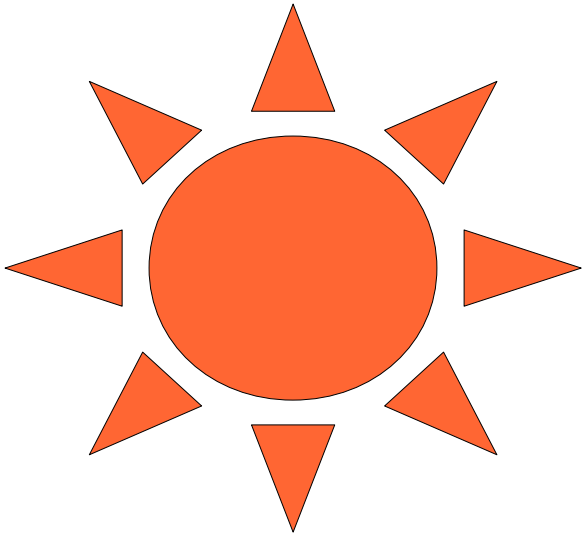
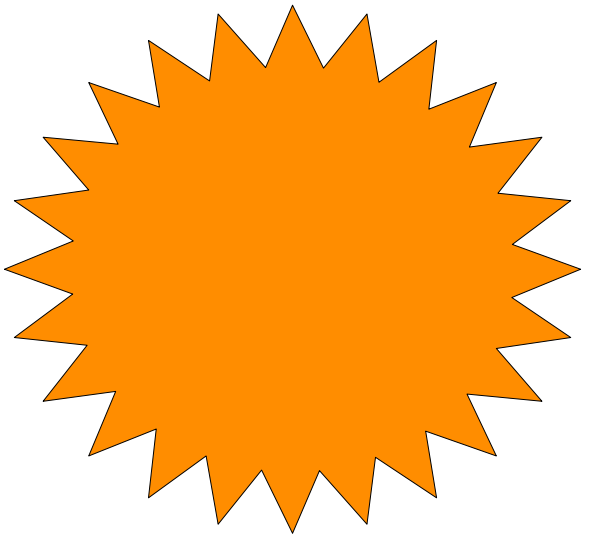
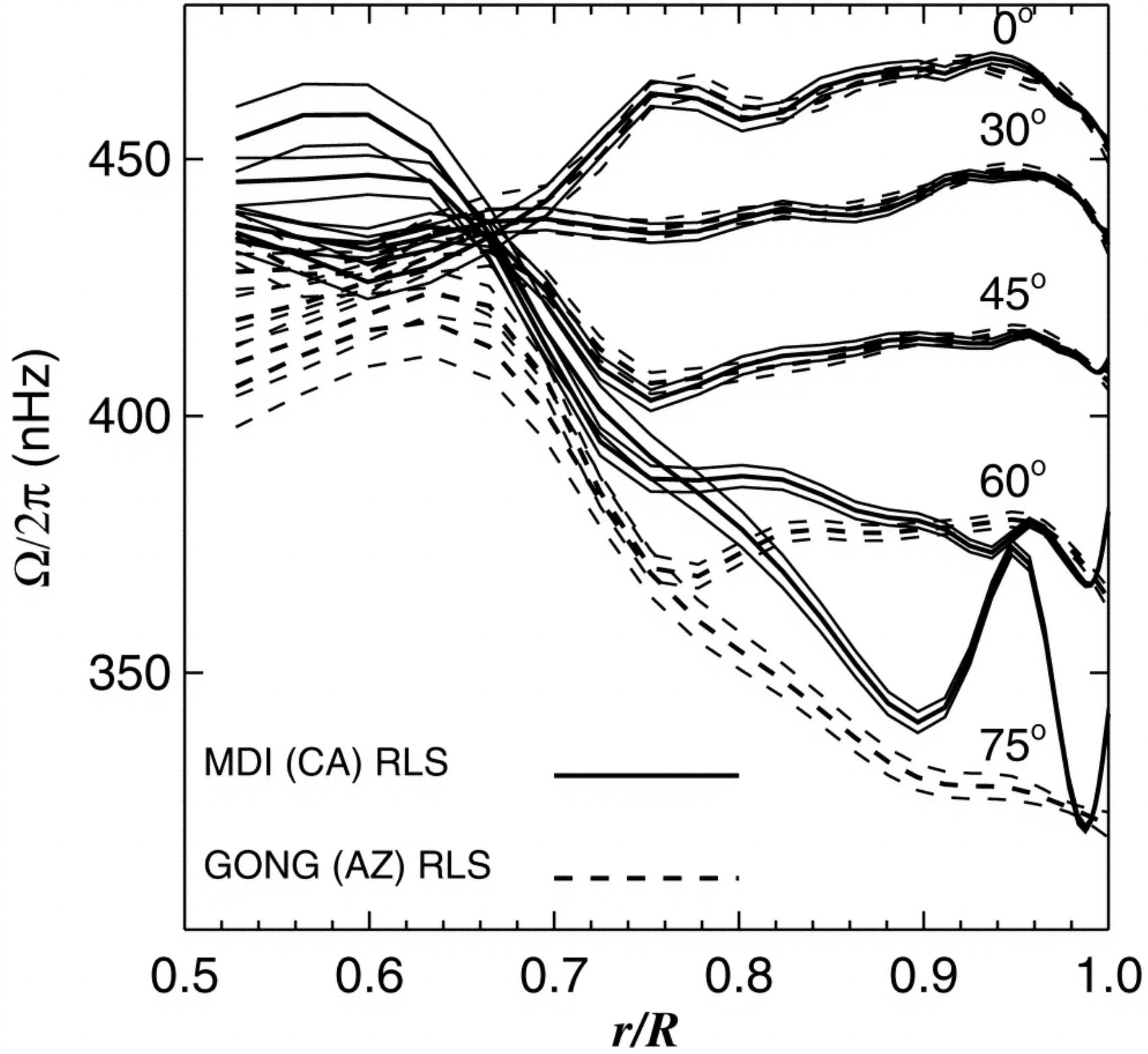


Leakage Matrices: Then and Now

Tim Larson
tplarson@sun.stanford.edu

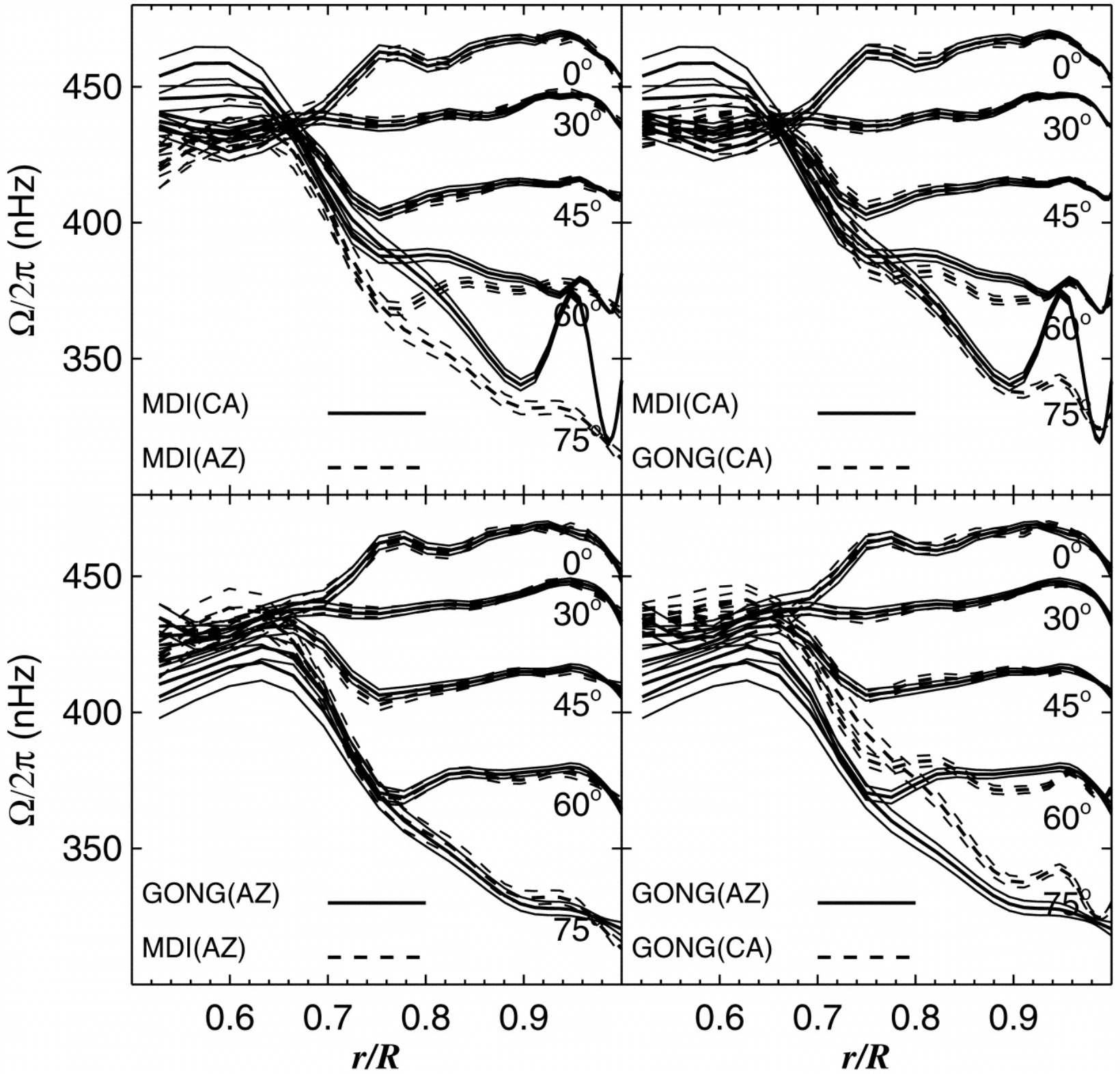
with figures and equations
shamelessly lifted from others

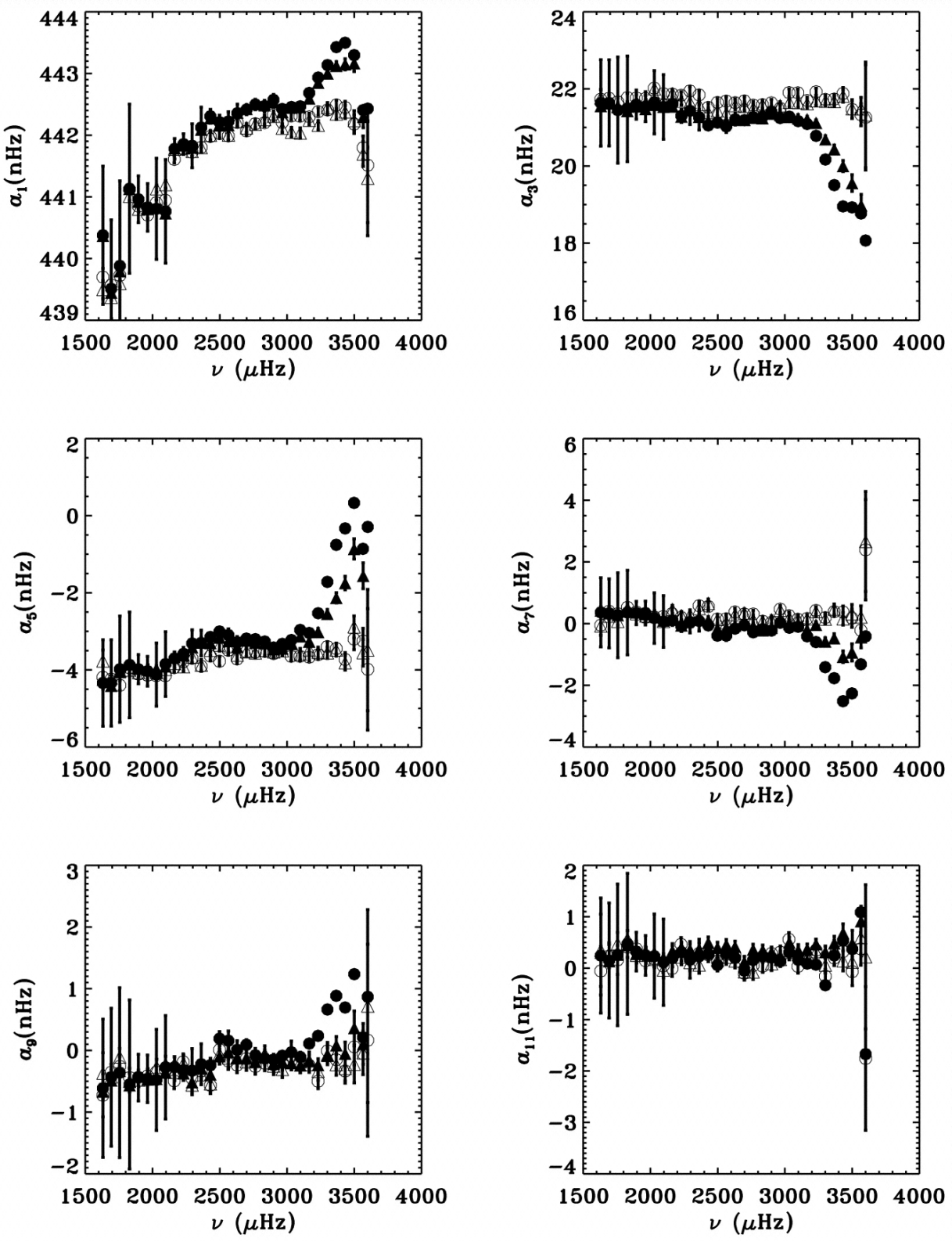


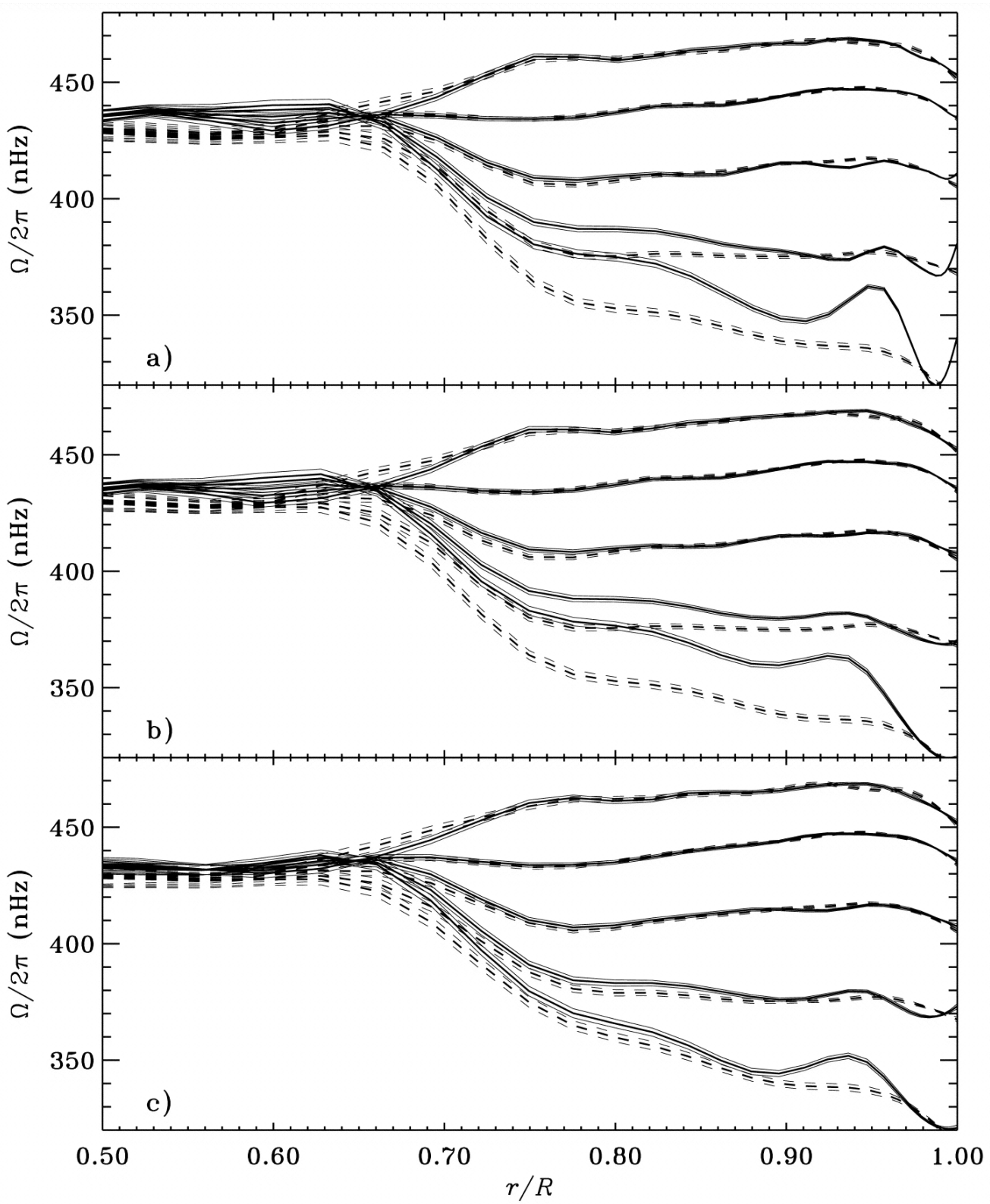

0°
30°
45°
60°
75°
450
400
350
Ω/2π (nHz)
MDI (CA) RLS
GONG (AZ) RLS
0.5
0.6
0.7
0.8
0.9
1.0
r/R

How I Got Started in Global Helioseismology

- Discrepancies with GONG
  - what doesn’t fix them in the remapping:
    - using the correct MDI plate scale, correcting for cubic distortion from the optics, P angle error, Carrington inclination error, and alleged tilt of the CCD
  - what doesn’t fix them in the fitting:
    - improved gapfilling, asymmetric line profiles, accounting for distortion of eigenfunctions by differential rotation and horizontal displacement at the solar surface


Ω/2π (nHz)
450
400
350
0°
30°
45°
60°
75°
MDI(CA)
MDI(AZ)
GONG(CA)
GONG(AZ)
0.6
0.7
0.8
0.9
r/R


a_1(nHz)
a_3(nHz)
a_5(nHz)
a_7(nHz)
a_9(nHz)
a_{11}(nHz)
ν (μHz)
1500 2000 2500 3000 3500 4000


450
400
350
Ω/2π (nHz)
a)
b)
c)
0.50
0.60
0.70
0.80
0.90
1.00
r/R

What is a Leakage Matrix Anyhow?

- Global helioseismology pipeline
  - apodize dopplergrams in image radius
  - remap to uniform grid in longitude and sin(latitude)
  - fourier transform in longitude, take inner product in latitude with associated legendre functions
  - form timeseries of resulting spherical harmonic coefficients, detrend and fill gaps
  - fit peaks in the fourier transform of these, taking leaks into account

$$o_{l,m}(t) = \int_{-1}^{1} \int_{-\pi/2}^{\pi/2} V_{\mathrm{obs}}(\phi, x, t) M_l^m(\phi, x) \mathrm{d}\phi \mathrm{d}x$$

$$= \int_{-1}^{1} \int_{-\pi/2}^{\pi/2} \sum_{n',l',m'} V_{n',l',m'}(\phi, x, t) M_l^m(\phi, x) \mathrm{d}\phi \mathrm{d}x$$

$$= \sum_{n',l',m'} \int_{-1}^{1} \int_{-\pi/2}^{\pi/2} \Big\{ P_{l'}^{m'}(x) \mathrm{Re}(a_{n'l'm'}(t) \mathrm{e}^{\mathrm{i}m'\phi})$$

$$\sqrt{1-r^2} \mathrm{Ap}(r) \frac{1}{\pi} Y_l^m(\theta, \phi) \Big\} \mathrm{d}\phi \mathrm{d}x$$

$$= \sum_{n',l',m'} \int_{-1}^{1} \int_{-\pi/2}^{\pi/2} \Big\{ \frac{1}{\pi} P_{l'}^{m'}(x) P_l^m(x)$$

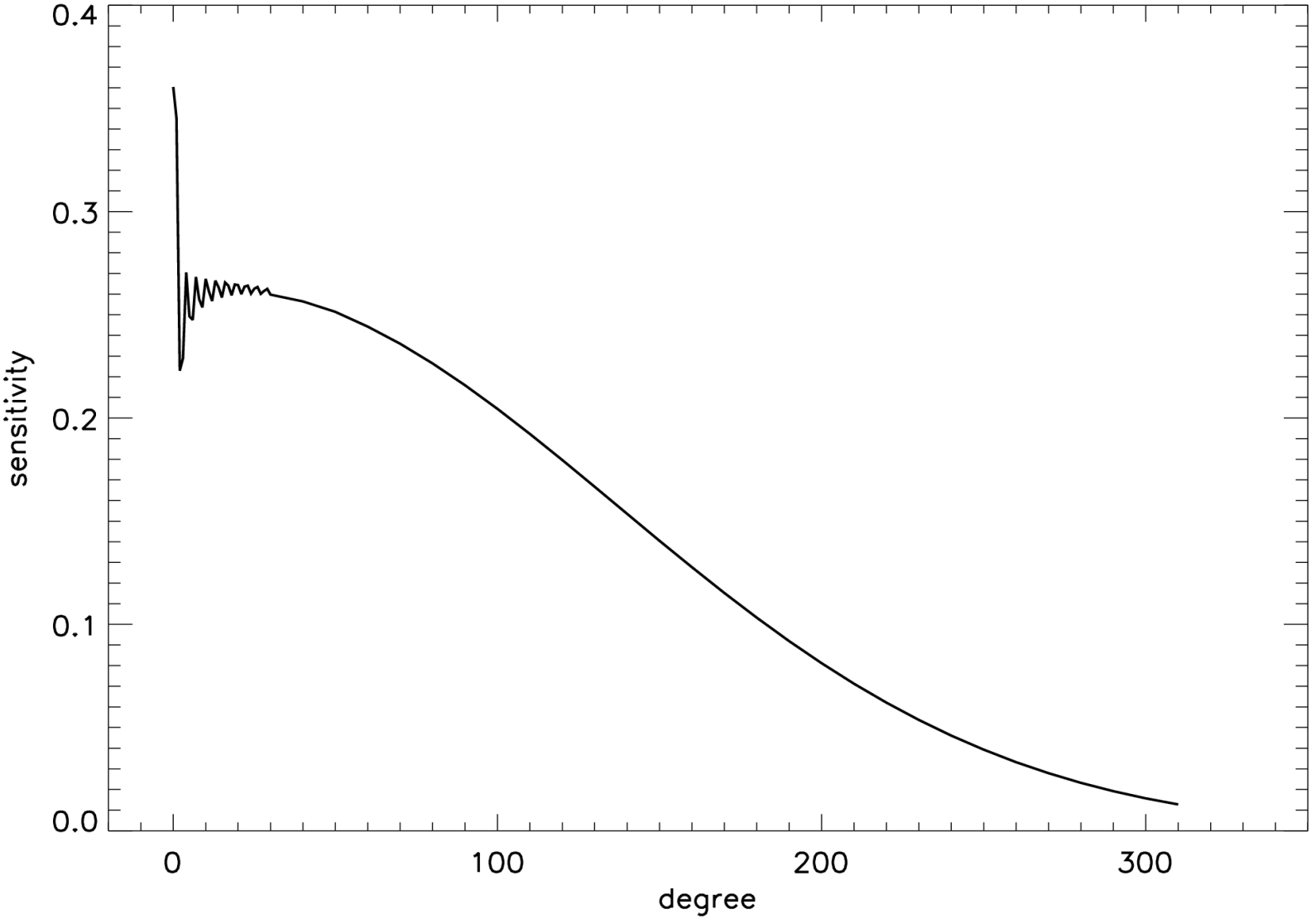
$$[\mathrm{Re}(a_{n'l'm'}(t)) \cos(m'\phi)$$

$$- \mathrm{Im}(a_{n'l'm'}(t)) \sin(m'\phi)]$$

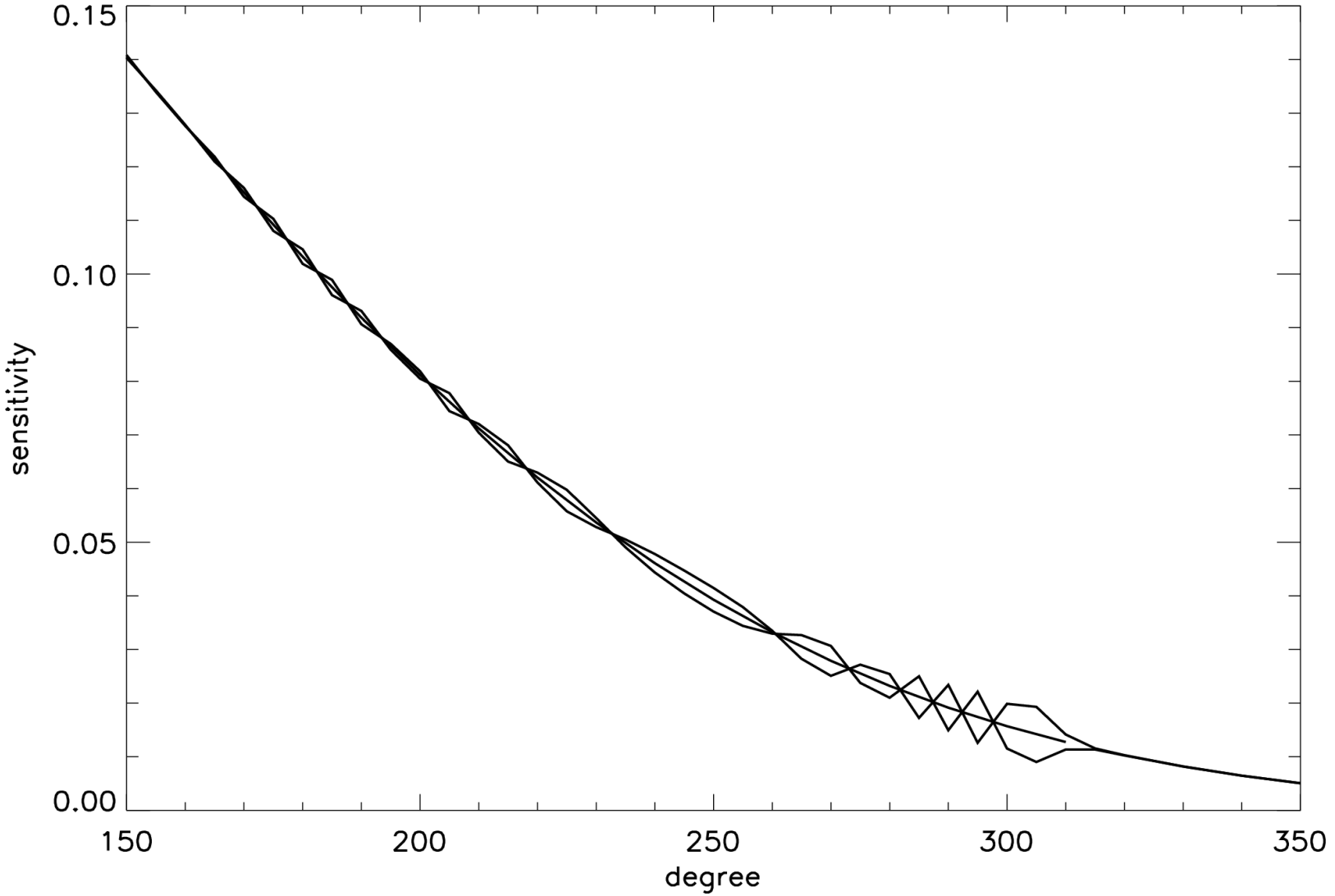
$$[\cos(m\phi) + \mathrm{i} \sin(m\phi)] \sqrt{1-r^2} \mathrm{Ap}(r) \Big\} \mathrm{d}\phi \mathrm{d}x$$

$$
\begin{aligned}
&= \sum_{n',l',m'} \int_{-1}^{1} \int_{-\pi/2}^{\pi/2} \Big\{ P_l^m(x) P_{l'}^{m'}(x) \mathrm{Ap}(r) \\
&\quad \sqrt{1-r^2} \left[\mathrm{Re}(a_{n'l'm'}(t)) \cos(m\phi) \cos(m'\phi) \right. \\
&\quad \left. - \mathrm{Im}(a_{n'l'm'}(t)) \sin(m\phi) \sin(m'\phi) \right] \Big\} \frac{1}{\pi} \mathrm{d}\phi \mathrm{d}x \\
&= \sum_{n',l',m'} \Big\{ c_{l,m,l',m'} \mathrm{Re}(a_{n',l',m'}(t)) \\
&\quad - \mathrm{i} c'_{l,m,l',m'} \mathrm{Im}(a_{n',l',m'}(t)) \Big\} \ .
\end{aligned}
$$

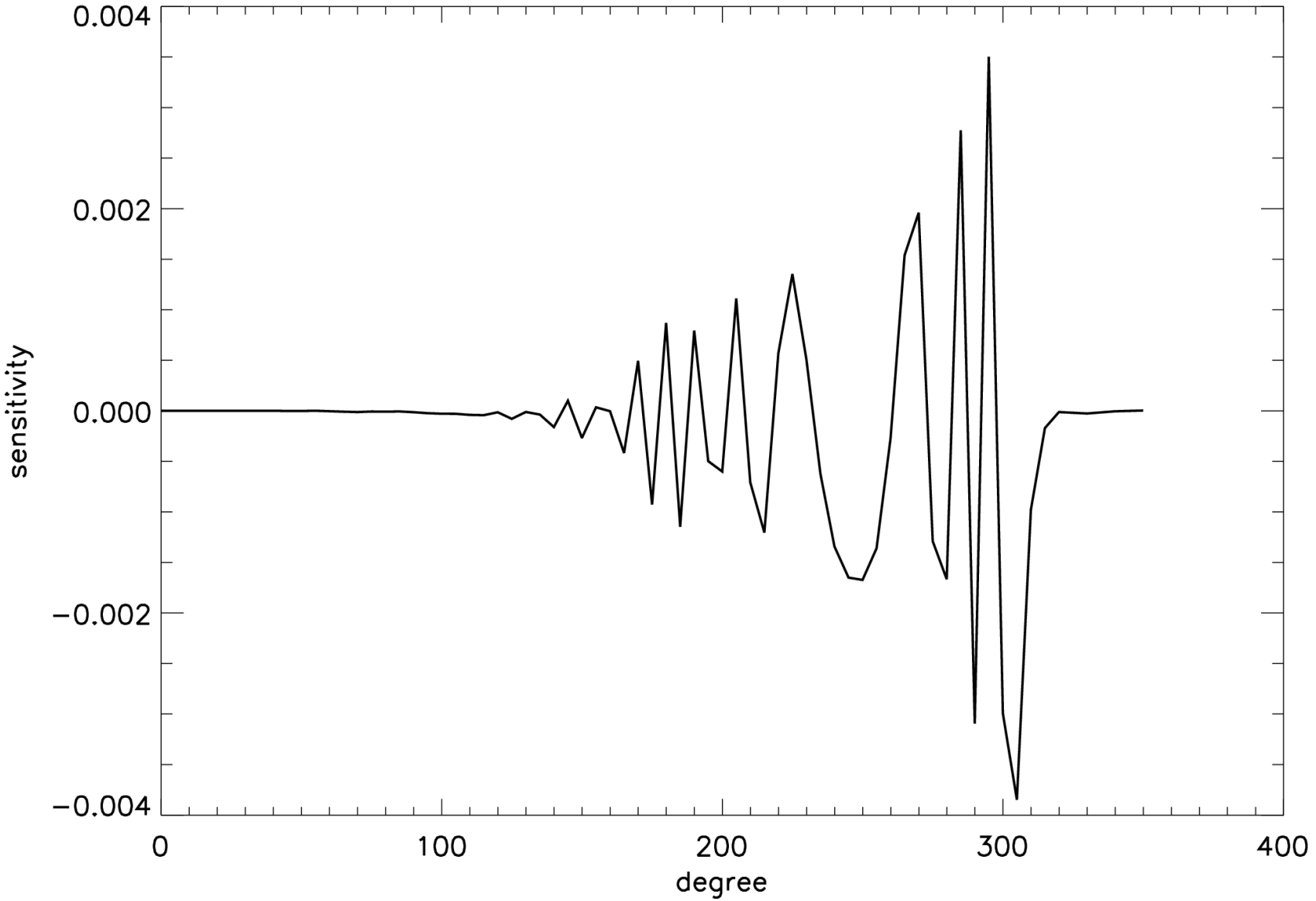
original leakage matrix, m=l



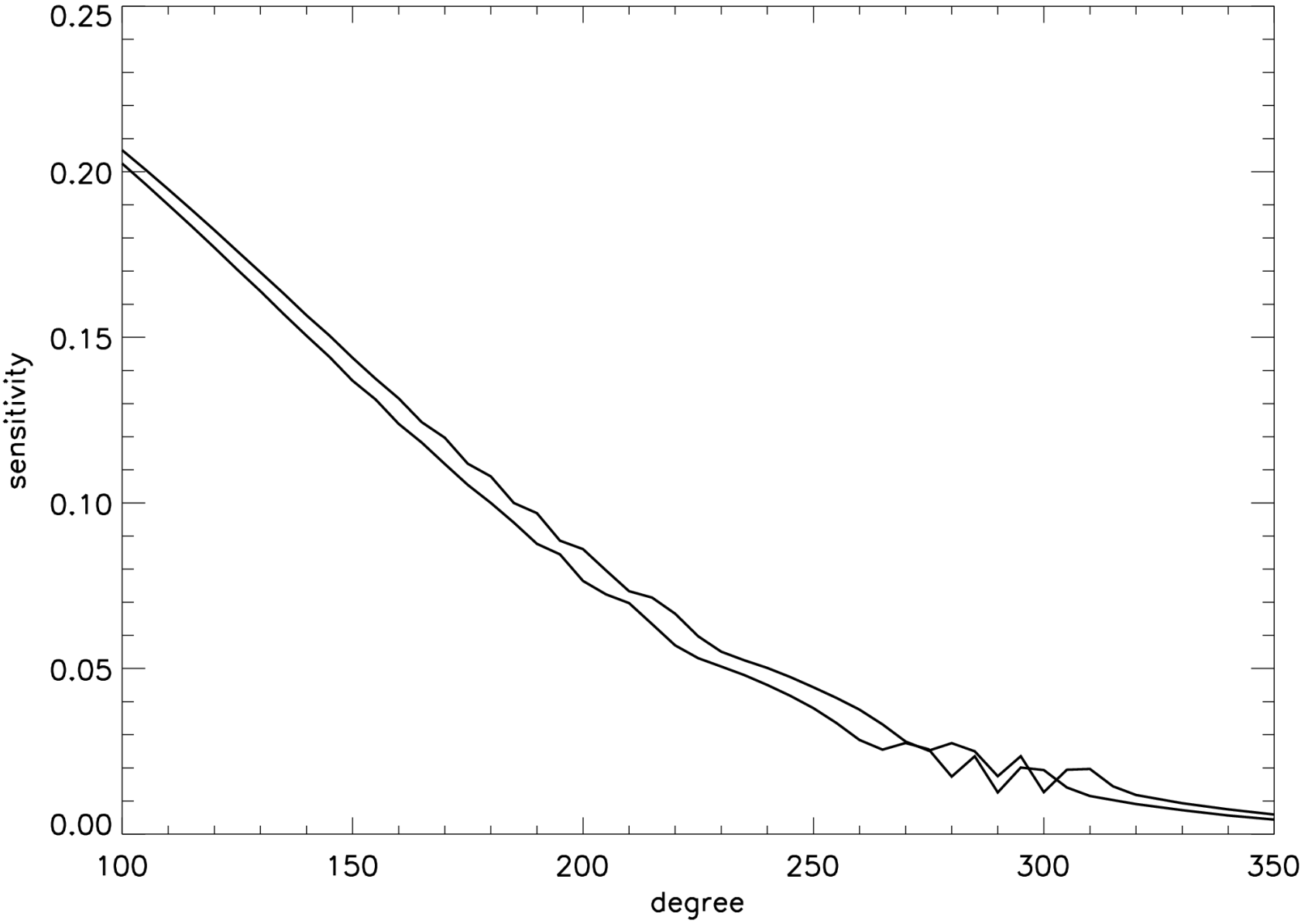
x offsets, m=l



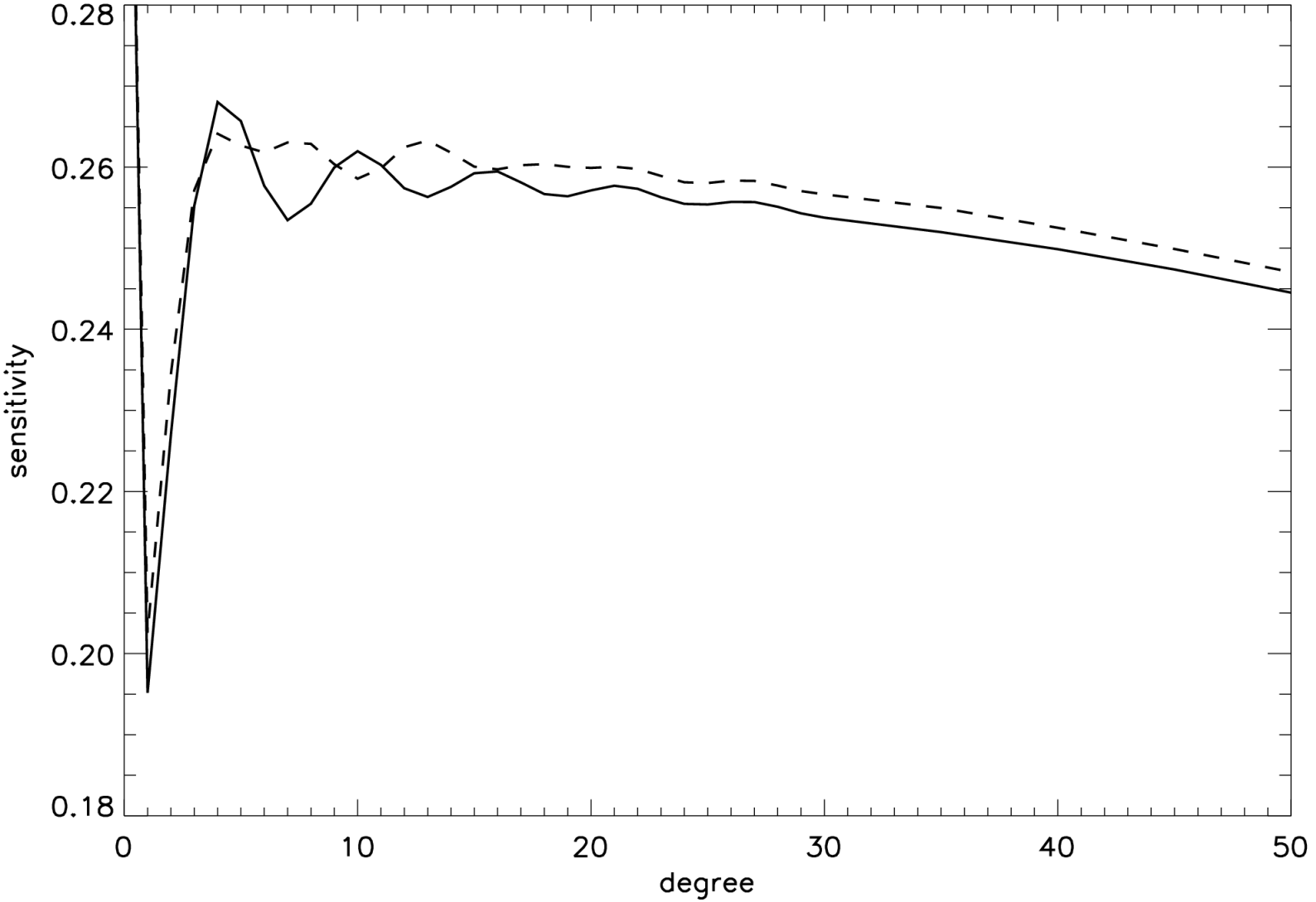
real to imaginary component, m=l



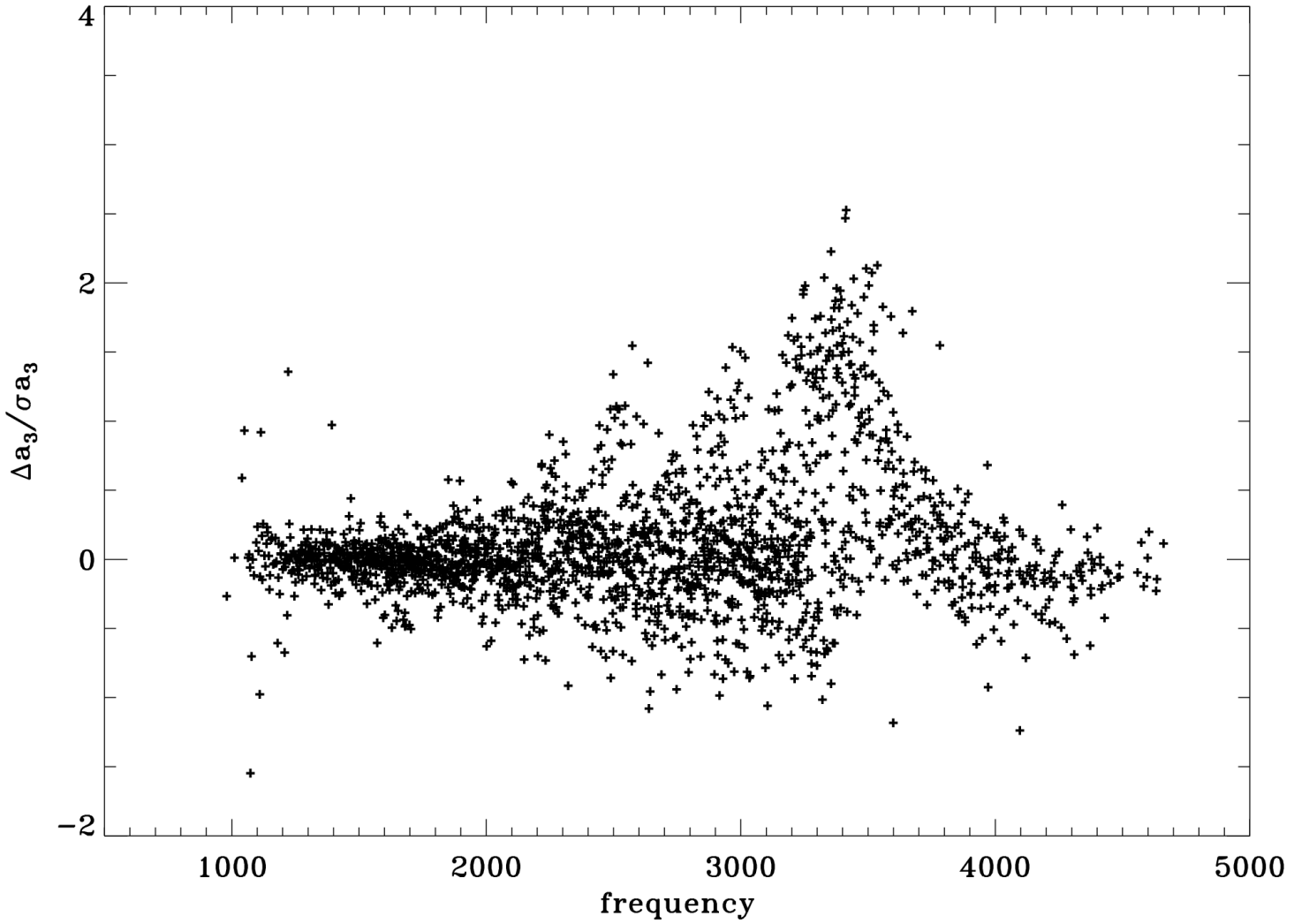
max (bottom) and min (top) observer distance, m=l



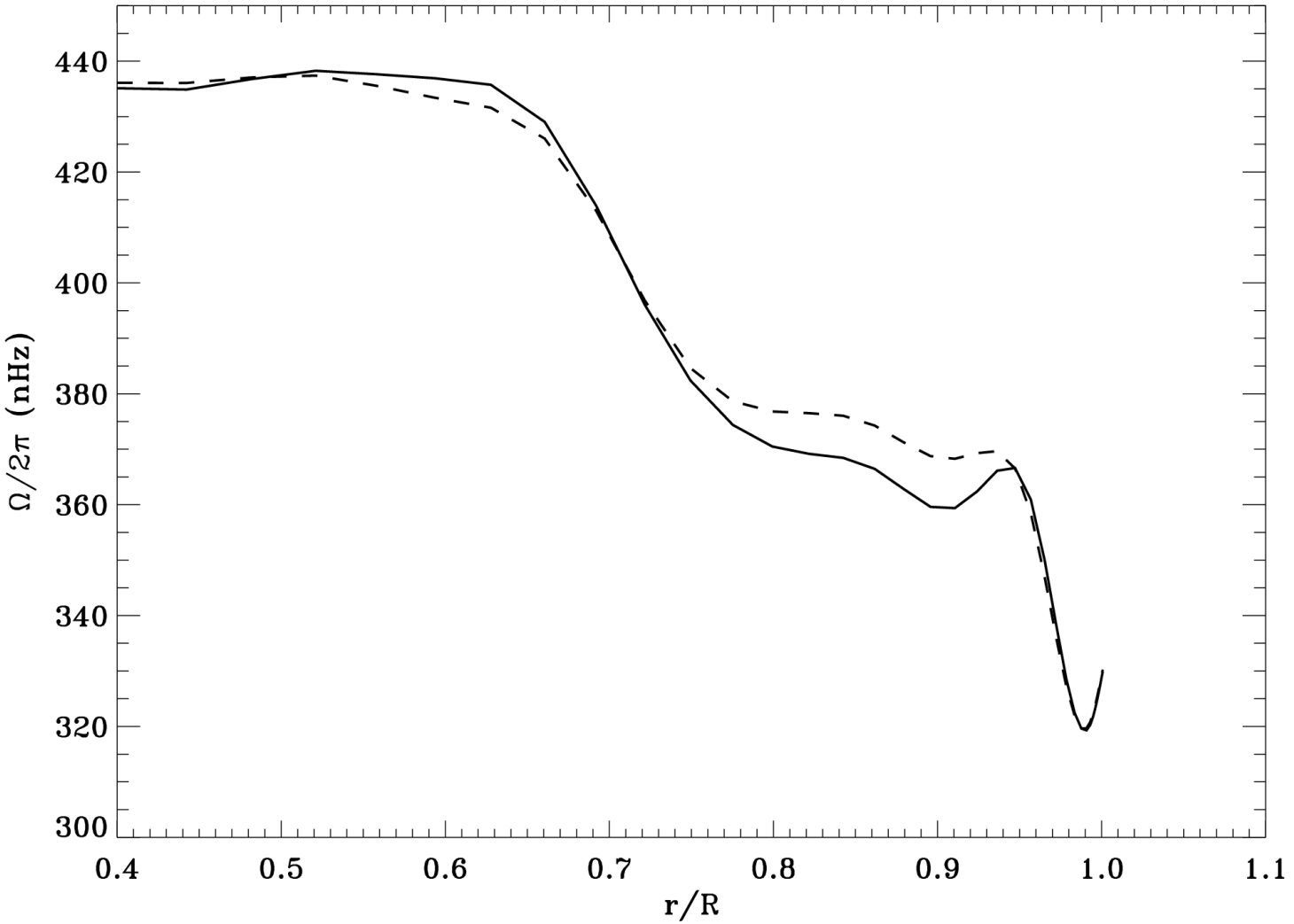
b angle = 0 (solid) and b angle = 7 (dashed), m=0



effect of b angle



rotation at 75 deg



What’s Next?

- Combine leaks from different pixel offsets
- Account for cross terms
- Convolve with point spread functions
- Try different apodizations to get more clues
  - different apodization radii and width
  - elliptical apodization
  - apodize in latitude/longitude rather than image radius