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Viscosity in the Solar Wind

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The effects of viscosity on a steady, radial, spherically symmetric solar wind with an embedded, non-radial magnetic field are reconsidered. The correct expression for the classical viscosity in the presence of a non-radial magnetic field is shown to be different from that used in the past, and a means of describing non-classical viscosity is presented. A physical interpretation of the classical and nonclassical descriptions of viscosity is provided, and observational inferences are used in discussing the nature and degree of viscous effects in the solar wind.

1. INTRODUCTION

Over the past two decades, a number of authors have considered the possible effects of viscosity on the solar wind. *Parker* [1963] pointed out that one can expect viscosity to play a relatively minor role in comparison with thermal conduction in determining the total solar wind energy flux and the basic dynamical character of the solar wind expansion. The subsequent suggestion [*Scarf and Noble*, 1965] that viscous wind models exhibit dynamical characteristics fundamentally different from those of inviscid models was shown [*Whang et al.*, 1966; *Axford and Newman*, 1966; *Newman and Axford*, 1967; *Summers*, 1980] to reflect an improper treatment of the higher order viscous equations, and Parker's original notion was found to be correct. *Wolff et al.* [1971] then suggested that viscosity might play a significant role in determining the proton temperature observed at the orbit of earth, through the conversion of a (small) fraction of the solar wind flow energy into internal energy of the protons. Despite considerable discussion of this suggestion [e.g., *Leer and Holzer*, 1972; *Brandt and Wolff*, 1973; *Holzer and Leer*, 1973; *Price et al.*, 1975] the question of what effect vicsocity has on the proton temperature remains open.

In the present paper, we reconsider the effects of viscosity on solar wind momentum and energy balance, but devote little attention to the role of viscosity in angular momentum transport [e.g., *Weber and Davis*, 1970] and shock structure [e.g., *Newman and Axford*, 1967; *Summers*, 1980]. First, we attempt to correct some misconceptions that have arisen from previous work [e.g., *Holzer and Axford*, 1970; *Weber and Davis*, 1970; *Wolff et al.*, 1971; *Hundhausen*, 1972; *Brandt and Wolff*, 1973; *Holzer and Leer*, 1973; *Summers*, 1978]. These misconceptions involve the mathematical description of classical viscosity in a magnetized plasma, the validity of the classical description, the relationship between viscosity and pressure anisotropy, and the description of nonclassical viscosity. We then consider the inferences that can be drawn from presently available solar wind observations concerning the role of viscosity in the solar wind.

Following completion of the present paper, we have become aware of an independent study of the same general subject by *Hollweg* [1985], and readers of the present paper are encouraged to examine Hollweg's paper (and a related paper by *Bravenec et al.* [1982]). It presents the mathematical derivations in a more compact form, which anyone wishing to study viscous effects in a different geometry will probably find more useful. Our presentation is designed to provide a physical feeling for viscous effects in the unstructured solar wind, and our mathematical formulation is tailored to that end.

2. CLASSICAL VISCOSITY

The classical description of viscosity in a strongly magnetized plasma [e.g., *Braginskii*, 1965] is represented in terms of the stress tensor, $\boldsymbol{\pi}_j = \rho_j\langle \mathbf{V}_j\mathbf{V}_j - \mathbf{I}V_j^2/3\rangle$, where ρ_j is the mass density of the jth particle species, V_j is the random component of the particle speed, $\mathbf{I}$ is the unit tensor, and the angle brackets denote an average over the particle velocity distribution function. (The "classical" description of dilatation viscosity includes only the effects of non-uniform expansion (or compression) and of collisional energy exchange among different translational degrees of freedom on the determination of the pressure anisotropy. A "non-classical" description of dilatation viscosity, as we use the term here, is one in which other physical effects are included in the determination of the pressure anisotropy: e.g., effects of wave-particle interactions. What we refer to as classical viscosity is sometimes referred to as Newtonian viscosity.) In a proton-electron plasma, protons generally provide the major contribution to viscosity: i.e., $\boldsymbol{\pi} = \boldsymbol{\pi}_p + \boldsymbol{\pi}_e \approx \boldsymbol{\pi}_p$, where the p and e subscripts refer to protons and electrons. Viscous effects in the momentum and energy balance of the steady, radial, spherically symmetric flow of such a plasma enter into a one-fluid description in the following way [e.g., *Braginskii*, 1965]:

$$\rho u \frac{du}{dr} = -\frac{dp}{dr} - (\nabla \cdot \boldsymbol{\pi})_r - \rho \frac{GM}{r^2} + \rho D \tag{1}$$

$$\frac{3}{2} pu \frac{d}{dr}(\ln T) = \frac{pu}{\rho}\frac{d\rho}{dr} - \boldsymbol{\pi}:(\nabla \mathbf{u}) + Q \tag{2}$$

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$$\frac{d}{dr}\left\{r^2\left[\rho u\left(\frac{1}{2}u^2+\frac{5}{2}\frac{p}{\rho}-\frac{GM}{r}\right)+(\boldsymbol{\pi}\cdot\mathbf{u})_r\right]+F\right\}=0 \qquad (3)$$

where (2) and (3) are alternative descriptions of energy balance, and mass balance is described, as usual, by $\rho u r^2 = \text{const}$. Here, the plasma density, pressure, temperature, and flow speed are represented by $\rho = \rho_e + \rho_p \approx \rho_p$, $p = p_e + p_p$, $T = (T_e + T_p)/2$, and $u = u_e = u_p$. The force-per-unit-mass D in (1) and the heating rate Q in (2) are source terms accounting for all the effects (including thermal conduction, in the case of Q) not explicitly appearing in these equations, and the energy flux F in (3) reflects the effects associated with D and Q.

The stress tensor can be written in terms of a rate-of-strain tensor [e.g., *Braginskii*, 1965] defined by

$$\mathbf{W} = \nabla\mathbf{u} + (\nabla\mathbf{u})^T - \tfrac{2}{3}(\nabla\cdot\mathbf{u})\mathbf{I} \qquad (4)$$

For a flow in which $\mathbf{u} = u\hat{e}_r$ and $\partial u/\partial\theta = \partial u/\partial\phi = 0$, the rate-of-strain tensor is diagonal in spherical coordinates (r, θ, ϕ) and has the form

$$W_{rr} = \frac{4}{3}\left(\frac{du}{dr}-\frac{u}{r}\right) \qquad (5a)$$

$$W_{\theta\theta} = W_{\phi\phi} = -\tfrac{1}{2}W_{rr} \qquad (5b)$$

The rate-of-strain tensor measures the rate of distortion of a fluid element, with the diagonal components reflecting the effects of expansion and contraction and the off-diagonal components (all zero in this case) reflecting the effects of shearing motion. Uniform, isotropic expansion or contraction does not change the shape of a fluid element and thus is not reflected in the rate-of-strain tensor: viz., in the case of radial, spherically symmetric flow, if the expansion rate in the direction of flow (du/dr) is the same as that normal to the flow (u/r), then $W_{rr} = W_{\theta\theta} = W_{\phi\phi} = 0$.

If there is a magnetic field lying in the $r-\phi$ plane and oriented at an angle α to the radial direction, $\mathbf{W}$ can be written as follows in the local orthogonal magnetic coordinate system $(b, \theta, \perp)$ in which the field is directed along the unit vector $\hat{e}_b$:

$$W_{bb} = W_{rr}(\tfrac{3}{2}\cos^2\alpha - \tfrac{1}{2}) \qquad (6a)$$

$$W_{\perp\perp} = -W_{rr}(\tfrac{3}{2}\cos^2\alpha - 1) \qquad (6b)$$

and $W_{\theta\theta}$ is given by (5*b*). Making use of (6) and *Braginskii*'s [1965] equation (2.21), we can write the stress tensor in the magnetic coordinate system and then transform it back to the spherical system to obtain

$$\pi_{rr} = -\eta_0\, W_{rr}(\tfrac{3}{2}\cos^2\alpha - \tfrac{1}{2})^2 + O(\omega_p{}^{-2}\tau_p{}^{-2}) \qquad (7a)$$

$$\pi_{\theta\theta} = \tfrac{1}{2}\eta_0\, W_{rr}(\tfrac{3}{2}\cos^2\alpha - \tfrac{1}{2}) \qquad (7b)$$

$$\pi_{\phi\phi} = \tfrac{1}{2}\eta_0\, W_{rr}(\tfrac{3}{2}\cos^2\alpha - \tfrac{1}{2})(3\cos^2\alpha - 2) + O(\omega_p{}^{-2}\tau_p{}^{-2}) \qquad (7c)$$

$$\pi_{r\phi} = \pi_{\phi r} = -\tfrac{3}{2}\eta_0\, W_{rr}(\tfrac{3}{2}\cos^2\alpha - \tfrac{1}{2})\sin\alpha\cos\alpha + O(\omega_p{}^{-2}\tau_p{}^{-2}) \qquad (7d)$$

where $\pi_{r\theta}$, $\pi_{\theta r}$, and $\pi_{\phi\theta}$ are all of order $(\omega_p\tau_p)^{-2}$, ω_p is the proton gyrofrequency, and τ_p is the proton Coulomb collision time. (In the corona and solar wind, $\omega_p\tau_p \gtrsim 10^5$, so terms of order $(\omega_p\tau_p)^{-1}$ are quite small.) (The reason for the existence of a relationship between the rate-of-strain tensor and the stress tensor is most evident in the case of shearing motion in the absence of a magnetic field. For example, if $\partial u_r/\partial\theta \neq 0$ $(W_{r\theta} \neq 0)$, information of the velocity gradient will be transmitted in the θ-direction by random particle motion (V_θ) over a scale-length of about one collision mean free path, and a consequent (tangential) stress $(\pi_{r\theta} \neq 0)$ will work to destroy the velocity gradient. When non-uniform expansion or contraction (i.e., $W_{rr} \neq 0$) drives a pressure anisotropy (e.g., $\pi_{rr} \neq \pi_{\theta\theta}$), collisional energy transfer between the r and θ degrees of freedom works to minimize the anisotropy, but the residual anisotropy gives rise to a correction to the scalar pressure gradient force.) The coefficient of viscosity, η_0, in (7) is given [e.g., *Braginskii*, 1965] by

$$\eta_0 \approx p_p\tau_p = 1.2\times 10^{-16}(20/\lambda)T_p{}^{5/2}\ \text{gm cm}^{-1}\ \text{s}^{-1} \qquad (8)$$

where λ is the Coulomb logarithm, and the correction to (8) arising from electron viscosity is smaller than $p_p\tau_p$ by a factor of $1.6\times 10^{-2}\,(T_e/T_p)^{5/2}$.

Using (7), we can now define an effective coefficient of viscosity, η, by

$$\eta = \eta_0(\tfrac{3}{2}\cos^2\alpha - \tfrac{1}{2})^2 \qquad (9)$$

and write the viscous terms appearing in (1)–(3) as follows:

$$(\nabla\cdot\boldsymbol{\pi})_r = -\tfrac{4}{3}\eta\left[\frac{d^2u}{dr^2}+\left(\frac{du}{dr}-\frac{u}{r}\right)\left(\frac{2}{r}+\frac{d\ln\eta}{dr}\right)\right] \qquad (10)$$

$$\boldsymbol{\pi}:(\nabla\mathbf{u}) = -\tfrac{4}{3}\eta\left(\frac{du}{dr}-\frac{u}{r}\right)^2 \qquad (11)$$

$$(\boldsymbol{\pi}\cdot\mathbf{u})_r = -\tfrac{4}{3}\eta u\left(\frac{du}{dr}-\frac{u}{r}\right) \qquad (12)$$

The form of (10)–(12) is exactly that of viscous terms in an unmagnetized solar wind or a wind with a radial magnetic field, except for the correction factor applied to η_0 in (9). Although this correction factor, $(3/2\cos^2\alpha - 1/2)^2$, approaches unity for a radial magnetic field, it is otherwise fundamentally different from the factor $\cos^2\alpha$ that has been used in the past to correct for the presence of a magnetic field [e.g., Wolff et al., 1971; Hundhausen, 1972]. (*Weber and Davis* [1970] do not explicitly use such a correction factor, because they consider explicitly only the azimuthal viscous force, which involves the $\pi_{r\phi}$ component of the stress tensor [cf. our eqn. (7*d*)]. Our equation (9) can, however, be derived from their equation (A.16) if a typographical error is corrected (viz., α^2 must be replaced by α^4 in the last term of the coefficient of the first component of their velocity gradient tensor). Note that u_ϕ, which is included in the Weber and Davis treatment is not important in the present discussion of the unstructured solar wind, because u_ϕ and u_r vary over comparable spatial scales, but $|u_\phi/u_r| \ll 1$.) Use of the factor $\cos^2\alpha$ has been justified by assuming that viscous momentum transport is analogous to the transport of heat by thermal conduction ($\cos^2\alpha$ is the proper correction factor for the coefficient of thermal conductivity). This not unreasonable justification is, however, invalid because the nature of dilatation viscosity (i.e., viscosity associated with expansion and contraction) is fundamentally different from that of other transport processes, such as thermal conduction or shear viscosity.

In the classical description, dilatation viscosity arises from a pressure anisotropy that is produced by the nonuniform expansion or contraction of a fluid element and is limited by collisional energy transfer among the three internal degrees of freedom characterizing the plasma [e.g., *Braginskii*, 1965; *Price et al.*, 1975]. In our spherical expansion, for example, if $du/dr < u/r$ and the magnetic field is radial, the pressure perpendicular to the magnetic field will tend to decrease (through expansion) more rapidly than the parallel pressure, and isotropy cannot be completely restored by collisions. Thus the

diagonal components of the stress tensor represent corrections to the plasma pressure tensor, **P**, that account for the small pressure anisotropy: i.e.,

$$\mathbf{P} = p\mathbf{I} + \boldsymbol{\pi} \tag{13}$$

In a strongly magnetized plasma ($\omega\tau \gg 1$, where ω and τ are the relevant particle gyrofrequency and momentum transfer collision time, as for protons in equation (7)), the magnetic field is far more effective in creating pressure isotropy perpendicular to itself than are collisions, and to order $(\omega\tau)^{-1}$, one can assume isotropy in the perpendicular directions (defined by $\hat{e}_\perp$ and $\hat{e}_\theta$ in the above context). It is the changing directions of isotropy (viz., $\hat{e}_\perp$ and $\hat{e}_\theta$) with changing α, not the inhibition of transport across the magnetic field, that gives rise to the magnetic correction factor applied to η_0 in (9), and this is the reason that the factor is $[(3\cos^2\alpha - 1)/2]^2$ rather than $\cos^2\alpha$.

The difference between these two correction factors is not generally small. For example, at $r = 0.4$, 1.0, and 1.4 AU, for a typical solar wind speed, $\cos^2\alpha \approx 0.86$, 0.5, and 0.33, while $((3\cos^2\alpha - 1)/2)^2 \approx 0.62$, 0.06, and 0. The vanishing of viscous effects when the magnetic field is some 55° from the radial results from the effective uniformity of the expansion produced by the isotropizing effect of the magnetic field. This point is clarified in the following section.

3. Anisotropic Pressure

It has frequently been stated or implied by solar wind researchers [e.g., *Holzer and Axford*, 1970; *Weber and Davis*, 1970; *Summers*, 1978] that the effects of pressure anisotropies and dilatation viscosity on solar wind momentum and energy balance are separate and distinguishable. Yet the preceding discussion indicates that these two effects are identical [e.g., *Braginskii*, 1965; *Price et al.*, 1975]. This point is clarified below, and in the process the criterion for validity of the classical description of viscosity is discussed, and a simple approach to the nonclassical description of viscosity is indicated.

If the pressure of the magnetized solar wind plasma is anisotropic such that the diagonal components (in the magnetic coordinate system) of the pressure tensor are $P_{bb} = p_\parallel$ and $P_{\perp\perp} = P_{\theta\theta} = p_\perp$, then the wind momentum equation can be written [e.g., *Schunk*, 1977]

$$\rho u \frac{du}{dr} = -\frac{d}{dr}(p_\parallel \cos^2\alpha + p_\perp \sin^2\alpha) - \frac{3}{r}(p_\parallel - p_\perp)(\cos^2\alpha - \tfrac{1}{3}) - \rho\frac{GM}{r^2} + \rho D \tag{14}$$

and the energy equations describing the parallel and perpendicular pressures of the jth particle species are [e.g., *Schunk*, 1977]

$$\tfrac{1}{2}p_{j\parallel} u \frac{d}{dr}(\ln T_{j\parallel}) = -p_{j\parallel}\left(\frac{du}{dr}\cos^2\alpha + \frac{u}{r}\sin^2\alpha\right) - \frac{1}{3\tau_j}(p_{j\parallel} - p_{j\perp}) + Q_{j\parallel} \tag{15}$$

$$p_{j\perp} u \frac{d}{dr}(\ln T_{j\perp}) = -p_{j\perp}\left[\frac{du}{dr}\sin^2\alpha + \frac{u}{r}(1 + \cos^2\alpha)\right] + \frac{1}{3\tau_j}(p_{j\parallel} - p_{j\perp}) + Q_{j\perp} \tag{16}$$

where it is assumed that $Q_{j\perp}$ describes the total perpendicular heating of the jth species, $p_\parallel/T_\parallel = p_\perp/T_\perp = p/T$, and $T = (T_\parallel + 2T_\perp)/3$. In both (15) and (16), the first term on the right side describes expansive cooling owing to wind acceleration (du/dr) and to geometric (flow-tube) expansion (u/r), with the orientation of the magnetic field determining how much each sort of expansive cooling affects the parallel or perpendicular pressure. In the limit of radial magnetic field ($\cos^2\alpha = 1$), the wind acceleration is responsible for parallel cooling and the geometric expansion for perpendicular cooling. In the limit of azimuthal magnetic field ($\cos^2\alpha = 0$) geometric expansion is responsible for parallel cooling and both acceleration and geometric expansion affect the perpendicular cooling. (Of particular interest is the case in which the field is oriented at about 55° to the radial ($\cos^2\alpha = 1/3$), when the rates of parallel and perpendicular cooling are identical (cf. last paragraph of section 2.) The second term on the right side of both (15) and (16) represents the Coulomb collisional transfer of energy between the parallel and perpendicular modes, and the third term represents the net source of heating for the mode. The heating (cooling) terms can contribute to an enhancement of the pressure anisotropy in various ways: e.g., through energy exchange with another particle species that has a more anisotropic pressure, through wave dissipation that preferentially enhances either parallel or perpendicular particle energy, through thermal conduction, etc. The heating terms can of course, similarly contribute to a decrease of the pressure anisotropy.

Multiplying (15) by $2/p_{j\parallel}$, subtracting it from (16) divided by $p_{j\perp}$, and using (8) to relate τ_j and η_{0j} leads to

$$(p_{j\parallel} - p_{j\perp}) = (p_{j\parallel}p_{j\perp}/p_j^2)\eta_{0j}\left\{-(3\cos^2\alpha - 1)\left(\frac{du}{dr} - \frac{u}{r}\right) + u\frac{d}{dr}[\ln(p_{j\perp}/p_{j\parallel})] + 2Q_{j\parallel}/p_{j\parallel} - Q_{j\perp}/p_{j\perp}\right\} \tag{17}$$

and adding (15) and (16) yields

$$\tfrac{3}{2}pu\frac{d}{dr}(\ln T) = -p_\parallel\left(\frac{du}{dr}\cos^2\alpha + \frac{u}{r}\sin^2\alpha\right) - p_\perp\left[\frac{du}{dr}\sin^2\alpha + \frac{u}{r}(1 + \cos^2\alpha)\right] + Q \tag{18}$$

It is readily shown that the momentum and energy balance equations (14) and (18), with the auxiliary equation (17), are the general form of which (1) and (2), with the auxiliary equations (10) and (11), are a special limiting case. (Note that (17) provides a link between the general CGL theory and classical transport theory [cf. *Marsch et al.*, 1983].) Consider the form of the stress tensor, which we recall is defined by $\boldsymbol{\pi} = \mathbf{P} - p\mathbf{I}$ [cf. (13)]. In magnetic coordinates, the diagonal components of this tensor are

$$\pi_{bb} = p_\parallel - p = \tfrac{2}{3}(p_\parallel - p_\perp) \tag{19a}$$

$$\pi_{\perp\perp} = \pi_{\theta\theta} = p_\perp - p = -\tfrac{1}{3}(p_\parallel - p_\perp) \tag{19b}$$

(Note that (19)–(26) apply equally well to the proton, electron, and total pressures and stress tensors.) Transforming to spherical coordinates, we obtain

$$\pi_{rr} = (\cos^2\alpha - \tfrac{1}{3})(p_\parallel - p_\perp) \tag{20a}$$

$$\pi_{\theta\theta} = -\tfrac{1}{3}(p_\parallel - p_\perp) \tag{20b}$$

$$\pi_{\phi\phi} = (\tfrac{2}{3} - \cos^2\alpha)(p_\parallel - p_\perp) \tag{20c}$$

$$\pi_{r\phi} = \pi_{\phi r} = (p_\parallel - p_\perp)\sin\alpha\cos\alpha \tag{20d}$$

From (13) and (20), we can determine the form of the pressure terms and stress tensor terms appearing in (1)–(3):

$$(\nabla \cdot \boldsymbol{\pi})_r = \frac{d}{dr}[(p_\parallel - p_\perp)(\cos^2\alpha - \tfrac{1}{3})] + \frac{3}{r}(p_\parallel - p_\perp)(\cos^2\alpha - \tfrac{1}{3}) \quad (21)$$

$$(\nabla \cdot \mathbf{P})_r = \frac{dp}{dr} + (\nabla \cdot \boldsymbol{\pi})_r \quad (22a)$$

$$= \frac{d}{dr}(p_\parallel \cos^2\alpha + p_\perp \sin^2\alpha) + \frac{3}{r}(p_\parallel - p_\perp)(\cos^2\alpha - \tfrac{1}{3}) \quad (22b)$$

$$\boldsymbol{\pi}:(\nabla\mathbf{u}) = (p_\parallel - p_\perp)(\cos^2\alpha - \tfrac{1}{3})\left(\frac{du}{dr} - \frac{u}{r}\right) \quad (23)$$

$$\mathbf{P}:(\nabla\mathbf{u}) = p\nabla \cdot \mathbf{u} + \boldsymbol{\pi}:(\nabla\mathbf{u}) \quad (24a)$$

$$= p_\parallel\left(\frac{du}{dr}\cos^2\alpha + \frac{u}{r}\sin^2\alpha\right) + p_\perp\left[\frac{du}{dr}\sin^2\alpha + \frac{u}{r}(1 + \cos^2\alpha)\right] \quad (24b)$$

$$(\boldsymbol{\pi} \cdot \mathbf{u})_r = u(\cos^2\alpha - \tfrac{1}{3})(p_\parallel - p_\perp) \quad (25)$$

$$(\mathbf{P} \cdot \mathbf{u})_r = pu + (\boldsymbol{\pi} \cdot \mathbf{u})_r \quad (26a)$$

$$= u(p_\parallel \cos^2\alpha + p_\perp \sin^2\alpha) \quad (26b)$$

It is clear from (22) and (24) that (1) and (2) are completely equivalent to (14) and (18). Furthermore, we see that if the pressure anisotropy is sufficiently small that $p_{j\parallel}p_{j\perp} \approx p_j^2$ and if the last three terms on the right side of (17) are negligible, then (20), (21), (23), and (25) reduce to the classical limit given in (7) and (10)–(12).

Evidently the classical description of dilatation viscosity can become invalid even when the plasma is strongly collision-dominated and the pressure anisotropy is arbitrarily small, contrary to the suggestions of *Holzer and Leer* [1973] and *Price et al.* [1975]. Such a breakdown of the classical description will occur whenever an effect included in the source terms $Q_\parallel$ and $Q_\perp$ of (17) becomes comparable in importance to non-uniform expansion in determining the pressure anisotropy. Examples of such effects (mentioned in the preceding section) are mechanical wave dissipation, collisional energy transfer and thermal conduction for which $2Q_\parallel \neq Q_\perp$. We see, however, that when the classical description of dilatation viscosity does break down, a satisfactory non-classical formalism is available in (1)–(3), (17), (21), (23), and (25). The existence of this non-classical formalism stands in sharp contrast to the situation we face when the classical description of thermal conduction breaks down [e.g., *Leer et al.*, 1982, and references therein].

4. OBSERVATIONAL INFERENCES

The formalisms developed in sections 2 and 3 allow us to use solar wind observations to determine both the accuracy of the classical description of dilatation viscosity and the relative importance of dilatation viscosity (or pressure anisotropy) in solar wind momentum and energy balance. The accuracy of the classical description can be estimated by comparing the following two parameters (cf. (17)):

$$A_{j1} = (p_{j\parallel} - p_{j\perp})p_j^2/(p_{j\parallel}p_{j\perp}) \quad (27)$$

$$A_{j2} = -\eta_{0j}(3\cos^2\alpha - 1)\left(\frac{du}{dr} - \frac{u}{r}\right) \quad (28)$$

If $A_{j1} \approx A_{j2}$, then the classical description is relatively accurate (fortuitously or otherwise) for the jth particle species. Regardless of the accuracy of the classical description, the simplest way to estimate the relative importance of dilatation viscosity (or pressure anisotropy) in solar wind momentum and energy balance is by comparing the magnitudes of the pressure terms with those of the stress tensor terms in (22*a*), (24*a*), and (26*a*). Three ratios facilitating this comparison are

$$C_3 = (\boldsymbol{\pi} \cdot \mathbf{u})_r/(pu) = (\cos^2\alpha - \tfrac{1}{3})(p_\parallel - p_\perp)/p \quad (29)$$

$$C_2 = [\boldsymbol{\pi}:(\nabla\mathbf{u})]/(p\nabla \cdot \mathbf{u}) = C_3\left(\frac{du}{dr} - \frac{u}{r}\right)\Big/\left(\frac{du}{dr} + \frac{2u}{r}\right) \quad (30)$$

$$C_1 = (\nabla \cdot \boldsymbol{\pi})_r/(dp/dr) = C_3\left\{\frac{3}{r} + \frac{d}{dr}[\ln(C_3 p)]\right\}\Big/\left[\frac{d}{dr}(\ln p)\right] \quad (31)$$

As mentioned in section 3, (21)–(26), and thus also (29)–(31), are applicable either to individual particle species pressures and stress tensors or to the total pressure and stress tensor. Hence, we can use (30), say, to discover the importance of dilatation viscosity (or pressure anisotropy) in the determination of the total solar wind temperature $[T = (T_e + T_p)/2]$, of the electron temperature alone, and of the proton temperature alone.

Since the ratios C_1, C_2, and C_3 measure only the relative magnitudes of viscous and pressure effects, a determination of the importance of the viscosity in the solar wind requires the additional consideration of the relative importance of pressure and other effects. For example, the pressure work term, pu, provides a measure of the advected thermal energy flux density (the enthalpy flux density is $5pu/2$) that can be converted

TABLE 1. Observationally Inferred Parameters (Applicable Between 0.3 and 1.0 AU)

Parameter	u, km s^{-1}				
	300–400	400–500	500–600	600–700	700–800
f, 10^8 cm^{-2} s^{-1}	4.0	4.0	4.0	3.0	3.0
$\bar{u}_p$, km s^{-1}	350	450	550	650	750
$T_{0p\parallel}$, 10^6 K	18	13	16	15	12
$a_{p\parallel}$	−1.03	−0.85	−0.80	−0.75	−0.69
$T_{0p\perp}$, 10^6 K	5.9	9.5	63	74	140
$a_{p\perp}$	−0.90	−0.86	−1.11	−1.08	−1.17
	$\bar{u}_t$, km s^{-1}				
	350		500		700
$T_{0t\parallel}$, 10^6 K	8.7		4.7		14
$a_{t\parallel}$	−0.69		−0.46		−0.62
$T_{0t\perp}$, 10^6 K	5.7		4.3		19
$a_{t\perp}$	−0.64		−0.49		−0.72

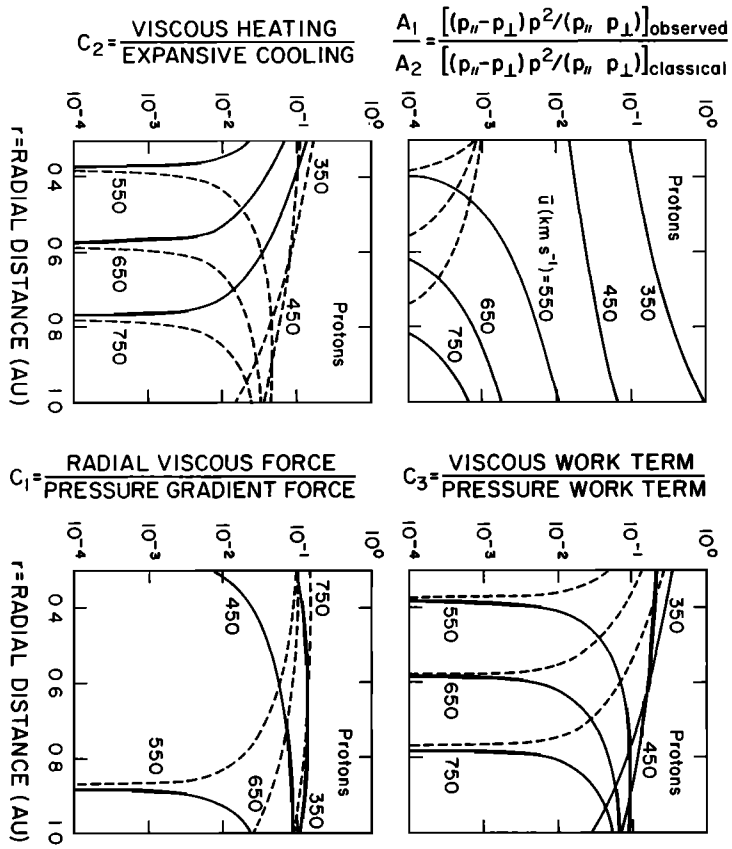

Fig. 1. Radial variation of the values for protons of A_1/A_2, C_1, C_2, and C_3 (cf. (27)–(31)), averaged over each of five solar wind flow speed intervals: $u < 400$, $400 < u < 500$, $500 < u < 600$, $600 < u < 700$, and $700 < u < 800$ km s^{-1}. The solid curves denote positive values and the dashed curves negative values of the parameters plotted. Each speed interval is denoted by its central speed, $\bar{u}$, and this central speed is used in all calculations for a given interval. The observational data used in determining $p_{p\parallel}$ and $p_{p\perp}$ are the results presented by *Marsch et al.* [1982*b*], and our analytic fits to their data are specified in Table 1. All calculations employ the assumptions that $|du/dr| \ll |u/r|$ and that $\rho = m_p f/(\bar{u}r^2)$, where f is the proton flux density at 1 AU, which we take [*Feldman et al.*, 1977] to be 4×10^8 cm^{-2} s^{-1} for $\bar{u} < 6 \times 10^7$ cm s^{-1} and 3×10^8 cm^{-1} for $\bar{u} > 6 \times 10^7$ cm s^{-1}. Within the observational uncertainties quoted by *Marsch et al.* [1982*b*], the largest values shown for C_1, C_2, and C_3 could be increased by as much as a factor of 2, but even with such an increase, the observationally inferred effects of dilatation viscosity are seen to be relatively minor in the range $0.3 < r < 1.0$ AU. In contrast, it is seen that the classically predicted viscous effects (obtained by multiplying C_1, C_2, and C_3 by A_2/A_1) can be quite substantial.

to flow energy through the action of the pressure gradient force. In the radial range considered ($0.3 < r < 1.0$ AU), the energy flux density (pu) represented by the pressure work term is always less than one tenth of the energy flux density carried by the flow ($\rho u^3/2$), so we must always multiply C_3 by a factor smaller than 0.1 to place an upper limit on the fractional contribution that the viscous work makes to the total wind energy flux. The same considerations clearly apply to the use of C_1, which is the ratio of the magnitudes of the radial viscous force and the pressure gradient force. The viscous heating rate, on the other hand, is compared with the expansive cooling rate in the ratio C_2, and the expansive cooling rate plays a dominant role in determining the radial evolution of the solar wind temperature (especially the proton temperature). Hence, C_2 can be taken as a direct measure of the importance of viscous heating in the thermal structure of the wind.

To calculate A_1, A_2, C_1, C_2, and C_3, we have made use of the Helios observations described by *Marsch et al.* (1982*a*, *b*) and by *Marsch and Richter* [1984], as well as a variety of other spacecraft observations summarized by Feldman et al. (1977); these other observations are employed only to specify the solar wind mass flux. (The Helios observations have been used in a related study by *Marsch et al.* [1983], in which the violation of adiabatic invariants for the protons and alphas between 0.3 AU and 1.0 AU is considered. The reader is encouraged to examine that paper in parallel with the present paper.) We use an analytic fit for the Helios temperature observations of the form

$$T = T_0(r/r_0)^a \tag{32}$$

where $r_0 = R_S$, and the fit can be considered reasonable only in the radial range $0.3 \lesssim r \lesssim 1.0$ AU. The observationally inferred parameters are shown in Table 1, where parameters relevant to both the proton pressure anisotropy (rows 1–7) and the total plasma pressure anisotropy (rows 1, 2, and 8–12) are given. (Note that the parameterization in Table 1 applies only in the range 0.3 AU $\leq r \leq$ 1 AU, so the anomalous coronal values that might be inferred from these parameters should be ignored.) In each flow speed range, the flow speed is taken to be $u = \bar{u}$ and the mass density is taken to be $\rho = m_p f r_E^2/(\bar{u}r^2)$, where m_p is the proton mass, r_E is the radius of the orbit of earth, and f is the proton flux density at 1 AU. The assumption implied by the expression for the mass density (viz., that $du/dr = 0$) is also used in calculating A_2, C_1, C_2, and

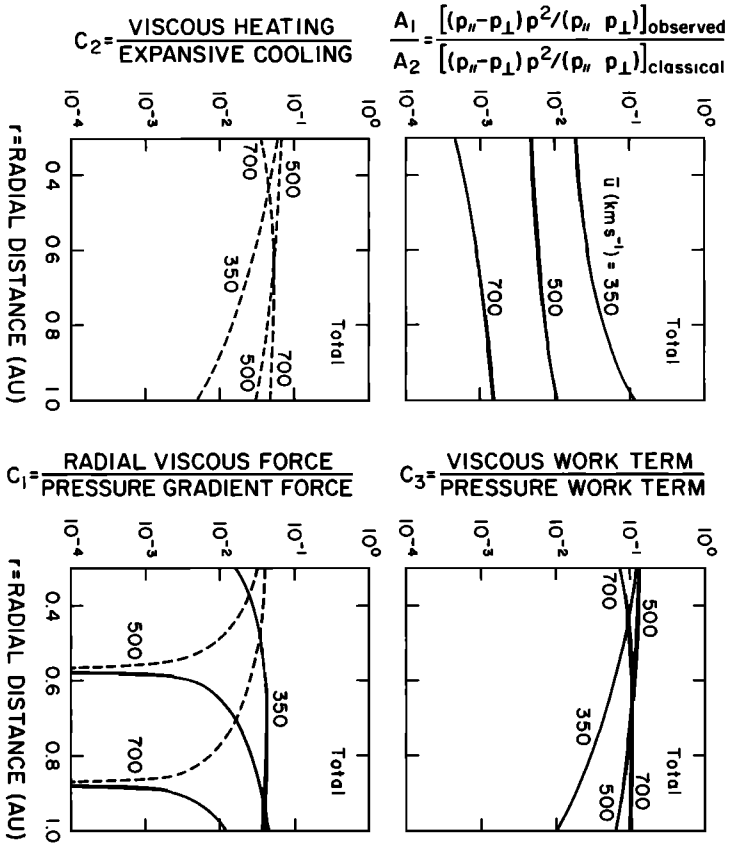

Fig. 2. Same as Figure 1, except that the total pressures $p_{\parallel}$ and $p_{\perp}$ (appropriate to a one-fluid description of a plasma composed of electrons, protons, and alphas) are used in calculating A_2, C_1, C_2, and C_3 [cf. *Marsch et al.*, 1982*a*, *b*; *Marsch and Richter*, 1984]. A_1 is calculated using a description of classical viscosity in a proton-electron plasma, in which the proton temperature is given by the observationally inferred "total" temperature: $T_p = (T_{\parallel} + 2T_{\perp})/3$. The use of a more refined classical description would not qualitatively change the inferences drawn from this figure, which are similar to those drawn from Figure 1. The observational uncertainties here are smaller than those quoted for Figure 1.

C_3. The results obtained from (27)–(31) and Table 1 are shown in Figures 1 and 2, for protons alone and for the plasma as a whole.

Figure 1 shows that the classical description of proton viscosity generally overestimates the proton pressure anisotropy by a substantial amount in the radial range $0.3 < r < 1.0$ AU. From use of the classical description alone one might infer that dilatation viscosity (or pressure anisotropy) plays a very important role in determining the proton temperature and a not insignificant role in determining the asymptotic flow speed, as can be seen in Figure 1 when the ratios C_1, C_2, and C_3 are multiplied by the factor A_2/A_1. The magnitudes of these classical viscous effects increase with increasing proton temperature (and thus with increasing solar wind flow speed). In contrast, the observed viscous (or pressure anisotropy) effect is seen generally (cf. C_1, C_2, and C_3 in Figure 1) to be quite small and to decrease with increasing proton temperature. Evidently, physical effects in addition to non-uniform flow tube expansion and Couloub collisional relaxation play a significant role in determining the solar wind proton pressure anisotropy.

It appears that proton viscosity (or pressure anisotropy) may become important inside $r = 0.3$ AU, but solar wind observations are not available in this region [cf. *Marsch et al.*, 1983]. It is clear, however, that one should avoid trying to estimate the effects of viscosity in this region with the classical description, because this description seems to become less accurate at smaller radial distances. The difficulty with the classical description, which always predicts viscous heating of the wind, is emphasized by the fact that observed pressure anisotropies near $r = 0.35$ AU indicate viscous cooling of the wind for relatively high flow speeds ($u \gtrsim 500$ km s^{-1}) and viscous heating for lower flow speeds ($u \lesssim 500$ km s^{-1}). The viscous (or pressure anisotropy) cooling is simply a consequence of $p_{\perp} > p_{\parallel}$, which results in an underestimate of the expansive cooling rate when a scalar pressure is used. In fact, viscous heating/cooling effects should generally be viewed as corrections to the scalar-pressure description of expansive cooling; thus heating associated with dilatation viscosity is really a reduction of expansive cooling, not an intrinsic heating of the expanding gas. (Note that the changes in sign of A_1/A_2, C_1, C_2, and C_3 in Figures 1 and 2 are not reflections of dramatic physical changes. For example, the change from $p_{\parallel} > p_{\perp}$ to $p_{\perp} > p_{\parallel}$ produces a sign change in A_1/A_2, C_2, and C_3 but merely reflects a continuous variation of the small viscous correction to the scalar pressure.)

The results shown in Figure 2 are qualitatively similar to those of Figure 1. The total pressure anisotropy includes effects of electron and alpha-particle pressure and the relative flow of alphas and protons (i.e., a single-fluid description of the plasma in which the total pressure is defined with respect to the mean flow speed is implied). The net result of these effects is to reduce the total pressure anisotropy below the proton pressure anisotropy, and thus to decrease the overall viscous effect on the single-fluid temperature below the effect of proton viscosity on the proton temperature.

5. SUMMARY AND CONCLUSIONS

Viscosity is associated with the existence of a non-zero stress tensor, and the stress tensor is that part of the pressure tensor that cannot be represented in terms of a scalar pressure. Thus, viscosity can be thought of as a correction to scalar pressure effects (e.g., pressure gradient force, expansive cooling) arising from off-diagonal components of the pressure tensor (shear viscosity) and non-uniform diagonal components of the pressure tensor (dilatation viscosity). The existence of non-uniform diagonal components is, of course, synonymous with the existence of a pressure anistropy. The term dilatation, which means expansion, is applied to the viscous effects associated with pressure anisotropy because in the classical description of dilatation viscosity it is assumed (implicitly) that the pressure anisotropy is created through nonuniform expansion (or contraction) of a fluid element. When effects other than nonuniform expansion (or contraction) and Coulomb collisional energy exchange (between parallel and perpendicular degrees of freedom) become important in controlling the pressure anisotropy, a more general description of momentum and energy balance (cf. (14)–(16)) must be used in place of that employing classical viscous terms.

Solar wind observations between 0.3 AU and 1.0 AU indicate that dilatation viscosity (or pressure anisotropy) has only a minor effect on the proton temperature and is quite negligible otherwise in this spatial range. In contrast, the classical description predicts a quite significant role for viscosity in this range, so it is clear that the classical description is not valid and the pressure anisotropy is controlled by something other than a balance between nonuniform expansion and Coulomb collisional energy exchange. Although one cannot rule out the possibility that dilatation viscosity becomes important inside 0.3 AU, it seems that the classical description is unlikely to be a reliable indicator of the importance of viscous effects in this, as yet, unobserved region of space.

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