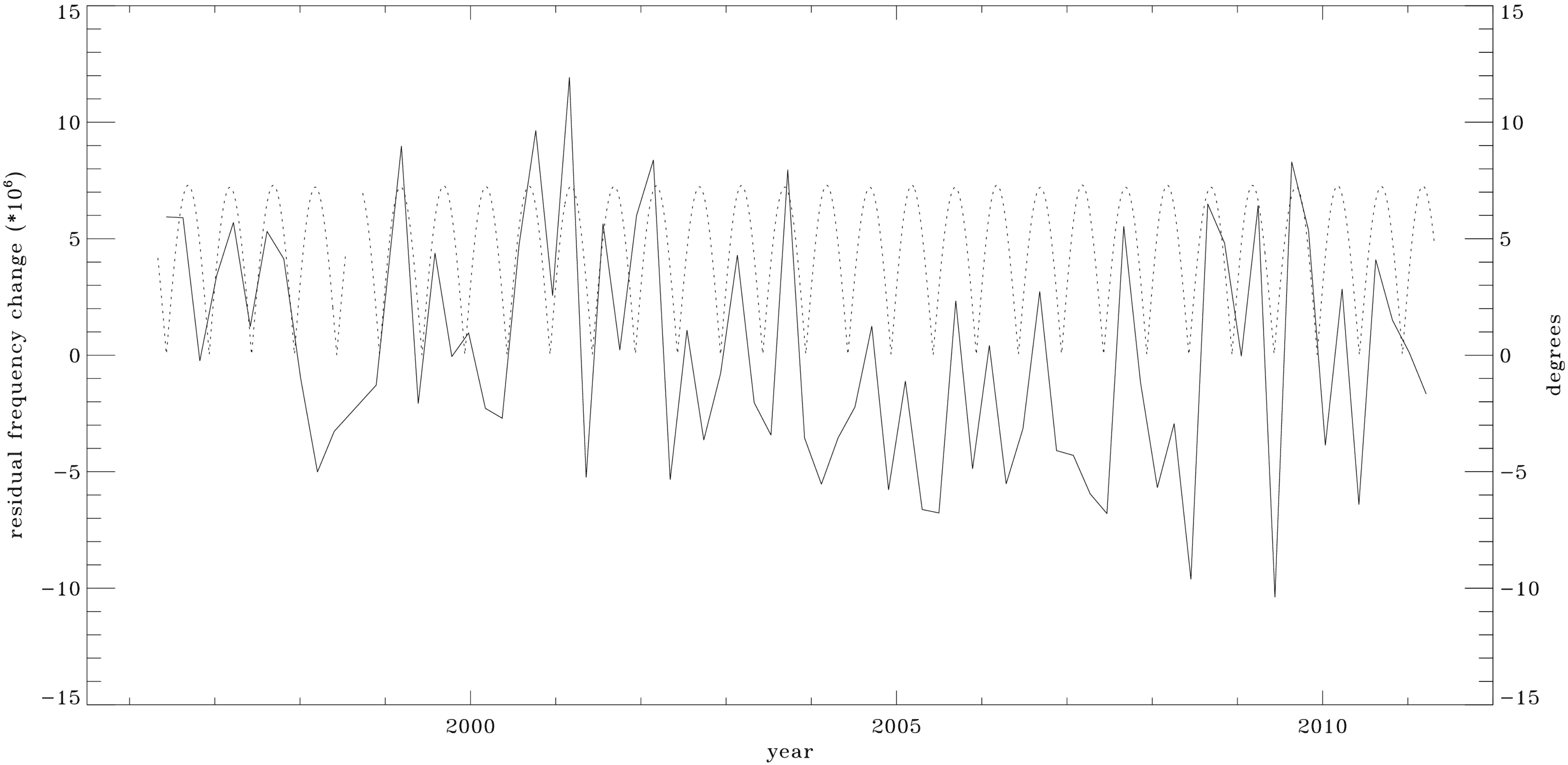
vw V proxy
normalized residuals
10
5
0
−5
−10
1
2
3
4
5
full disk
normalized residuals
10
5
0
−5
−10
1
2
3
4
5
ν (mHz)



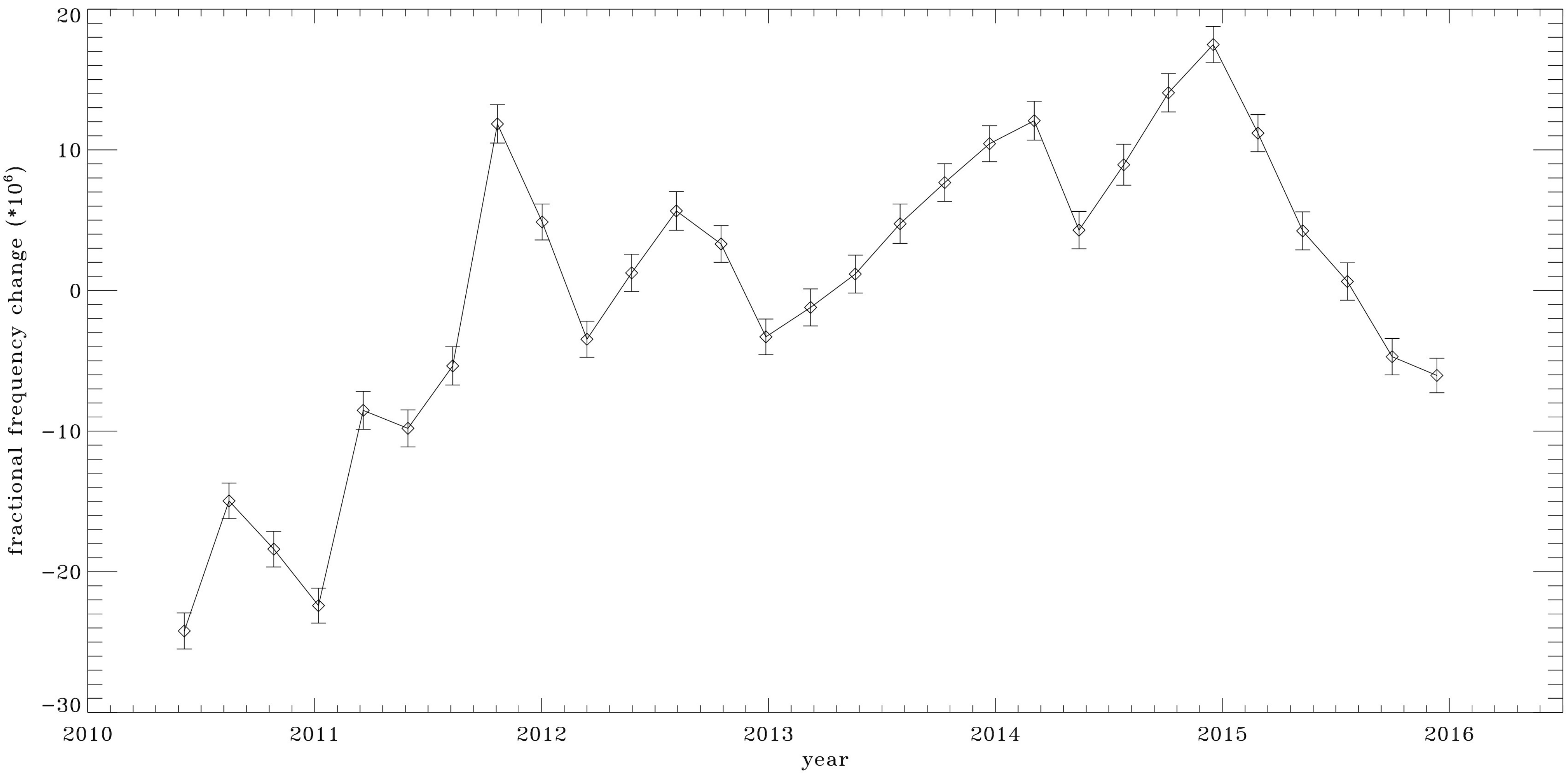
400
380
360
340
320
300
Ω/2π (nHz)
0.70
0.75
0.80
0.85
0.90
0.95
1.00
r/R
full disk
vw V proxy

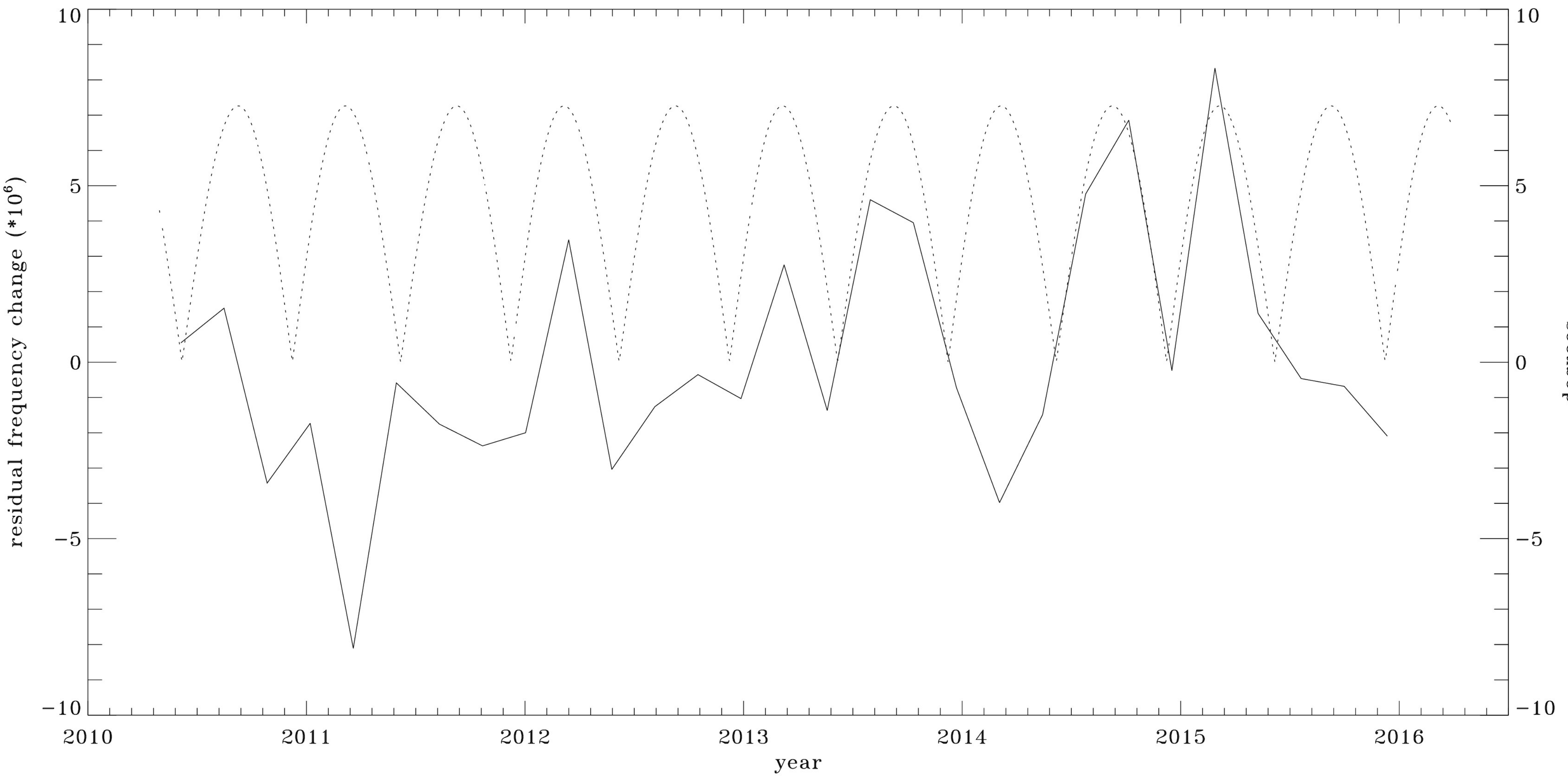


60
40
20
0
−20
−40
fractional frequency change (*10⁶)
2000
2005
2010
year

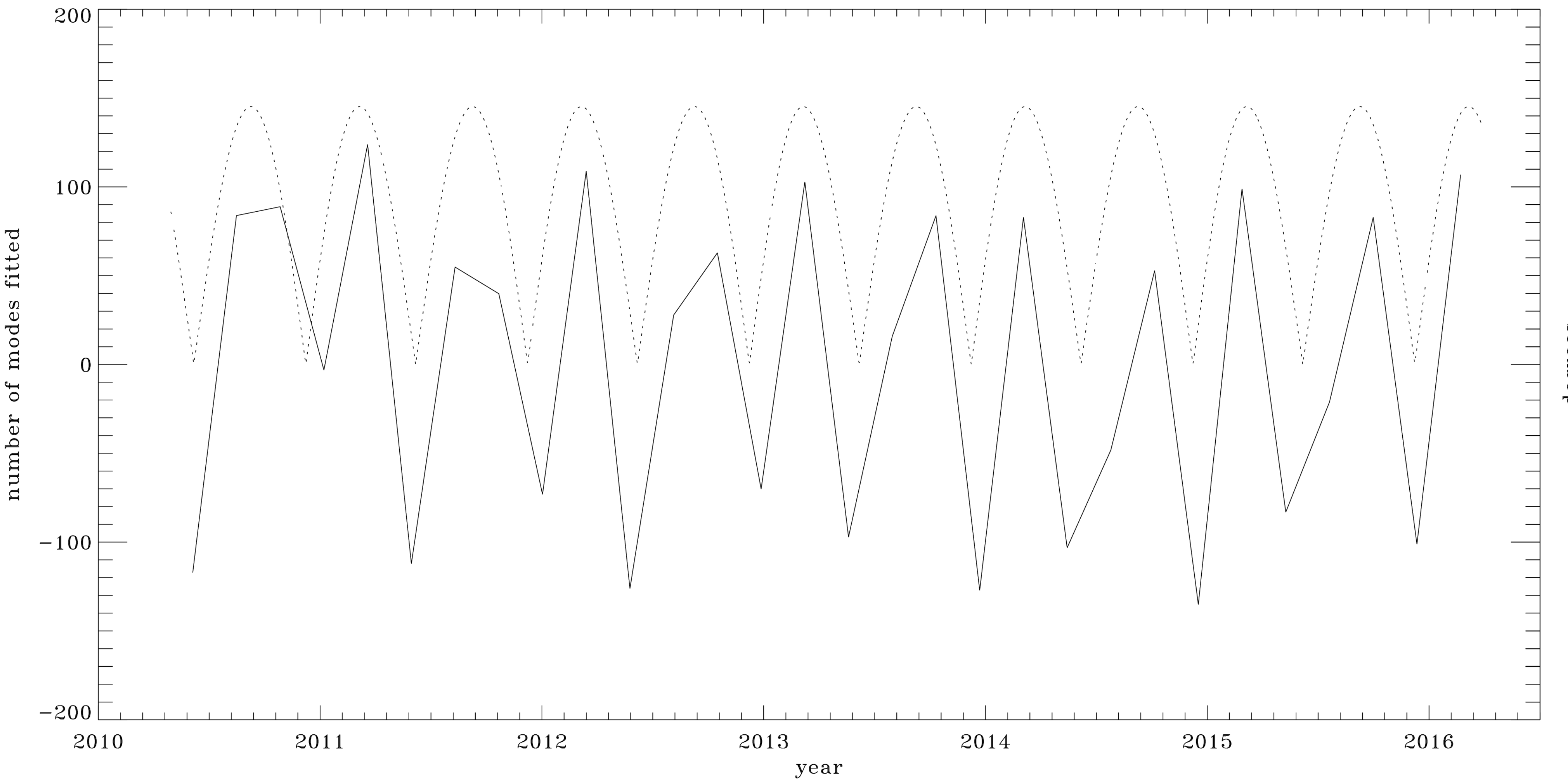


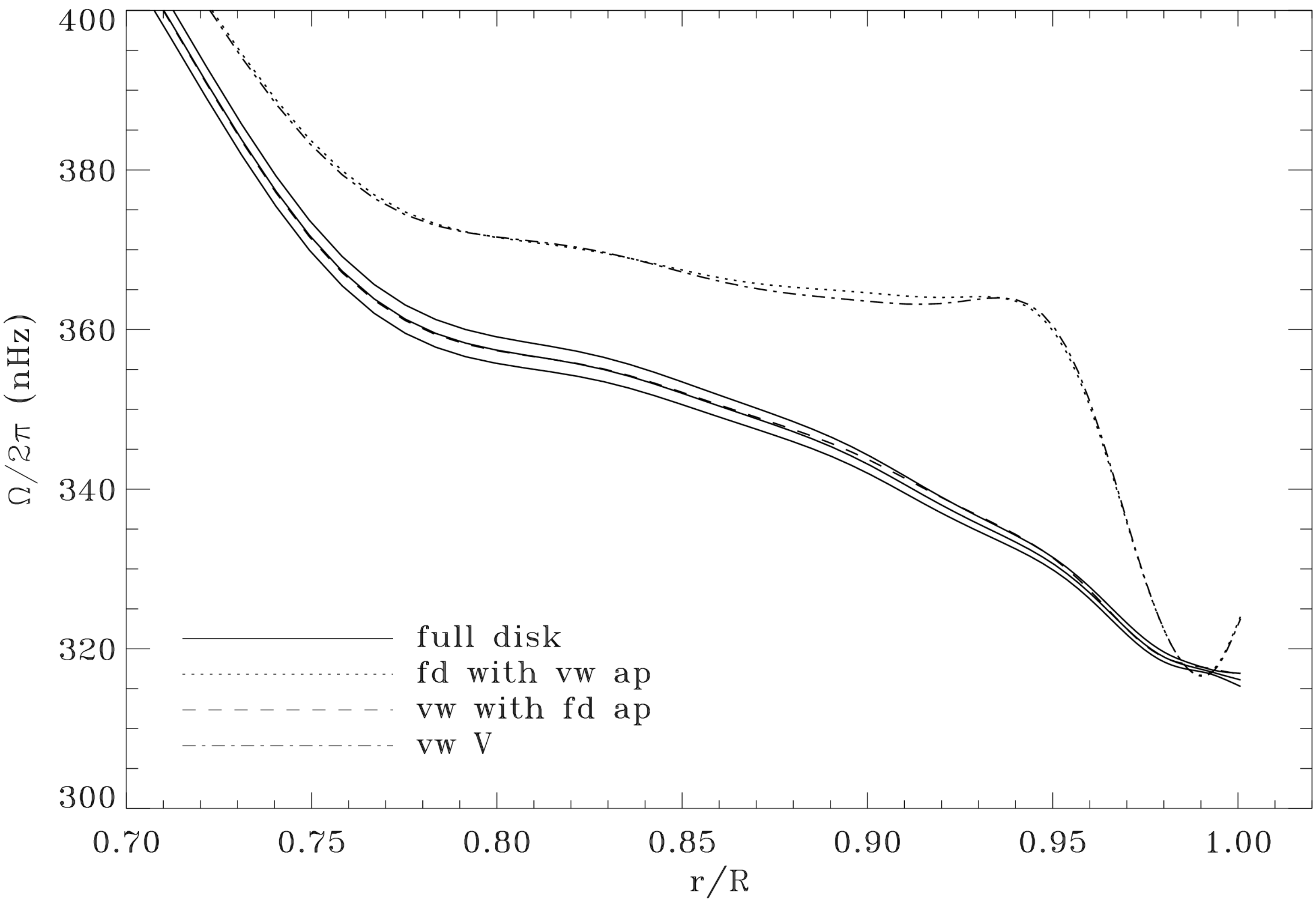
15
10
5
0
−5
−10
−15
residual frequency change (*10^6)
degrees
2000
2005
2010
year


20
10
0
−10
−20
−30
fractional frequency change (*10^6)
2010
2011
2012
2013
2014
2015
2016
year

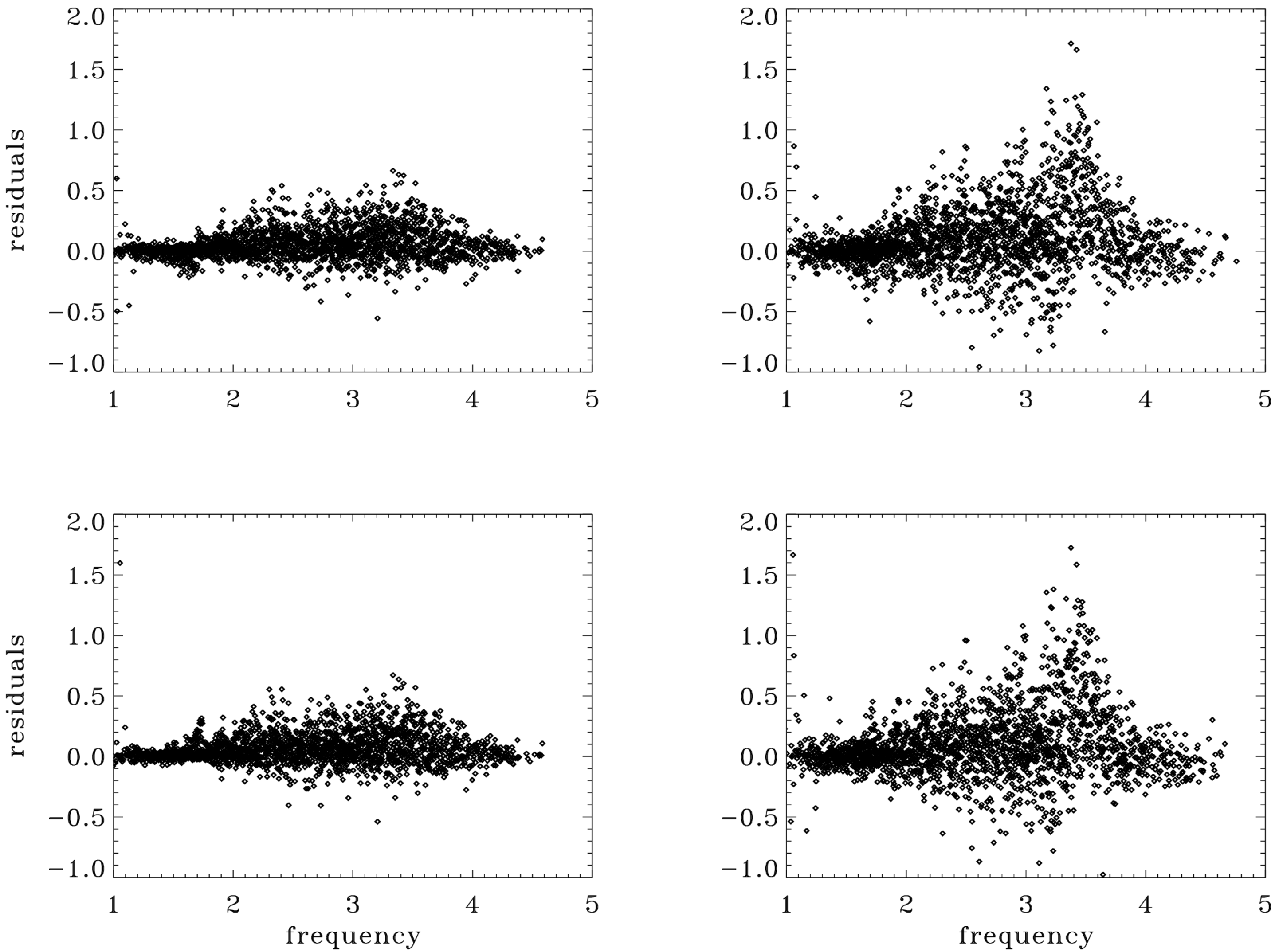


10
5
0
−5
−10
2010
2011
2012
2013
2014
2015
2016
year
residual frequency change (*10^6)
degrees


200
100
0
−100
−200
number of modes fitted
degrees
2010
2011
2012
2013
2014
2015
2016
year



400
380
360
340
320
300
0.70
0.75
0.80
0.85
0.90
0.95
1.00
r/R
Ω/2π (nHz)
full disk
fd with vw ap
vw with fd ap
vw V


residuals
frequency
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
1
2
3
4
5

Conclusions

- none of the changes we applied to the vw_V analysis removed the bump or the jet, but the horns are gone

- changing the apodization does get rid of them