SYSTEMATICS (S. Vorontsov)

My approach to hunting for systematics in tabulated rotational splittings can hardly be more straightforward or waterproof. For a particular degree s of the rotational splitting coefficient a_s, $s = 1, 3, 5, \ldots$, I am just plotting the a_s-values for all the multiplets versus the position of the inner turning point. Due to high-frequency asymptotic properties of solar p modes, the results shall collapse close to a single curve. Slight deviations from this common behaviour can be caused by regions of rapid variation of the rotational velocity with depth on a scale short compared with radial wavelength (e.g. base of the convection zone), or by its sharp variation very near the surface, in a thin layer spanned by the (frequency-dependent) upper turning points.

In the a_1 to a_9 plots which you can see, I compare Tim and Jesper's (Stanford) results (on the left) with those obtained by Sylvain (on the right) from the same data and with using the same (Jesper's) leakage matrix. Horizontal axis is dimensionless acoustic radius of the inner turning point (one can use anything which varies monotonically with $(l+1/2)/\nu$). The base of the convection zone is at about $t_1 = 0.4$. To make them comparable with a_s, Sylvain's "CG-coefficients" need proper rescaling. To bring an equivalent to a_s, his CG-coefficient of degree s has to be multiplied with

$$(-1)^{k+1}\left(\frac{2s+1}{2}\right)^{1/2}\frac{(l-1)!}{(l-k)!}\frac{(2l+1)!!}{(2l+s)!!}\frac{s!!}{(k-1)!}, \quad s = 2k-1, \quad k = 1, 2, \ldots.$$

Here, $n!!$ designates the product of all positive integers of the same parity up to and including n (e.g., $5!! = 1 \times 3 \times 5$). This recipe came from my calculation, based on Sylvain's explanation of how he defines his CG-coefficients. The scaling coefficient which is needed to get a_1 is $(3/2)^{1/2}$. Note that this recipe is not applicable to rescaling even coefficients.

To address more multiplets, I am using 360d data sets. To reduce random errors, I am averaging consecutive data sets. Simple arythmetic (i.e. unweighted) average is implemented, to address mean solar rotation over the entire time interval. To avoid bias, only those modes which are present in all the consecutive data sets are taken into account. First 11 of the MDI "Medium-l" (vw) 360d data sets were used in the average (the idea was to address solar rotation averaged over the activity cycle). The results are shown by the panels of the upper row. For HMI full-disc (fd) velocity measurements (panels in the middle row), 6 first 360d data slots were averaged (Tim has already tabulated the 7th, but I only see 6 in Sylvain's data files). In each figure, a single panel of the bottom row shows the result obtained with Sylvain's analysis of 22 consecutive GONG 360d data sets (it is hard to believe that we already have two solar cycles covered by the GONG measurements...). I am displaying the results of symmetric fits only (rotational splittings are not influenced much by accounting for asymmetry). Stanford data is that obtained with polynomial fits of degree 18. As Sylvain limits his polynomial approximation by degree 9, I am only addressing the rotational splitting coefficients up to and including a_9.

In all the plots, red points correspond to multiplets with centroid frequencies below 2mHz, green—with frequencies between 2 and 3mHz, blue is for frequencies above 3mHz. Shown in black are a_1, a_3 and a_5 calculated directly from a rotating solar model with internal rotation specified by a result of one of my old rotational inversions, targeted at measuring the internal rotation at solar activity minimum. The synthetic coefficients (black) are shown for all the modes which are present in the corresponding observational average (shown in colour; and for these modes only).

What do we see? Look at MDI vw data first. Stanford results: obvious and perfectly measurable systematics (higher-frequency ends of fitted p-mode ridges show up as regular repetitive "horns"), changing sign between consecutive a-coefficients, with magnitude of up to 2nHz or even higher. Any decent rotational inversion will reject this measurement: there is no regular solution which could fit the data with any sensible likelyhood (I do not draw errorbars, but magnitude of random errors can well be judged from the random scatter on the plots). Sylvain's result is apparently free (or almost free) of this problem. Another advantage of the Sylvain's result is significantly better coverage of lower-frequency multiplets.

Now HMI fd. Stanford result: detectable systematics have gone! (or almost gone, there are some signatures probably left in a_7 and a_9). Random scatter of the plotted averages is smaller than that of Sylvain's, which indicates smaller random errors in splitting measurements. A closer look at plots with Sylvain's results reveal some signatures of systematics. On the other hand, the amount of lower-frequency multiplets which were fitted successfully by Sylvain is significantly bigger (as with MDI data). I would refrain from judging which one of the two data sets can provide better absolute measurement of the internal solar rotation.

In measuring "torsional oscillations", one obviously has to address the results of both the techniques, as it is reasonable to believe that systematics are largely time-independent and will thus cancel when taking the splitting differences.

Addressing the origin of the striking differences which arise when the same input data is analyzed with Stanford and Sylvain's techniques may help to improve both.