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Reduction of Systematic Errors in Global Mode Analysis

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Abstract

In spite of the unprecedented success of the MDI Medium-l Program, the global mode analysis pipeline is known to contain errors. Physical effects such as line asymmetry, horizontal displacement at the solar surface, and distortion of eigenfunctions have been ignored, as well as a wide array of instrumental effects. Additionally, certain improvements in the pipeline algorithm itself are possible, most notably in the gap-filling of time series. Perhaps unsurprisingly, some features of the results seem to be the effect of systematic errors. The most remarkable of these features are an annual variation in f-mode frequency changes, a bump in the normalized residuals of the a-coefficients around 3.4 mHz, and polar jets in the inversions. In this poster we discuss the application of a variety of corrections to the analysis, the resulting changes in the mode parameters, and the effect on the magnitude of systematic errors. We also describe new software tools that have been created to simplify and generalize the pipeline, making it of greater utility to anyone interested in global mode analysis.

The Pipeline

Our work begins with MDI medium-l Dopplergrams. These are essentially velocity images of the Sun's surface with a resolution of about 200x200 pixels taken at a cadence of one minute. Each image is decomposed into its spherical harmonic components, up to $l = 300$, by performing a spherical harmonic transform (SHT). We start by apodizing in image radius and remapping to a uniform grid in longitude and sin colatitude. This remapping must take into account the relative positions and velocities of Sun and observer, and also provides an opportunity to correct for certain instrumental errors. An inner product is then taken between each image and the set of target spherical harmonic modes. It must be noted, however, that since we can only see slightly less than half of the Sun, the modes cannot be perfectly separated by this process and leak into each other. The next step is to rearrange the data into timeseries, typically of length 72 days, one for each target l and m. Gaps in the timeseries are identified, the timeseries is detrended, and all gaps up to a certain size are filled. A Fourier transform is then performed and the transforms themselves (rather than the power spectra) are fitted using a maximum likelihood minimization and taking into account the correlations (leaks) between the modes. This part of the analysis has acquired the moniker of peakbagging; it yields the peak frequency, amplitude, and linewidth for each l and n it was able to fit. Modes with different m are assumed to have the same amplitude and linewidth, and a frequency given by a polynomial in m/l, the coefficients of which are also fit for during the peakbagging. Once we have obtained all these mode parameters, we can perform an inversion to obtain (among other things) the angular velocity of the Sun as a function of latitude and radius.

The Corrections

The following table summarizes all the corrections to the above procedure that we have made to date. Each subsequent correction includes all previous ones.

	Abbr.	Takes into account...
1	wood	Woodard effect: distortion of eigenfunctions by differential rotation
2	horiz	Horizontal displacement at the solar surface
3	rad	Correct radius of the sun in SHT's
4	dist	Cubic distortion from optics in leakage matrix
5	pang	Correct P-angle in SHT's
6	cinc	Correct Carrington inclination in SHT's
7	tilt	Tilt of CCD, cubic distortion now corrected in SHT's
8	gaps	New algorithm for gap-filling and detrending
9	asym	Asymmetric line profiles

The first two effects are handled in the peakbagging through the leakage matrix, which quantifies how much the modes leak into one another. Although the Woodard effect tells us that the spherical harmonics are not really eigenfunctions of the oscillation equations in the presence of differential rotation (Woodard, 1989), the true eigenfunctions can still be expressed as a sum over spherical harmonics. The standard pipeline approximates the displacement at the solar surface as being purely radial, which is to say that it ignores the horizontal components in the leakage matrix. Although this is a good approximation for high-order p-modes, f-modes and low-order p-modes have significant horizontal displacement. The third correction is the first one to require repeating the SHT. The standard pipeline uses a fixed value for the solar radius in pixels, but it has been found that this value in fact varies over time. The next four corrections deal with instrumental issues, with the exception of the Carrington inclination correction, which uses an updated value for the orientation of the solar rotation axis. Although one can account for the cubic distortion using a modified leakage matrix, it is more correct to account for it in the SHT; we tried both methods. The last correction is the most tenuous; it is difficult to get convergence of the mode parameters during peakbagging. Results from the asymmetric fits should still be regarded as preliminary.

The effect of these various corrections on the mode frequencies is summarized in the figures below. For each correction we reanalyzed two years of data. To summarize each analysis, we took the average over the two years by taking the modes that were fit at least 70% of the time and fitting a second-order polynomial to each parameter. We then took the average of the fit. Each plot shows the difference between one analysis and a previous one for the modes they had in common (fit in 70% of both analyses).

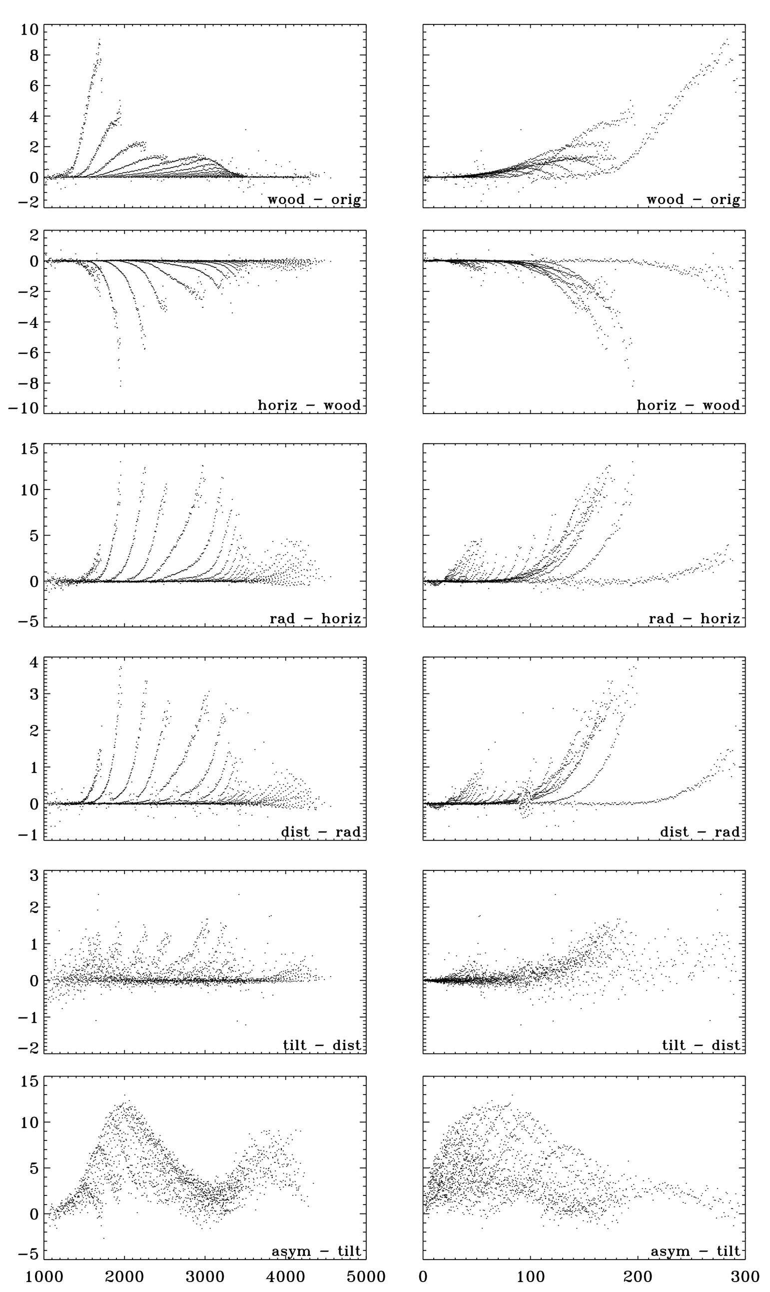

Figure 1: Frequency differences between selected analyses in units of standard deviation, on the left as a function of frequency in μHz, on the right as a function of l.

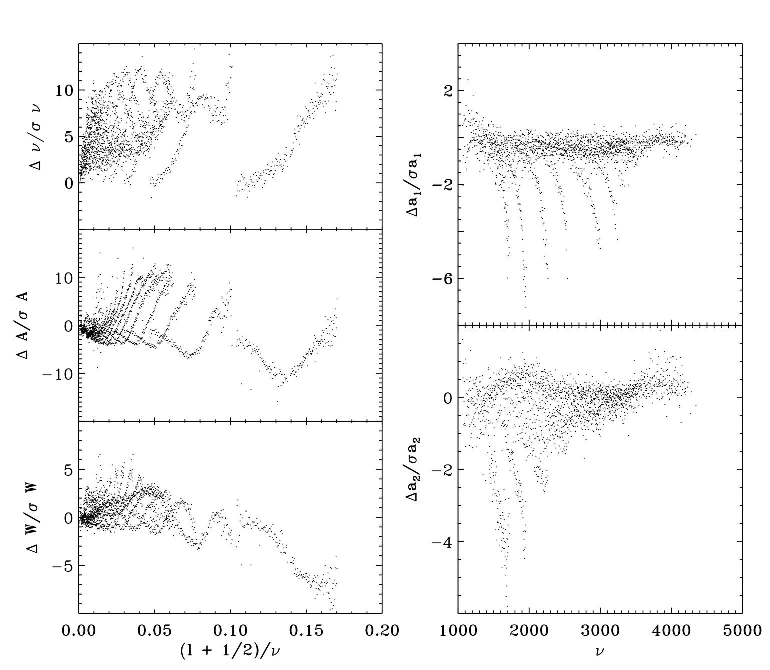

Figure 2: Comparison of original and final analyses for frequency, amplitude, linewidth, a_1 and a_2.

The Problems

Looking at a plot of fractional change in the seismic radius of the Sun as a function of time (Figure 3), one sees something strange. The first thing to notice is a variation in sync with the solar cycle, but one can also discern a smaller variation with a period of almost exactly one year (Antia et al., 2001). It is believed that this annual term is an artifact of the analysis, which is to say there must be some systematic error for which we are not correcting.

The relative change in seismic radius is proportional to the relative change in f-mode frequency. To explore this phenomenon, in the case of each correction we fit a function of the form $f(t) = A\sin(\omega_{yr}t) + B\cos(\omega_{yr}t) + Ct + D$ to the fractional change in average f-mode frequency, with t measured

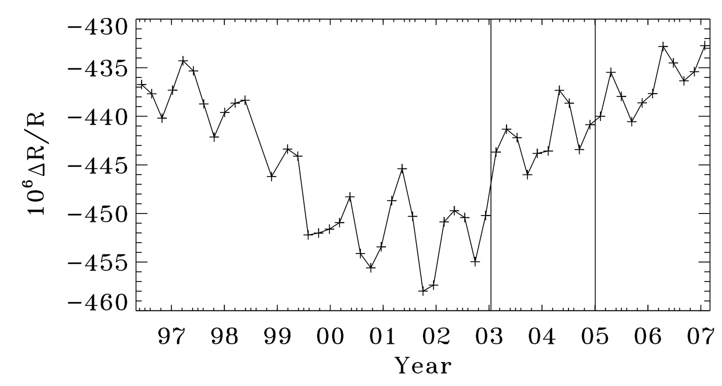

Figure 3: Change in seismic radius of Sun. The vertical lines mark the two years over which we repeated the analysis.

in days and $\omega_{yr} = 2\pi/365.25$, which assumes that the solar cycle variation can be represented by a straight line over this interval. We quantify the error by the magnitude of the annual component, $\sqrt{A^2 + B^2}$. The results are summarized in the phase diagram below.

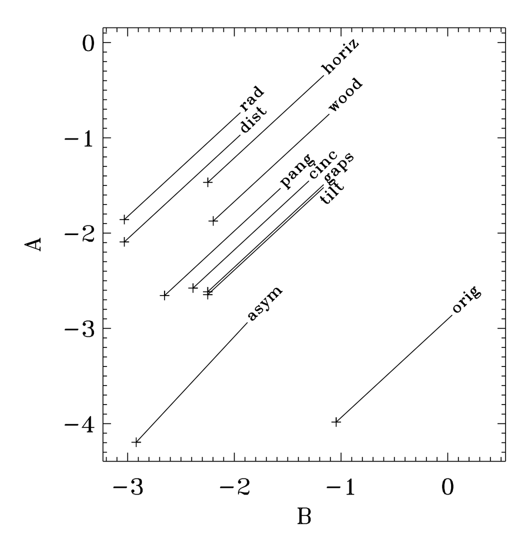

Figure 4: Phase diagram for frequency changes. Plusses mark the initial point, straight lines indicate the change made by the Doppler correction.

Our initial results were somewhat disheartening; none of the corrections seemed to move us much closer to the origin, and some even increased the magnitude of the annual component. It then occurred to us that the mode frequencies themselves were Doppler-shifted by the relative motion of the Sun and spacecraft. Applying this correction and refitting the frequency changes moved the points as indicated in the diagram.

Another anomaly in the analysis shows up in the inversion of the a-coefficients. After the inversion is performed, we can tell what a-coefficients our derived rotation profile would yield. Taking the difference of these output coefficients with the coefficients we input and dividing by the error should yield points normally distributed around zero. What is observed instead is a rather striking bump around 3.4 mHz (Figure 5).

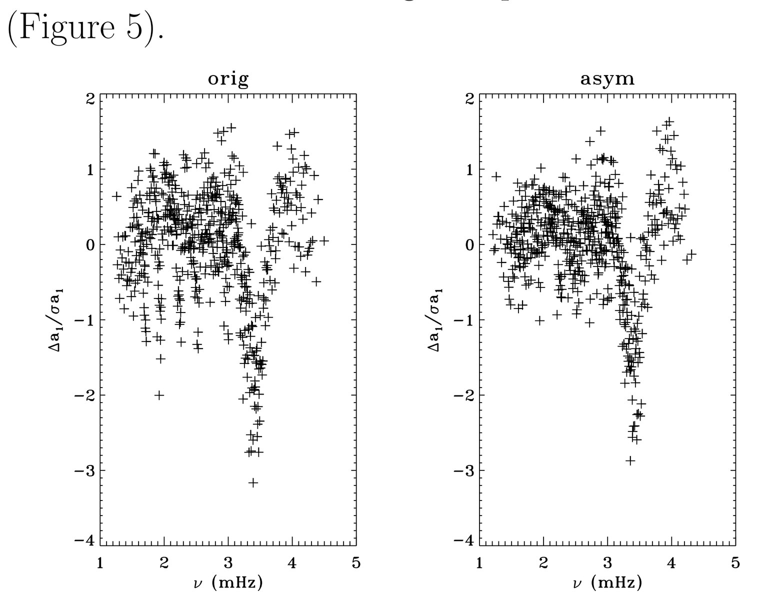

Figure 5: Normalized residuals of a_1 for the original analysis and after all corrections. These plots have been binned in frequency for clarity.

To study the bump, which appears in all the odd a-coefficients, we chose to fit a Gaussian to it. Unfortunately this revealed that no set of corrections made any significant change in the bump, although one can see from the figure that the "horns" have been reduced. In every case and for all coefficients, the peak frequency of the bump is always about 3.4 mHz. We plan to do a detailed study of the power spectrum to see if any features can be correlated with this frequency. There is also some indication that the bump may have a slight time dependence.

Another problem with the inversions can be seen in the rotation profile itself (Figure 6). The feature

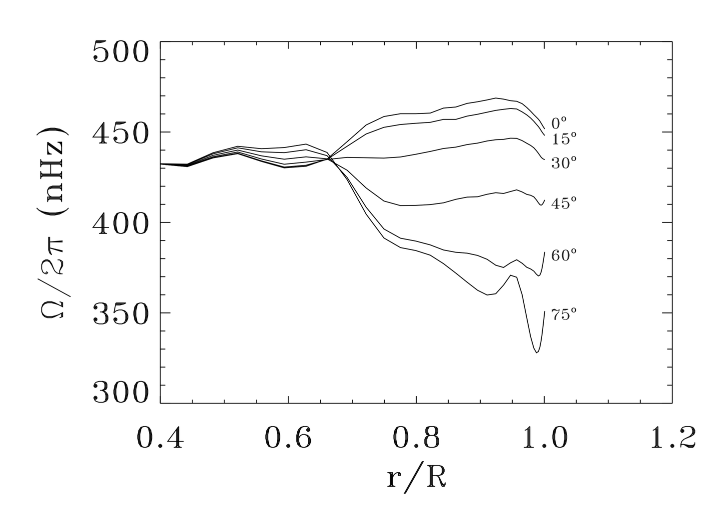

Figure 6: Rotation rate of the solar interior at selected latitudes as a function of fractional radius.

that is believed to be spurious is the polar jet, a spike in the rotation rate at high latitudes. Although it could possibly be a real feature of the Sun, discrepancies with the GONG project make this unlikely (Schou and Howe et al., 2002). One might be tempted to assume that the polar jet is related to the bump in the a-coefficients, but inversions of mode sets below 3 mHz still show this feature. Although the corrections we applied make some changes in the rotation profile in the region $0.5 < r/R < 0.9$, all our analyses still show the jet.

Conclusions

We found that several of the corrections we made resulted in highly significant changes in the mode parameters. Not counting the asymmetric fits, the corrections taken all together also made a reduction in the magnitude of the annual component in the f-mode frequency changes. But the largest breakthrough in that regard was the application of a Doppler correction to the mode frequencies, which made the greatest reduction.

We had somewhat less success treating the anomalies in the inversions. The bump in the a-coefficients and the polar jet remained unaffected. Hence we have been able to rule out nine effects as their possible cause. The deviations in mode parameters at the end of the ridges (the horns), however, were greatly reduced.

Perhaps our greatest accomplishment in the course of this work has been the development of new software to streamline the pipeline. It is now possible to proceed from the Dopplergrams all the way through the inversions using a single tool. Arbitrary lengths and start times of the timeseries may be selected. Location of gaps and handling of processing errors have been automated. Hence repetition of the analysis making whatever changes desired has been facilitated.

Future Work

The first order of business is to finalize the results from the asymmetric fits. This is likely to require inserting a method for rejecting bad modes between iterations of the peakbagging. Such an improvement is likely to affect the fits in the other analyses as well.

The Doppler correction to the mode frequencies is now applied after the end of the normal processing. To properly account for this effect from the beginning, we need to correct the timeseries for the fact that due to time dilation, a minute on the spacecraft is not a minute on the Sun.

We also plan to use different length timeseries and more fully explore the discrepancies with the GONG project. We may also have the opportunity to validate our analysis using simulated helioseismic data (Hanasoge et al., 2006). If the reader has any further suggestion for corrections, please contact the authors.

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