The Effect of Image Apodization on Global Mode Parameters and Rotational Inversions

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It has long been known that certain systematic errors in the global mode analysis of data from both MDI and HMI depend on how the input images were apodized. Recently it has come to light, while investigating a six-month period in f-mode frequencies, that mode coverage is highest when B_0 is maximal. Recalling that the leakage matrix is calculated in the approximation that $B_0=0$, it comes as a surprise that more modes are fitted when the leakage matrix is most incorrect. It is now believed that the six-month oscillation has primarily to do with what portion of the solar surface is visible. Other systematic errors that depend on the part of the disk used include high-latitude anomalies in the rotation rate and a prominent feature in the normalized residuals of odd a-coefficients. Although the most likely cause of all these errors is errors in the leakage matrix, extensive recalculation of the leaks has not made any difference. Thus we conjecture that another effect may be at play, such as errors in the noise model or one that has to do with the alignment of the apodization with the spherical harmonics. In this poster we explore how differently shaped apodizations affect the results of inversions for internal rotation, for both maximal and minimal absolute values of B_0.

Apodizations

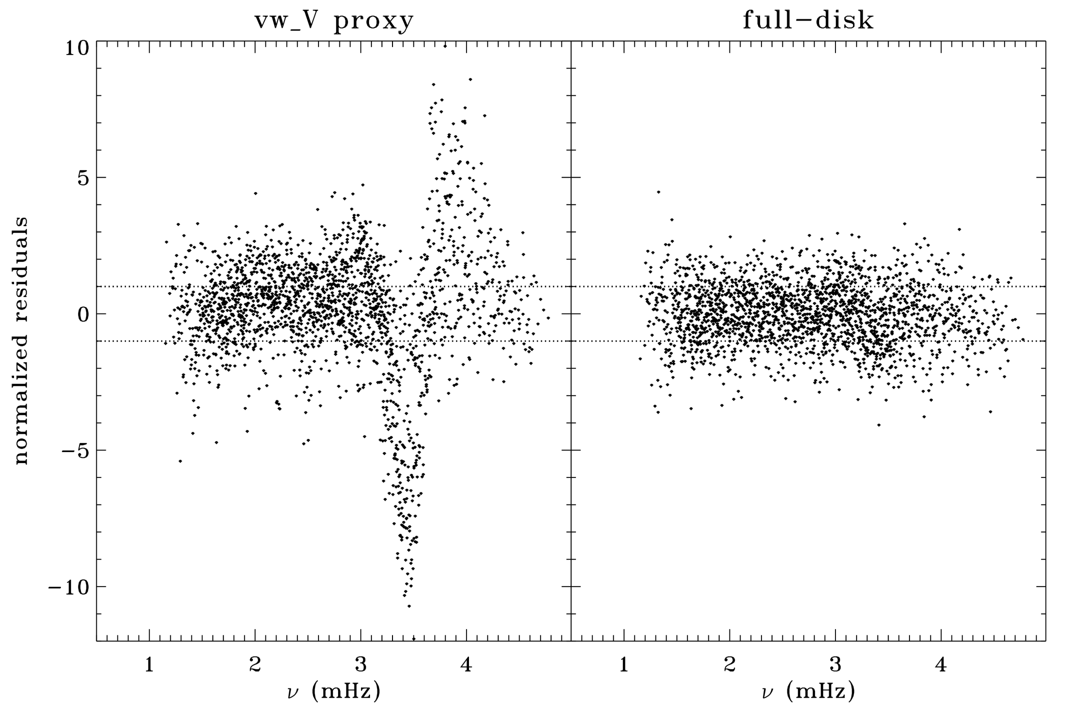
Velocity data from MDI were generally apodized in two different ways. The full-disk data were apodized with a cosine curve in fractional image radius from 0.90 to 0.95. The so-called medium-l data (hereafter vw_V) were apodized in the same way from 0.83 to 0.87. Although the effect of the apodization should be fully accounted for in the leakage matrix, our investigation revealed disturbing differences. We have analyzed the HMI data using these same apodizations. For the vw_V, we first rebinned by a factor of 4 and then convolved with the same gaussian used by MDI.

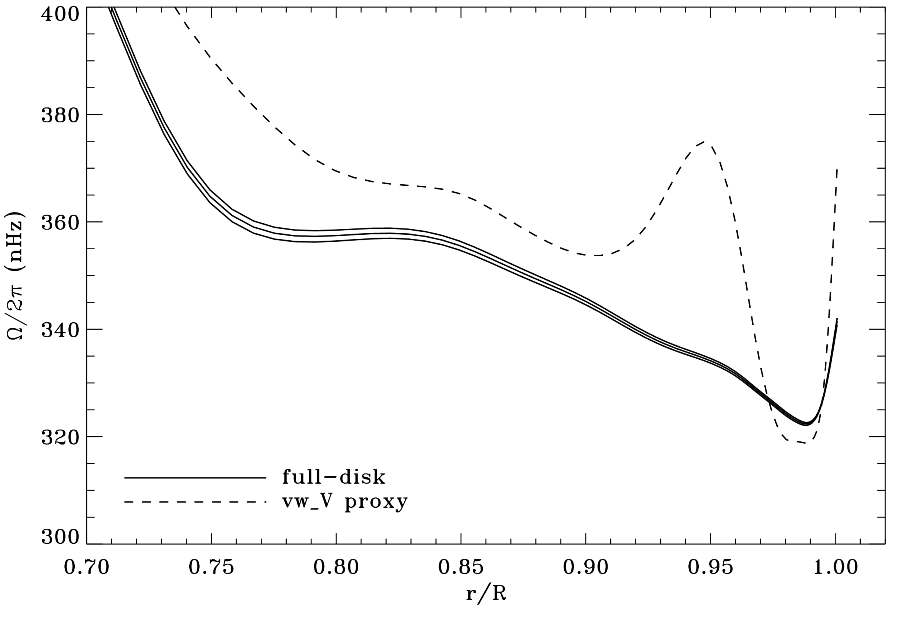
Problems

Top plot shows residuals of a_1 resulting from both analyses.

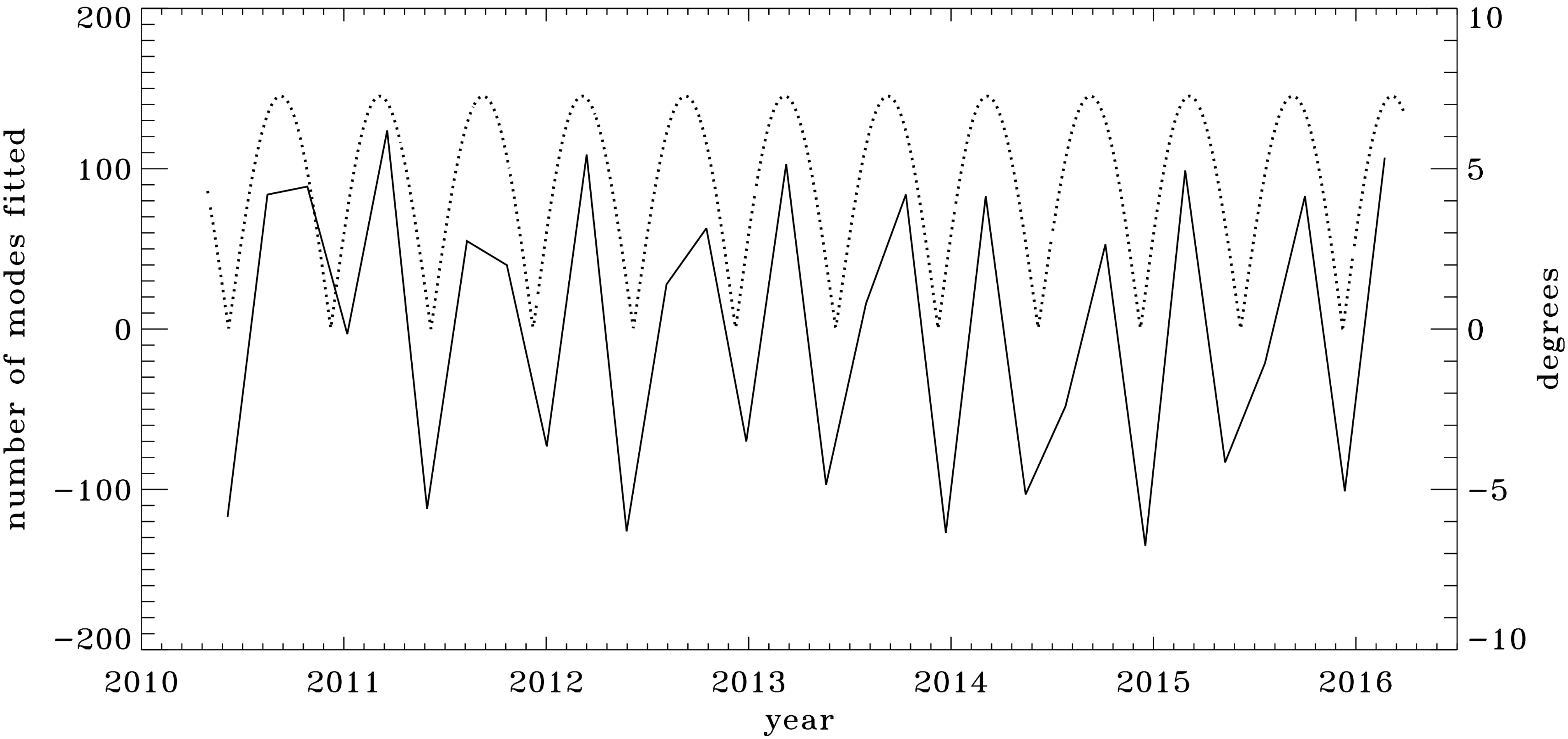
Bottom plot shows internal rotation at 75° latitude. Solid lines show the full-disk analysis and its error; errors for the other analysis were similar

Both plots represent an average over six years of HMI analyzed in 72-day timeseries.



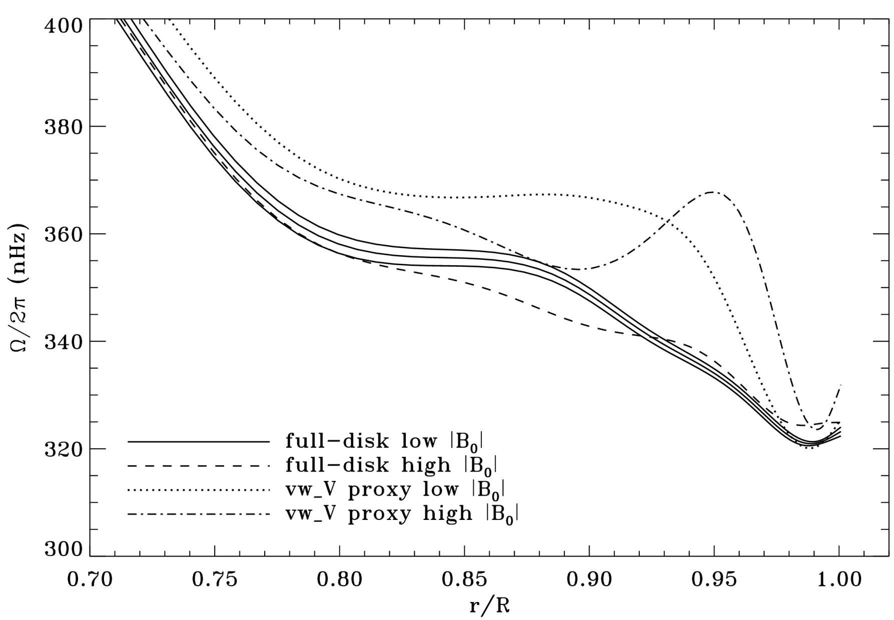


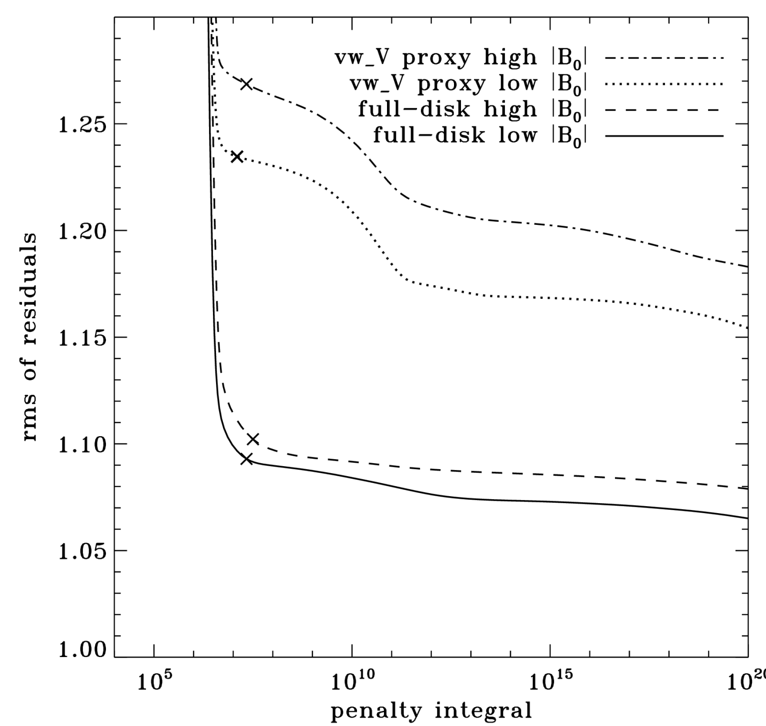
Effect of B_0



Number of modes fitted relative to the mean for the vw_V proxy, with $|B_0|$ overplotted. Correlation coefficient is 0.95.

Recalling that the leakage matrix is calculated in the approximation that $B_0=0$, it can only come as a surprise that we fit more modes when the leakage matrix is most incorrect. The full-disk analysis also saw this six-month oscillation, but with a much smaller magnitude. Although the vw_V proxy fit more modes than the full disk for high $|B_0|$, it fit far fewer for low $|B_0|$. As the following plots show, we later discovered that the high-latitude inversion results depend on $|B_0|$ as well.

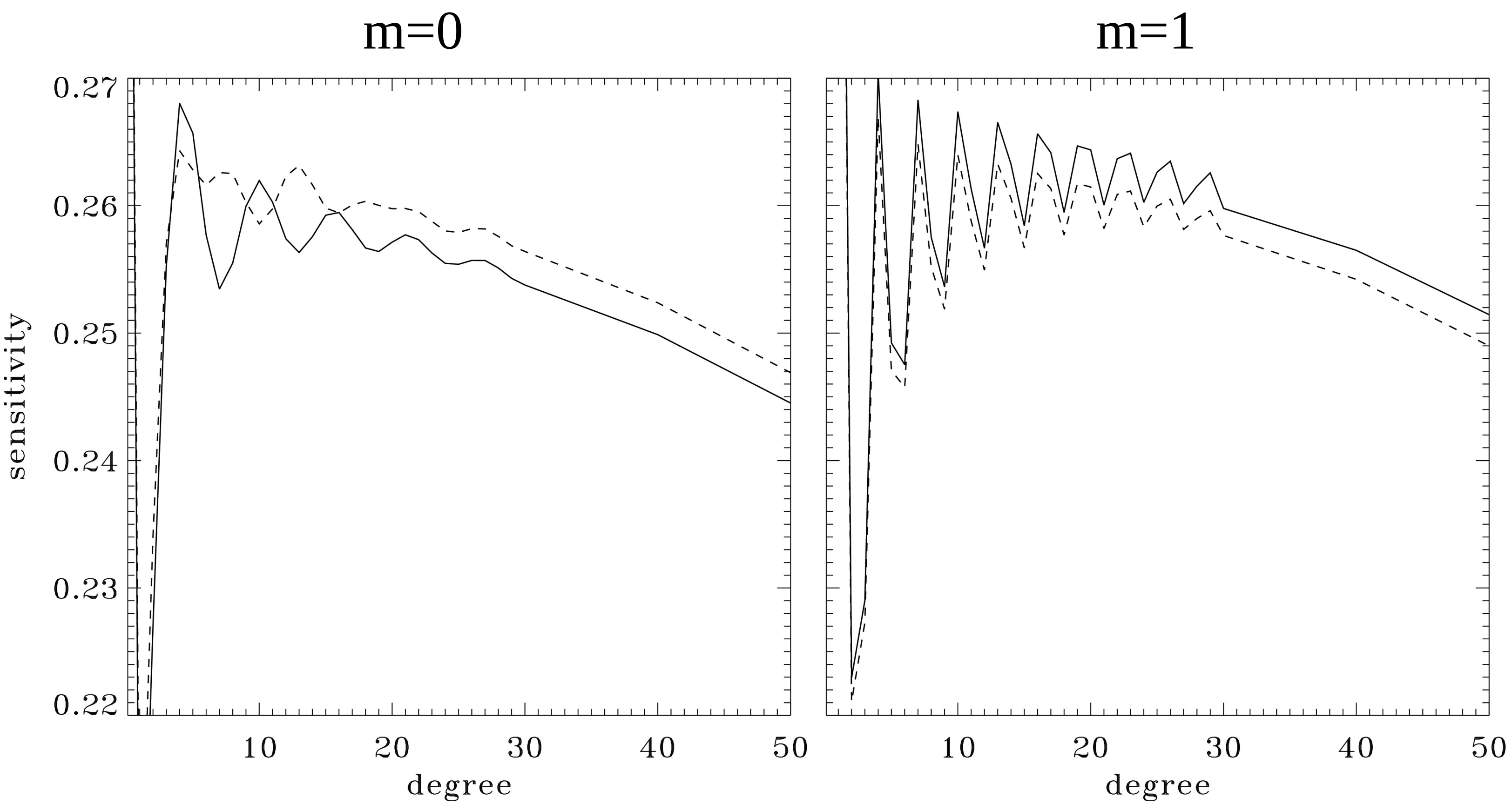




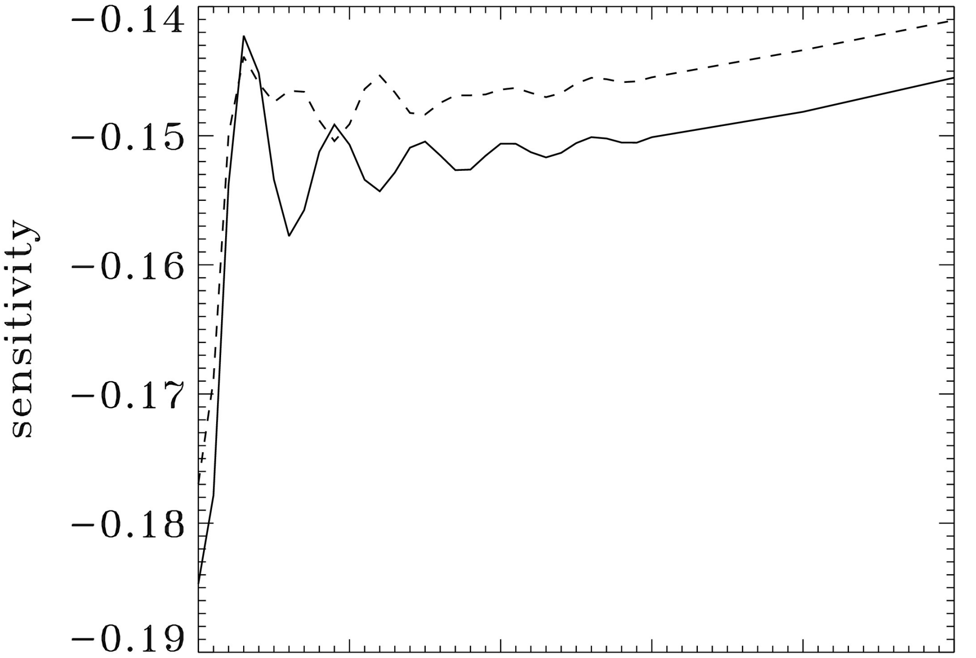
Although we fit more modes for high $|B_0|$, the residuals were lower for low $|B_0|$. We believe the "bump" seen in the residuals of odd a-coefficients is reflected in the "knee" in the tradeoff curves for the vw_V proxy.

In both cases, the high-latitude rotation rate differed significantly between the two apodizations. The jet is seen only in the vw_V proxy at high $|B_0|$. For the full-disk analysis, the rotation rate has an upturn near the surface only for low $|B_0|$, although the vw_V analysis shows an upturn at all times. Both features are clearly visible in the averaged results shown above. Repeating the inversion of the high-$|B_0|$ interval using only modes common with the low-$|B_0|$ interval still resulted in the jet, but the two analyses were then in agreement near the surface with an intermediate radial gradient.

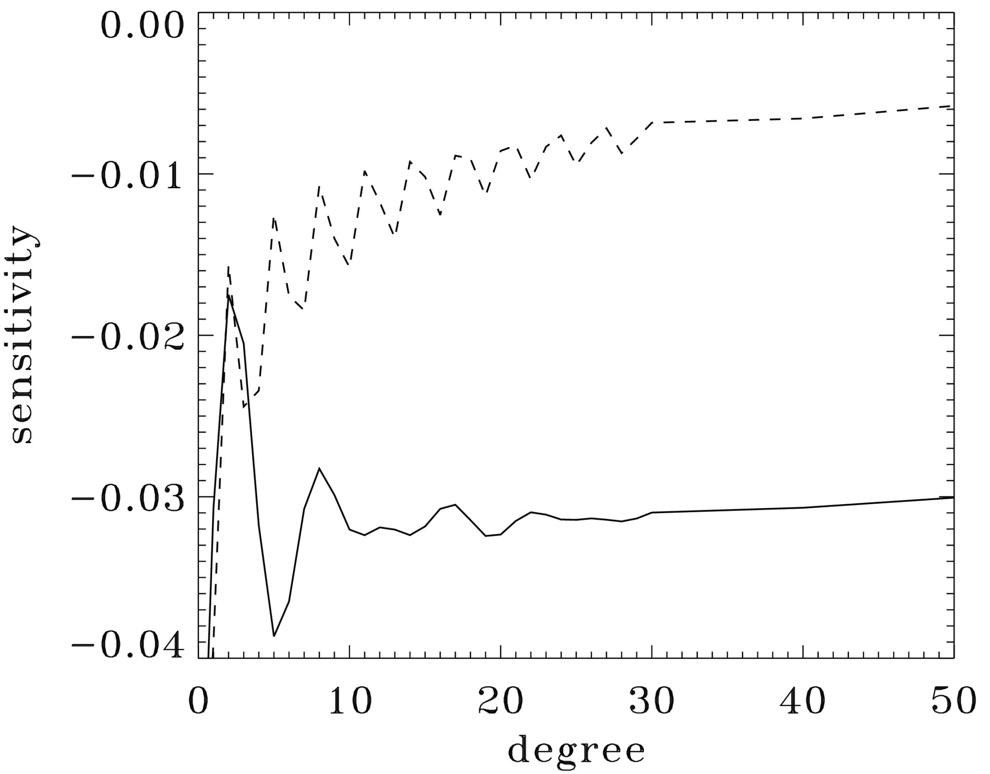
One wonders what would happen if we used a leakage matrix calculated for the actual value of B_0 averaged over an interval, in this case -6.79°. Unfortunately, this would not necessarily be an improvement, since the fitting code always assumes the leaks from $\Delta l+\Delta m$ odd, which arise from north-south asymmetry, are zero. Nonetheless, we refit the high-$|B_0|$ interval using such a matrix. The change in the leaks themselves are shown for the original vw_V apodization in the following plots.



Sensitivity to target mode. Solid lines show original leakage matrix, dashed lines show leaks calculated for high $|B_0|$.



Leaks for m=0 and Δl=2, Δm=0. Solid line shows original, dashed line shows leaks for high $|B_0|$.



Leaks for high $|B_0|$, Δl=1, Δm=0. Solid line shows m=0, dashed line shows m=l. Original leaks are identically zero.

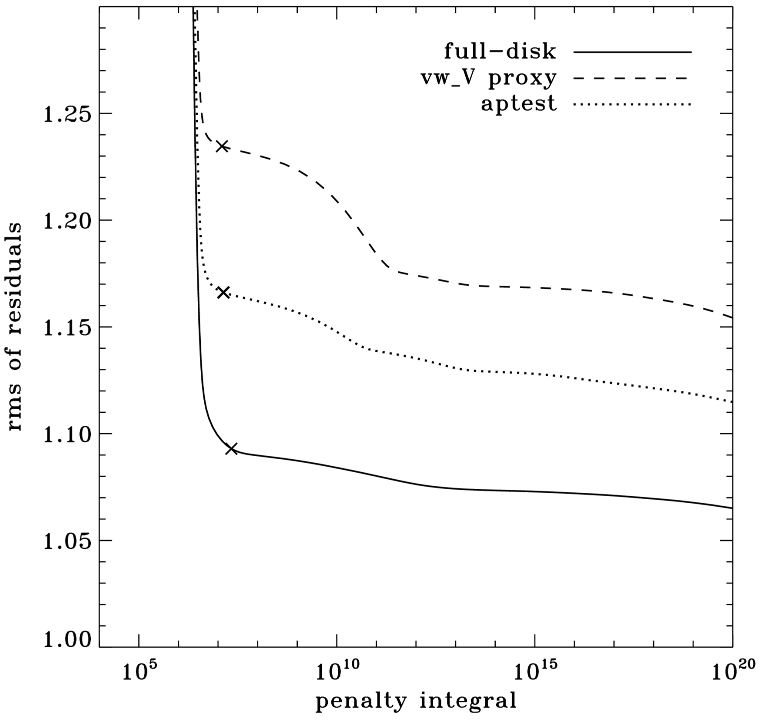
New Apodizations

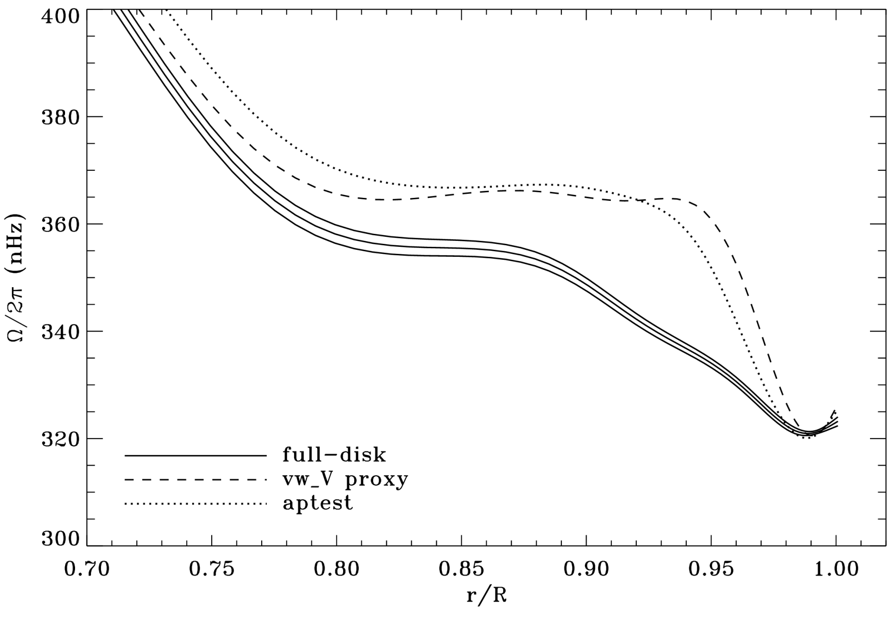
Since the vw_V apodization, for reasons unknown, seems problematic, we have tried other apodization shapes that would fit within the MDI vw_V crop radius of 0.9R. In each case we have chosen to preserve the extent of the apodization in longitude at the equator. For each apodization, we refit the low and high $|B_0|$ interval, and for the latter we also repeated the fit using the leakage matrix described above. The following plots show the resulting tradeoff curves and high-latitude rotation rates, along with those from the original vw_V and full-disk analyses.

(Please note, with my apologies, that line styles were inadvertently switched in the rotation plots between the test and original vw_V apodizations.)

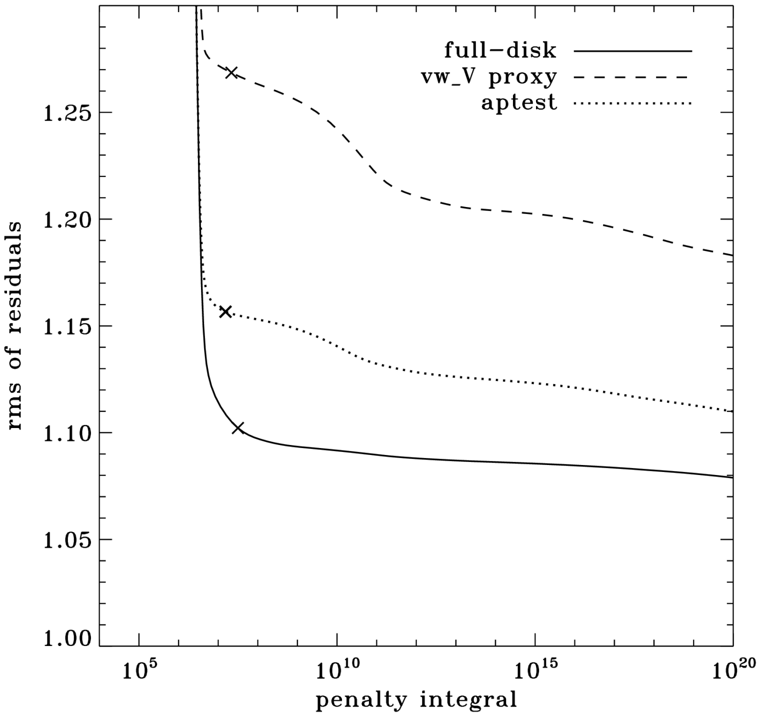
Our first test was to apodize in longitude from 54.1° to 60.5° with a hard cutoff in latitude at 60°. This would extend outside the crop radius at the corners, so an additional hard cutoff at an image radius of 0.88R was also imposed.

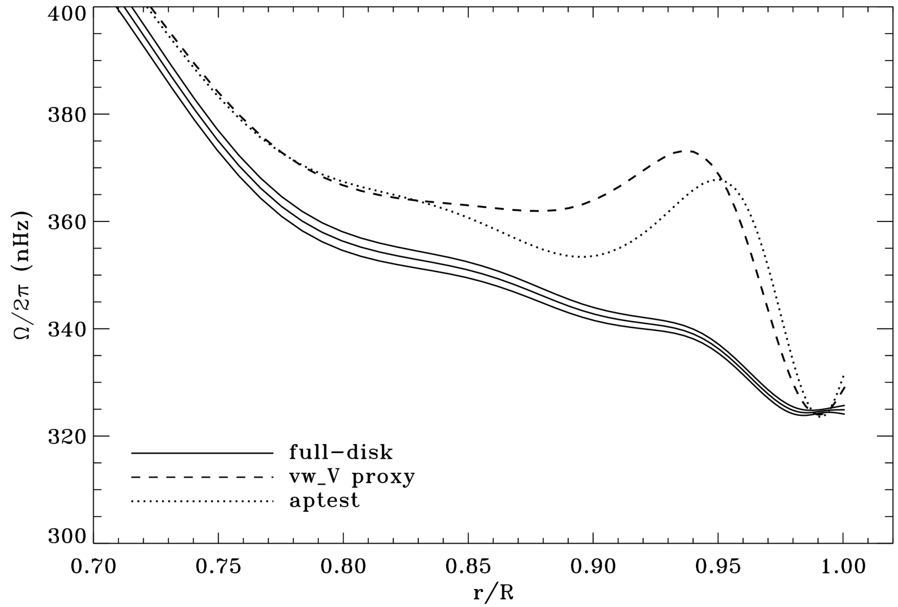
Low $|B_0|$



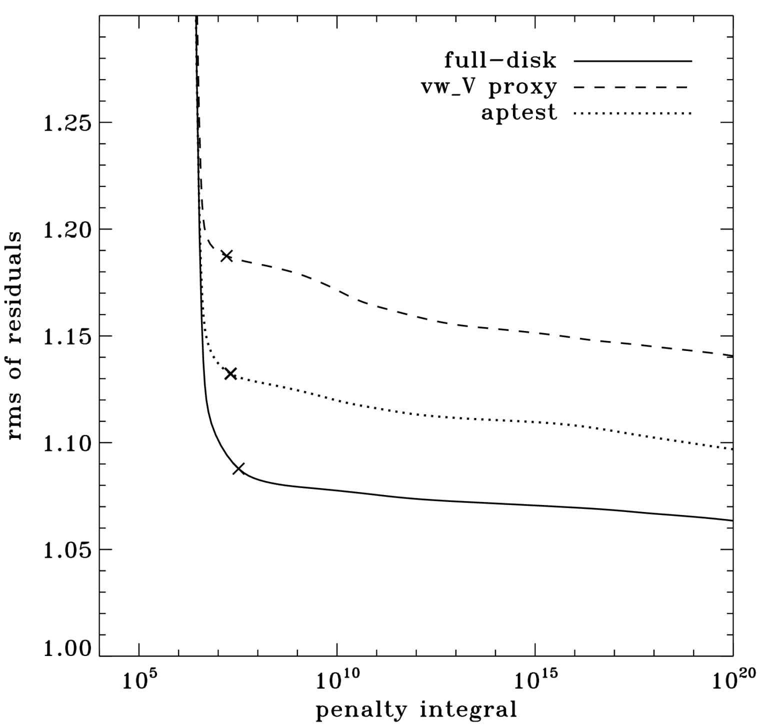


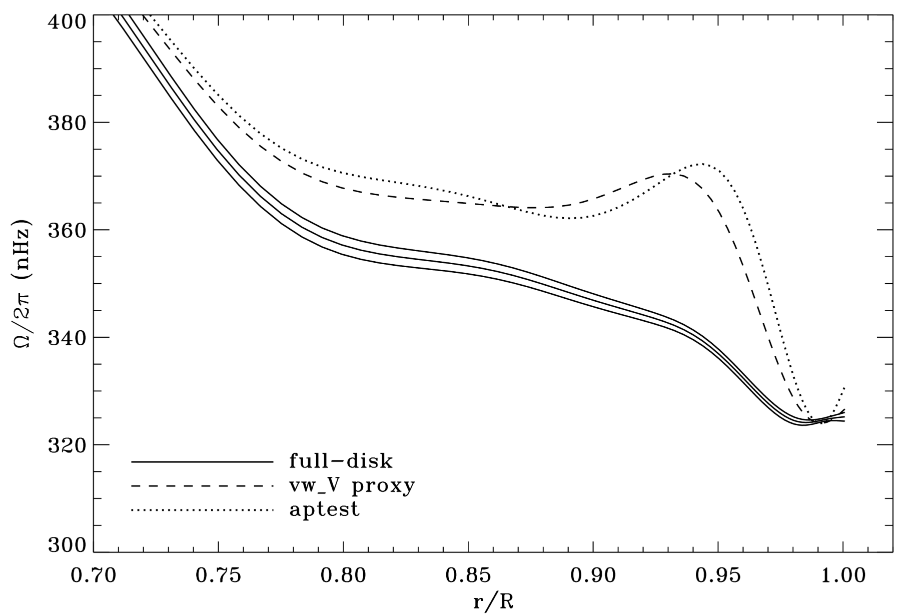
High $|B_0|$





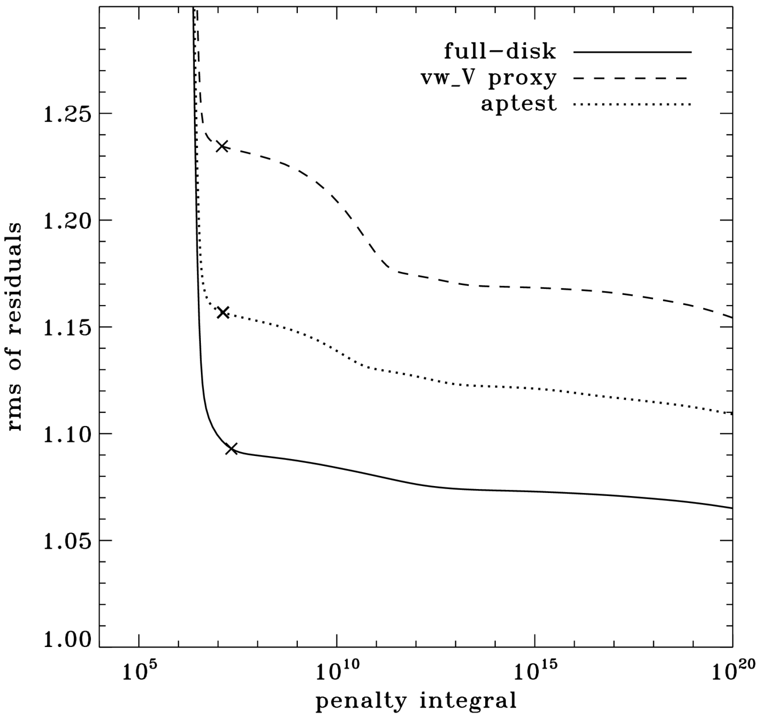
New Leaks

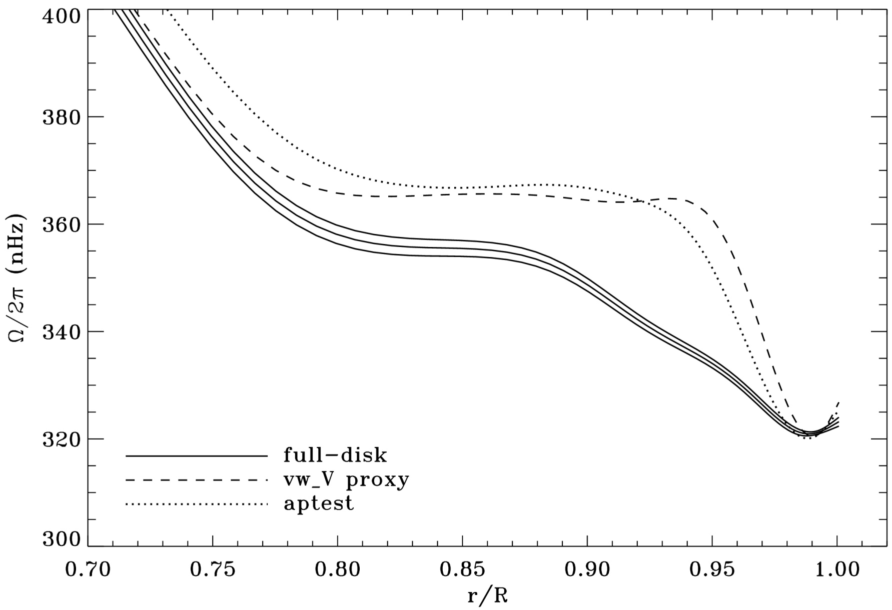




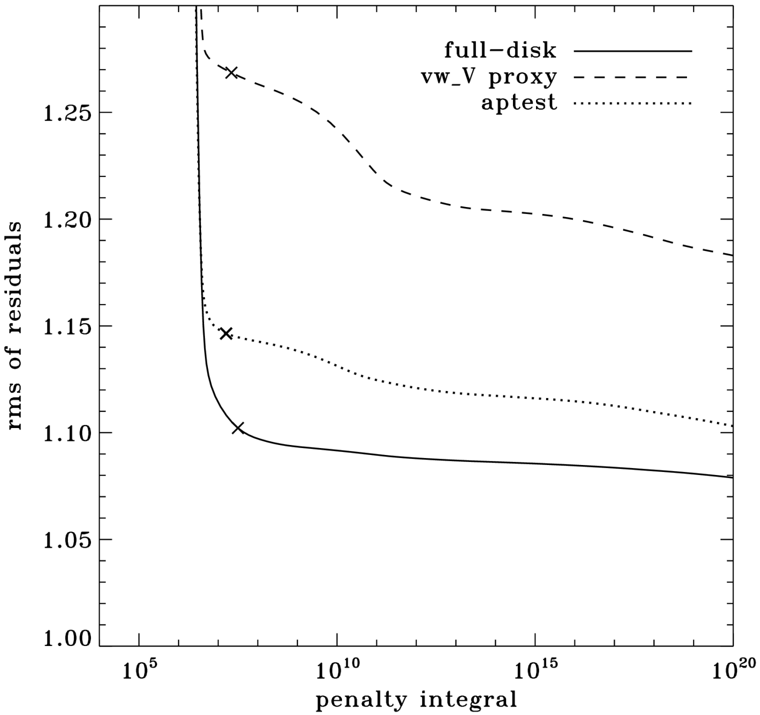
These results were mildly encouraging. Since the hard cutoff in latitude would be expected to spread power around, our next test apodized in latitude the same as in longitude, leaving the hard cutoff at r=0.88R.

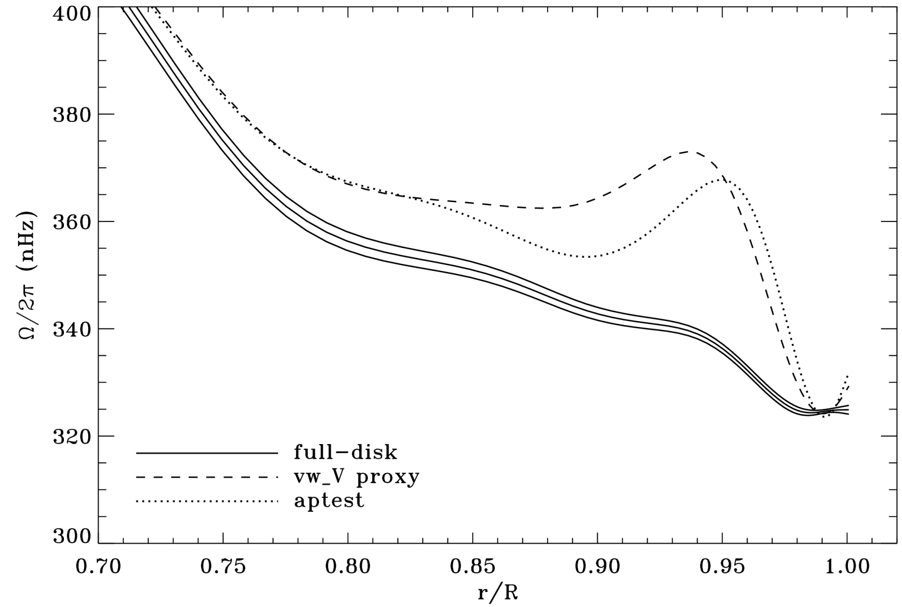
Low $|B_0|$



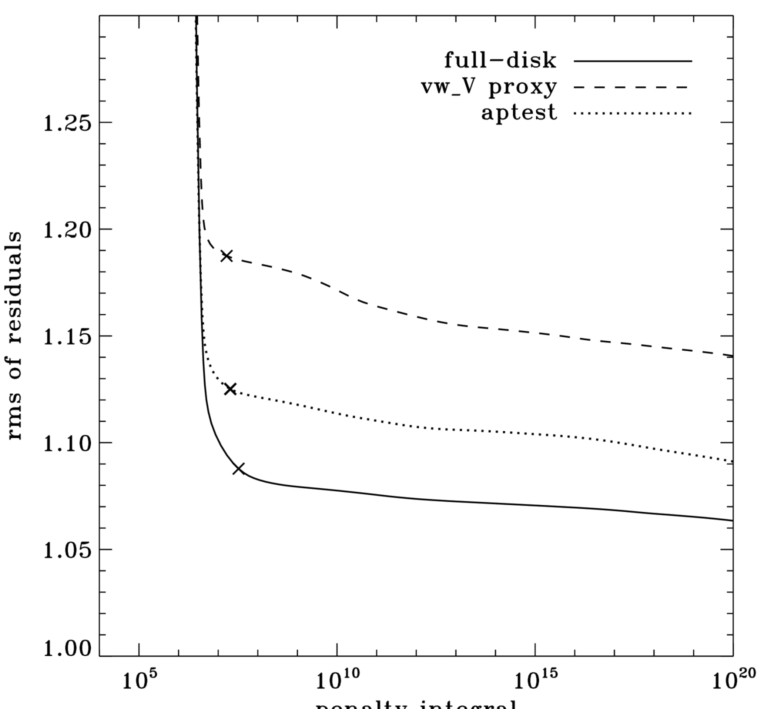


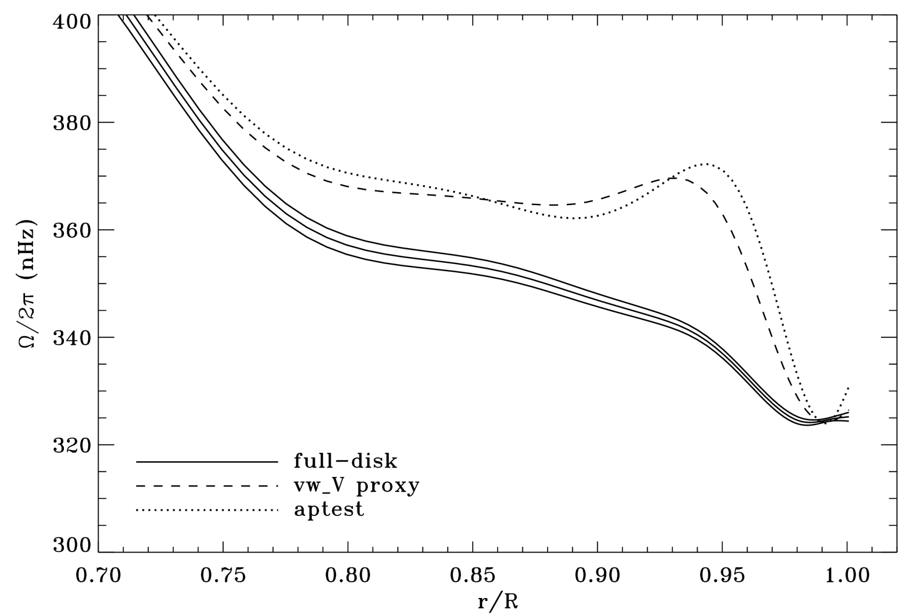
High $|B_0|$





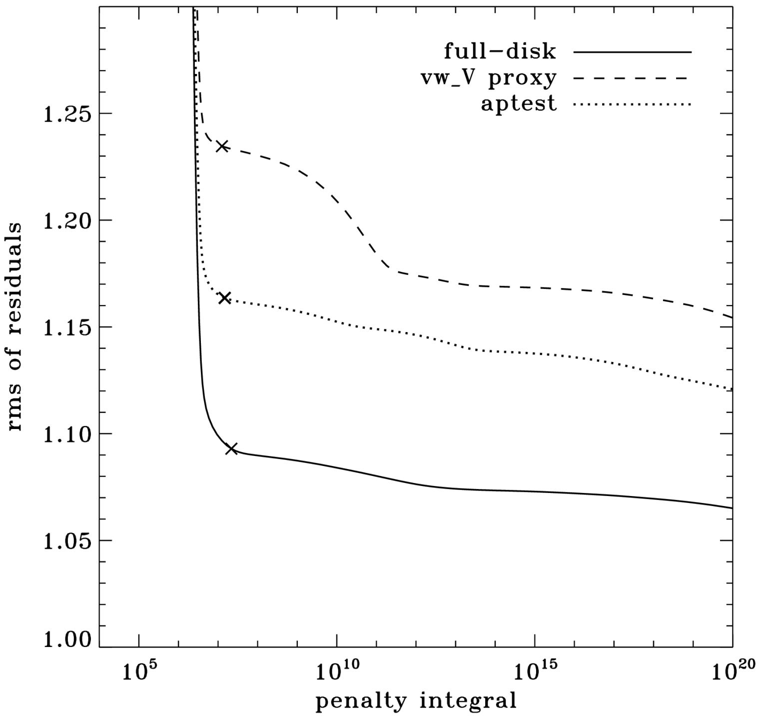
New Leaks

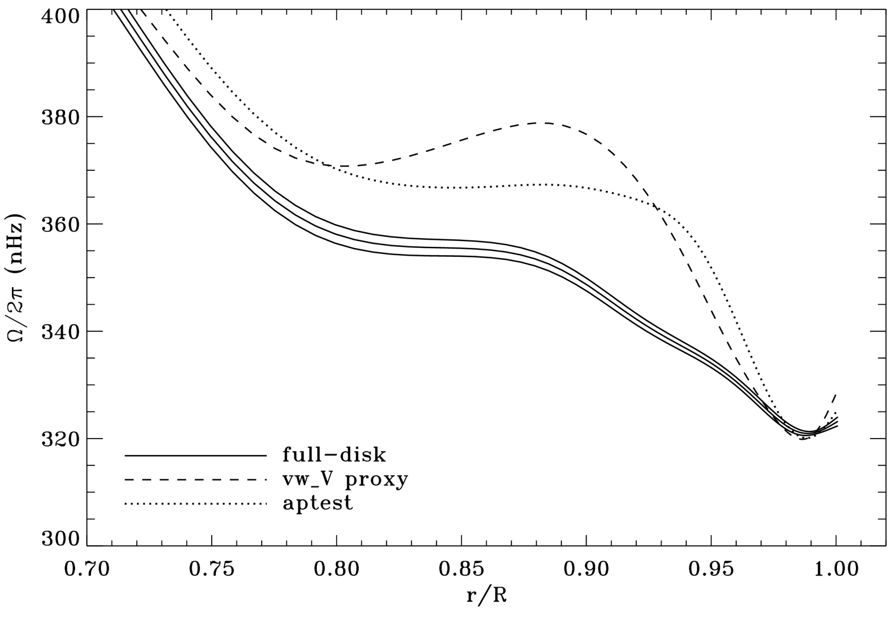


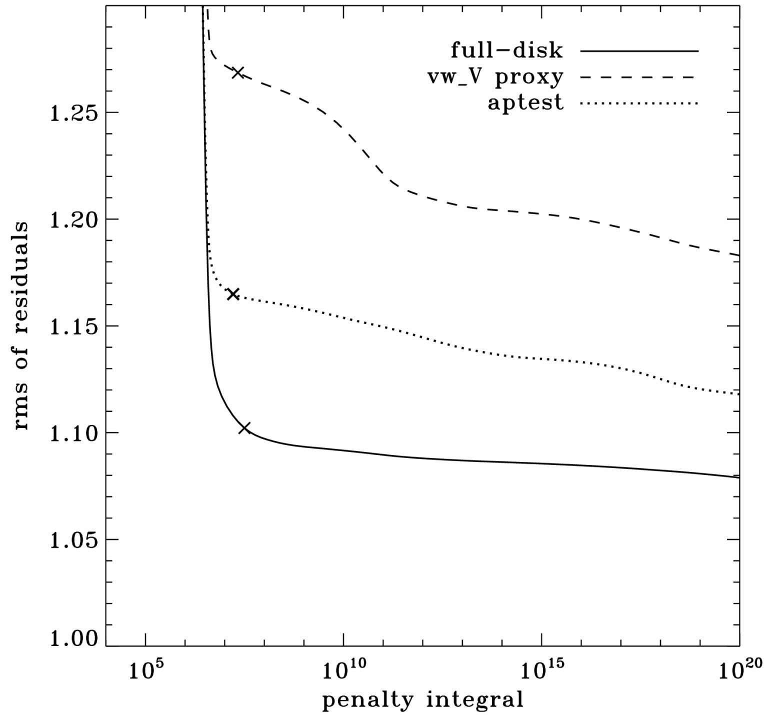


The residuals got even lower, but there was virtually no change in the rotation rates. For our next test we apodized more tightly in longitude and latitude, from 50.0° to 54.4°. This time we also apodize in fractional image radius from 0.87 to 0.88.

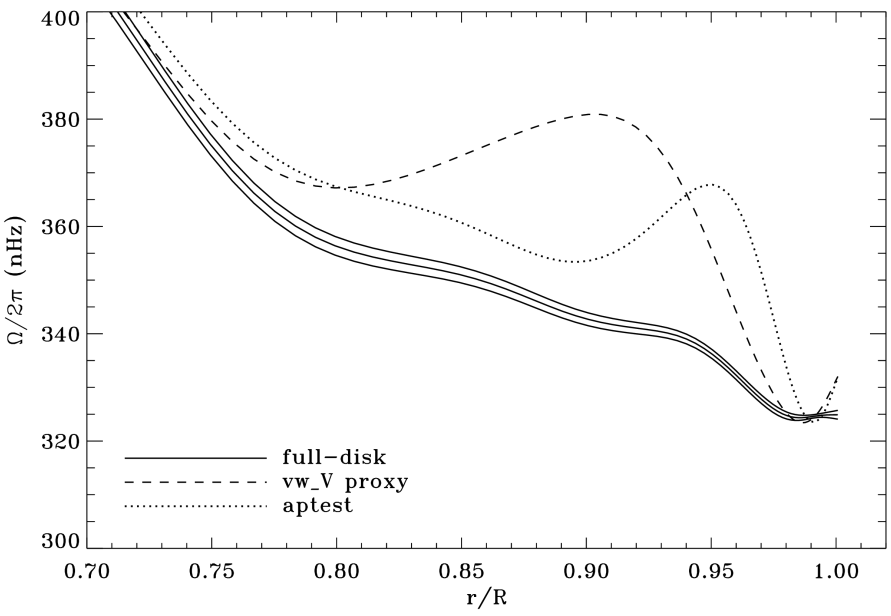
Low $|B_0|$

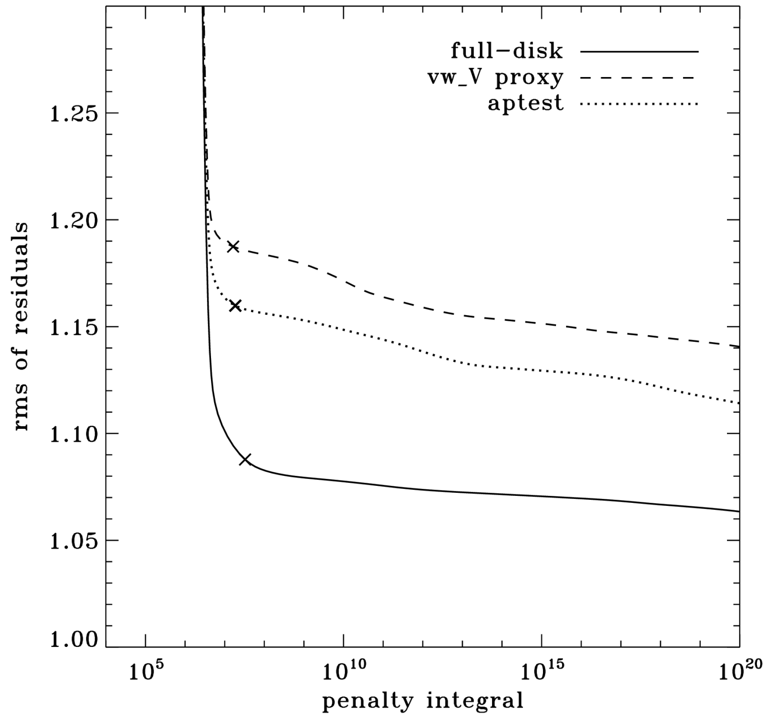




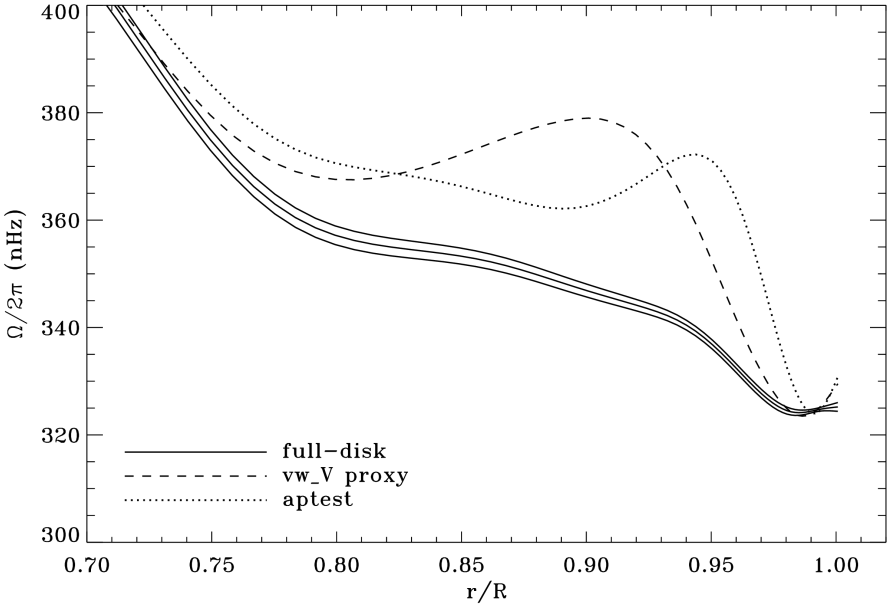


High $|B_0|$



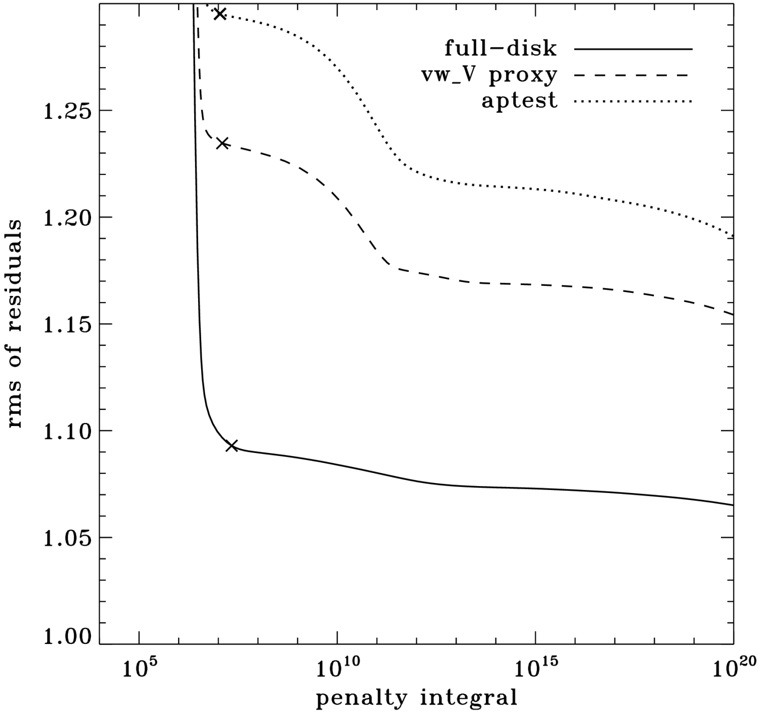


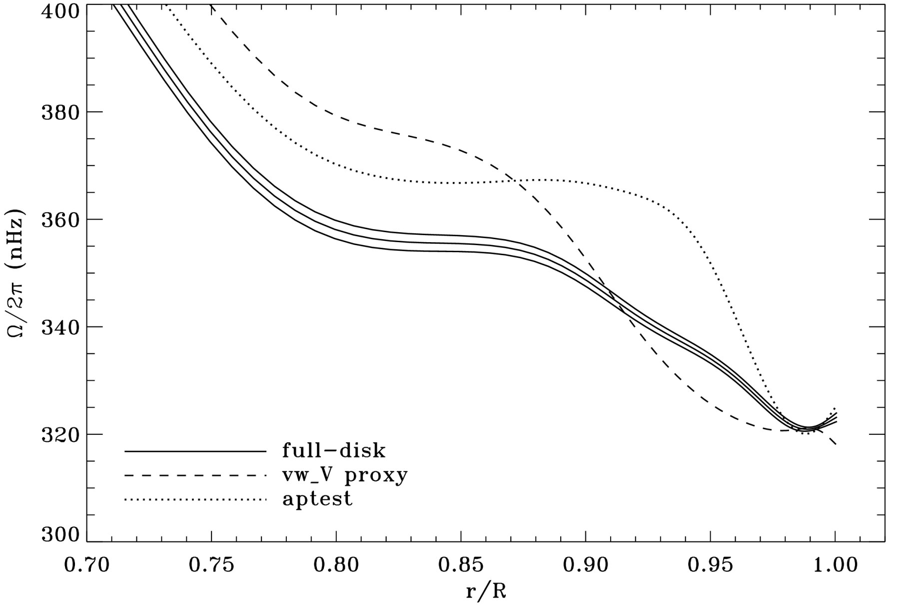
New Leaks

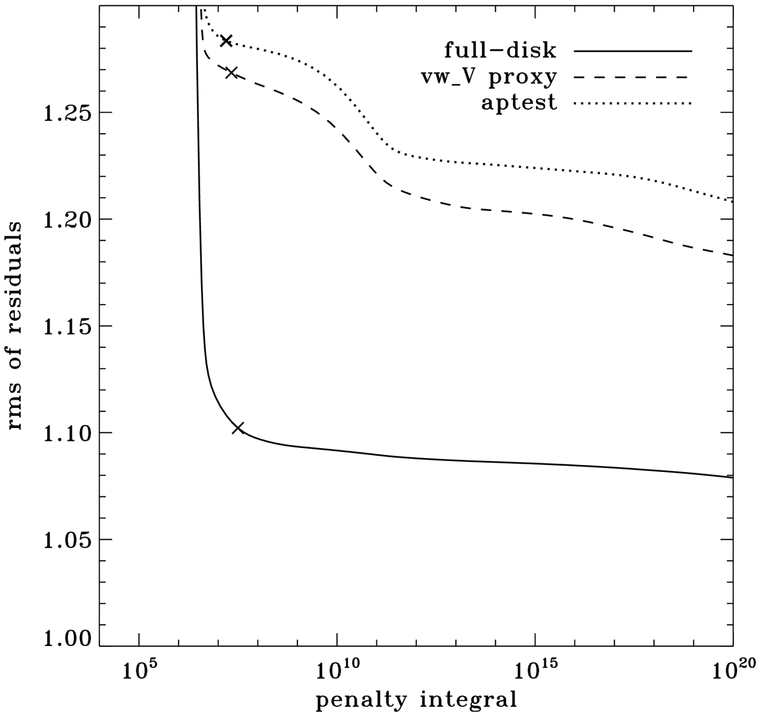


The residuals were marginally increased, and this apodization placed the jet deeper, higher, and wider. As a final test, since the code allows it, we tried an elliptical apodization, compressing by 0.9 in the horizontal direction. This implied a wider (less steep) apodization in the vertical direction.

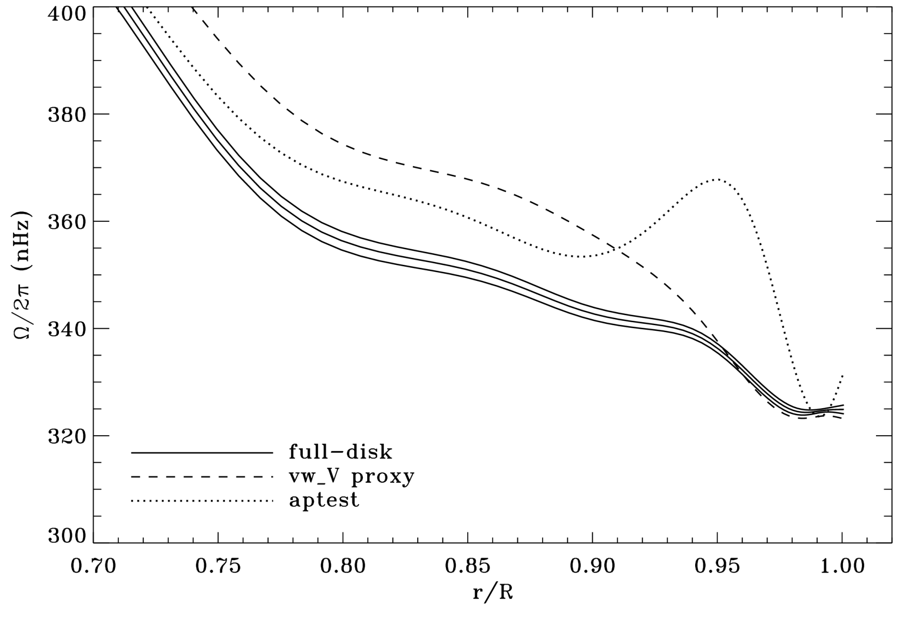
Low $|B_0|$

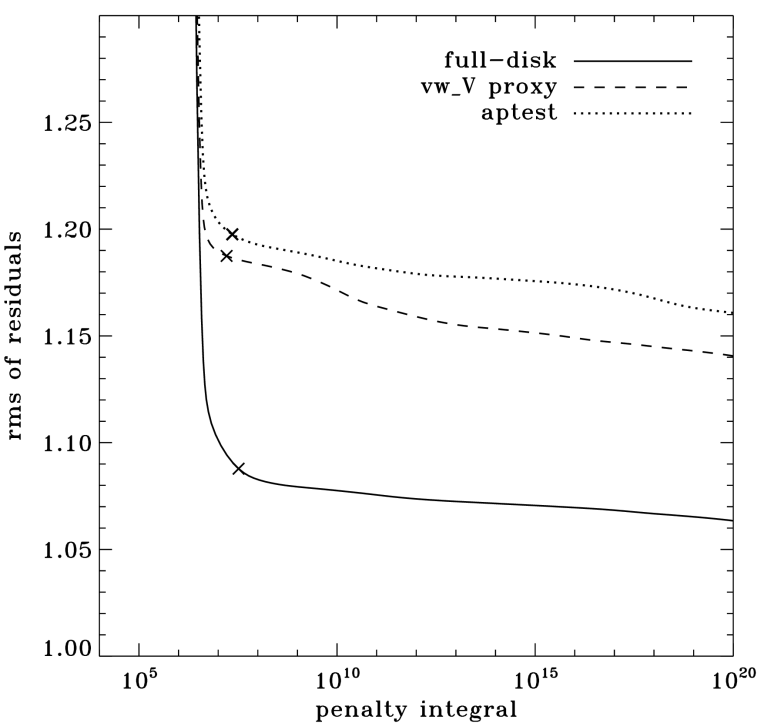




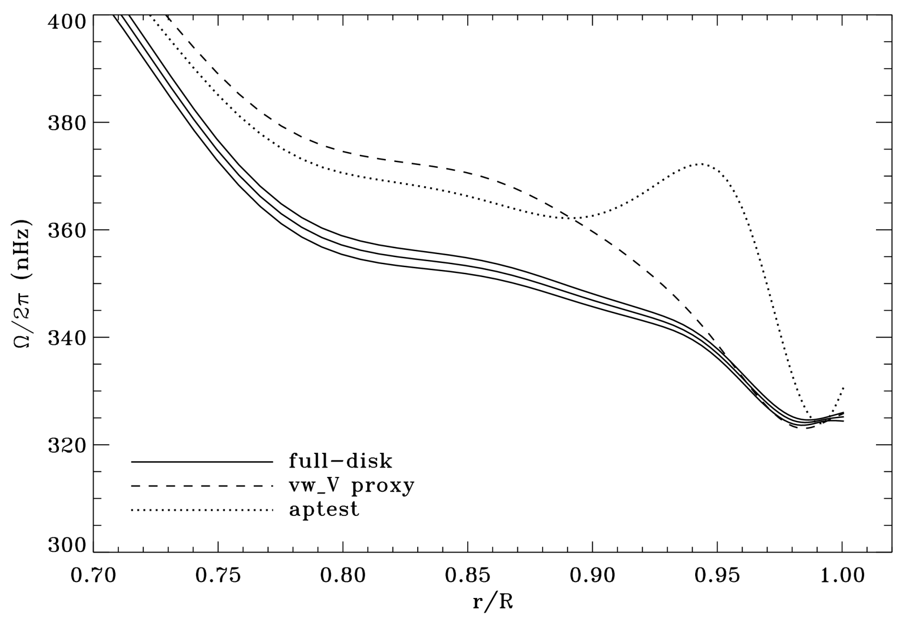


High $|B_0|$





New Leaks



Although this last test resulted in residuals even higher than the original, for the high-$|B_0|$ interval the rotation rates came into agreement with the full disk near the surface.

We are perhaps no closer to understanding how the apodization affects the results in the first place, but these tests give one hope that some of the systematic errors in the vw_V analysis might be reduced. In particular, it seems the bump in the residuals of odd a-coefficients might be nearly eliminated.

Summary

Using new leaks generally decreased the residuals and the bump for the vw_V data, and improved agreement in the near-surface radial gradient.

Apodizing in longitude/latitude also decreased the residuals and the bump. When using the larger apodizations and a leakage matrix for the actual value of B_0, the rotation rates did not differ substantially.

Elliptical apodization increased residuals, but improved agreement with full disk near the surface.

Future Work

All inversions presented here used identical tradeoff parameters, but these might be varied for the different analyses.

All mode fitting used leaks up to a maximum Δl of 6, but increasing this might improve the results of the different apodizations.

The effect of the width of the frequency interval used for mode fitting on the results of the different apodizations should be explored.