Improvement of Inversion Solutions of the Elliptic Cone Model for Frontside Full Halo CMEs

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1 Short title: IMPROVEMENT OF SOLUTIONS OF ELLIPTIC CONE MODEL
Abstract. The fact that the inversion solution calculated from the original
inversion equation system of the elliptic cone model in Zhao (2008) cannot be used
to reproduce non-disk Type C frontside full-halo CMEs suggests that the assumption
made for establishing the inversion equation system is questionable. By improving the
assumption, a new inversion equation system of the elliptic cone model is established.
Comparison of the new inversion solution obtained from the new inversion equation
system with the old one shows that the new inversion solutions are significantly improved
over the old ones. Both disk and non-disk Type C halo CMEs can be well reproduced
using the new inversion solutions.
1. Introduction

Coronal mass ejections (CMEs) are believed to be driven by free magnetic energy stored in field-aligned electric currents, and the magnetic configuration of most, if not all, CMEs is thus expected to be CME ropes, i.e., magnetic flux ropes with two ends anchored on the solar surface (e.g. Riley et al., 2006). Most limb CMEs appear as planar looplike transients with a radially-pointed central axis and a constant angular width. The existence of full halo CMEs, i.e., those CMEs with an apparent (sky-plane) angular width of 360°, implies that the looplike transients are three-dimensional (Howard et al., 1982). Both looplike and halolike CMEs show the evidence of the CME rope configuration.

The bright structures characterizing coronal mass ejections (CMEs) observed by coronagraphs in the plane of sky are the photospheric light scattered by CME electrons along the line-of-sight. Outlines of many, if not most of, full halo CMEs are ellipse-like. A conical shell (or cone) model, i.e., a hollow body which narrows to Sun’s spherical center from a round, flat base was suggested to be similar to 3-D CME structures and to qualitatively understand the formation of the elliptic halo CMEs (Howard et al., 1982). The circular cone model has been used to invert geometrical and kinematical properties of 3-D CMEs from observed apparent geometrical and kinematical properties of 2-D elliptic halo CMEs (Zhao et al., 2002; Xie et al, 2004).

Ellipse-like halo CMEs can be classified into Types A, B, and C based on whether the minor (Type A) or major (Type B) axis of ellipse-like halos passes or not (Type C) through the solar disk center (See Figure 1 of Zhao, 2008) (Zhao08, hereafter). It has been shown that the circular cone model can be used to produce only Type A FFH
CMEs, and thus invert the 3-D properties only for Type A FFH CMEs (Zhao, 2005).

The outer boundary of the top (or leading) part of CME ropes may be better approximated by an ellipse than a circle. The elliptic cone model has been developed to invert the model parameters that characterize the propagation direction, size, shape and orientation of 3-D rope-like CMEs [Zhao, 2005; Cremades and Bothmer, 2005]. We had established an inversion equation system for the elliptic cone model trying to invert the model parameters for all three types of ellipse-like CMEs (Zhao08). It is found, however, that although the inversion equation system can be used to invert model parameters for all three types of ellipse-like CMEs, the ellipses reproduced using the inverted model parameters can well match Types A and B, but only a part of Type C FFH CMEs, the so called “disk” halo CMEs, of which the associated near-surface activity occurred near the solar disk center (See Figures 6, 7, 8 of Zhao08).

The present paper establish an new inversion equation system so that the non-disk Type C FFH CMEs, as well as Types A, B, and disk Type C, can be correctly inverted.

2. Expressions for Observed and Modeled Halo CMEs

In the Heliocentric Ecliptic coordinate system $X_hY_hZ_h$ with $X_h$ axis pointing to the Earth, $Y_h$ axis to the west, and $Z_h$ axis to the north, the plane $Y_hZ_h$ is the sky-plane where the halo CMEs occur due to the projection of the conical shell-like 3-D CMEs. The direction of central axis of 3-D CMEs is expressed here by the sky-plane latitude $\beta$ and longitude $\alpha$, instead of the commonly used ecliptic-plane latitude and longitude. The sky-plane latitude $\beta$ denotes the angle from the plane $Y_hZ_h$ to the central axis and the longitude $\alpha$ between projection of the central axis on the the plane $Y_hZ_h$ and the west axis $Y_h$. 
2.1. 6 Model Parameters and Expressions for Modeled Halos

An elliptic cone can be expressed in the coordinate system $X_cY_cZ_c$ with its origin colocated with the origin of the $X_hY_hZ_h$ system, and the $X_c$ axis aligned with the central axis of the elliptic cone, and expressed by the sky-plane latitude and longitude, $\beta$ and $\alpha$. The base of the elliptic cone is parallel to the the plane $Y_cZ_c$ normal to the $X_c$ axis (see Figure 1). The axis $Y_c$ is the intersection between the plane $Y_hZ_h$ and the plane $Y_cZ_c$. In addition, four more model parameters, $R_c$, $\omega_y, \omega_z$, and $\chi$ are needed to characterize the elliptic cone base in $X_cY_cZ_c$ system. As shown in Figure 1, parameter $R_c$ denotes the distance from Sun’s spherical center to the center of the cone base; $\omega_y$ and $\omega_z$ are the half angular width covered by two semi-axes of an elliptic base, $SA_{yb}$ and $SA_{zb}$ respectively; $\chi$ is the angle between the semi-axis $SA_{yb}$ and the $Y_c$ axis.

Given a set of values for five model parameters $R_c, \omega_y, \omega_z, \chi, \beta$, a modeled halo on the plane $X_c'Y_c'$ can be obtained by the transformation of the rim of the elliptic cone base from coordinate system $X_cY_cZ_c$ to $X_c'Y_c'Z_c$ and from $X_cY_cZ_c$ to $X_c'Y_c'Z_c'$ (see Zhao08 for the details),

$$
\begin{bmatrix}
 x_{cb}' \\
 y_{cb}' \\
 z_{cb}'
\end{bmatrix} =
\begin{bmatrix}
 \cos \beta & \sin \beta \sin \chi & -\sin \beta \cos \chi \\
 0 & \cos \chi & \sin \chi \\
 \sin \beta & -\cos \beta \sin \chi & \cos \beta \cos \chi
\end{bmatrix}
\begin{bmatrix}
 R_c \\
 R_c \tan \omega_y \cos \delta_b \\
 R_c \tan \omega_z \sin \delta_b
\end{bmatrix}$$

(1)

where the symbol $\delta_b$ is the angle of radii of an elliptic base relative to $SA_{yb}$ axis and increase clockwise along the rim of the elliptic base from $0^\circ$ to $360^\circ$ (Note Expression (1) here slightly differs from Expression (5) in Zhao08 because the typos existed there).
2.2. 5 Halo Parameters and Expressions for Observed Halos

An elliptic halo observed by coronagraphs on the plane $Y_hZ_h$ as shown by the white ellipse in Figure 2, can be expressed using 5 halo parameters, $D_{se}$, $\alpha$, $SA_{xh}$, $SA_{yh}$, and $\psi$. Here $D_{se}$ denotes the distance from solar disk center to halo center. The axis $X'_c$ ( $Y'_c$ ) is aligned with (perpendicular to) $D_{se}$. Parameter $\alpha$ is the angle between $X'_c$ axis and $Y_h$ axis, increasing clockwise from $Y_h$. Parameters $SA_{xh}$ and $SA_{yh}$ are the semi-axes of elliptic halos adjacent to $X'_c$ and $Y'_c$ respectively. Parameter $\psi$ denotes the angle between axes $SA_{yh}$ and $Y'_c$ and increases clockwise from $Y'_c$.

The 5 halo parameters characterize the location of the center of ellipse-like halos ($D_{se}$, $\alpha$), the size and shape ($SA_{xh}$, $SA_{yh}$) and orientation ($\psi$) of the halos. By using four halo parameters $D_{se}$, $SA_{xh}$, $SA_{yh}$, and $\psi$, the 2-D elliptic halo on the plane $X'_cY'_c$, can be expressed

$$
\begin{bmatrix}
x'_{ch} \\
y'_{ch}
\end{bmatrix}
= 
\begin{bmatrix}
D_{se} \\
0
\end{bmatrix}
+ 
\begin{bmatrix}
\cos \psi & \sin \psi \\
-\sin \psi & \cos \psi
\end{bmatrix}
\begin{bmatrix}
SA_{xh} \sin \delta_h \\
SA_{yh} \cos \delta_h
\end{bmatrix}
\hspace{1cm} (2)
$$

The symbol $\delta_h$ in equation (1) is the angle of radii of elliptic halos relative to $SA_{yh}$ axis, and increases clockwise along an elliptic rim from 0° to 360°. Here $\alpha$ is both the model parameter and the halo parameter.

3. Derivation of New Inversion Equation System

By comparing the like items between Expressions (1) and (2) and setting $\triangle \delta \delta_h = \delta_b + \Delta \delta$, the relationship between elliptic cone model parameters and elliptic
CME halo parameters can be established

\[
\begin{align*}
R_c \cos \beta &= D_{se} \\
R_c \tan \omega_y \sin \beta \sin \chi &= SA_{xh} \cos \psi \sin \triangle \delta + SA_{yh} \sin \psi \cos \triangle \delta \\
-R_c \tan \omega_z \sin \beta \cos \chi &= SA_{xh} \cos \psi \cos \triangle \delta - SA_{yh} \sin \psi \sin \triangle \delta \\
R_c \tan \omega_y \cos \chi &= -SA_{xh} \sin \psi \sin \triangle \delta + SA_{yh} \cos \psi \cos \triangle \delta 
\end{align*}
\]

(3)

All model (halo) parameters occur in left (right) side of the equation system (3). In Zhao08, we obtained equation systems (8)-(10) and the inversion equation system (11) there by simply assuming \( \triangle \delta \simeq \psi - \chi \). However, the assumption and the equation systems (8)-(11) may be questioned because the model parameters obtained from the inversion equation system (11) can not be used to reproduce non-disk Type C halo CMEs. As shown in the right columns of Figures 3, the \( \chi' \) and \( \psi \) depend on \( \beta \) as well as \( \chi \), and we have \( \delta_h + \psi \simeq \delta_h + \chi' \). As \( \beta \) approaches 90°, \( \chi' = \chi \), and \( \psi = \chi \). Thus we have \( \triangle \delta \simeq \chi' - \psi \simeq \chi - \psi \). By replacing \( \triangle \delta \) in Equation (3) with \( \chi - \psi \), the similar equation systems to (8) – (10) of Zhao08 can be established as follows.

\[
\begin{align*}
R_c \cos \beta &= D_{se} \\
(R_c \tan \omega_y \sin \beta - a) \tan \chi &= b \\
-R_c \tan \omega_z \sin \beta + b \tan \chi &= a \\
R_c \tan \omega_y - b \tan \chi &= c 
\end{align*}
\]

(4)

where

\[
\begin{align*}
a &= SA_{xh} \cos^2 \psi + SA_{yh} \sin^2 \psi \\
b &= (-SA_{xh} + SA_{yh}) \sin \psi \cos \psi \\
c &= SA_{xh} \sin^2 \psi + SA_{yh} \cos^2 \psi 
\end{align*}
\]

(5)

For Types A and B FFH CMEs, \( \psi = 0 \) and \( \chi = 0 \), and for disk Type C FFH CMEs,
\( \psi \simeq \chi \), equation systems (4), (5) become

\[
\begin{align*}
R_c \cos \beta &= D_{se} \\
-R_c \tan \omega_z \sin \beta &= SA_{xh} \\
R_c \tan \omega_y &= SA_{yh}
\end{align*}
\]

(6)

and when \( \omega_y = \omega_z \), the number of model parameters equals the number of halo parameters, equation system (6) reduce to the inversion equation system for the circular cone model (Xie et al., 2004).

The new inversion equation system for the elliptic cone model can be derived from Equation system (4)

\[
\begin{align*}
R_c &= \frac{D_{se}}{\cos \beta} \\
\tan \omega_y &= \frac{[(a + c \sin \beta) + \sqrt{(a - c \sin \beta)^2 + 4 \sin \beta b^2}]}{(2R_c \sin \beta)} \\
\tan \chi &= \frac{(R_c \tan \omega_y - c)}{b} \\
\tan \omega_z &= \frac{-(a - b \tan \chi)}{R_c \sin \beta}
\end{align*}
\]

(7)

4. Comparison of New With Old Inversion Solutions

As the same as the old inversion equation system, the inversion equation system (7) shows that if the sky-plane latitude \( \beta \) can be specified, the four unknown model parameters, \( R_c, \omega_y, \omega_z \) and \( \chi \), can be uniquely determined by coefficients \( a, b, \) and \( c \) that can be calculated using Equation system (5) from observed four halo parameters, \( D_{se}, \)

\( SA_{xh}, SA_{yh} \) and \( \psi \).

Zhao08 suggested the one-point approach to find out the candidate model parameter \( \beta \) for FFH CMEs on the basis of the information included in the measured parameter \( \alpha \) and the disk location of flares associated with the CMEs (See Section 5 of Zhao08 for
the details). The candidate sky-plane latitudes $\beta$ for one Type A, one Type B, and four Type C FFH CMEs have been calculated in Zhao08.

Using the candidate sky-plane latitudes $\beta$ and the new inversion equation system we find the four unknown model parameters for the six events. As expected, the new inversion solutions obtained using the new inversion equation system for Types A, B, and disk Type C FFH CMEs are basically the same as that obtained using the old inversion equation system in Zhao08, since for these types FFH CMEs, both $\Delta \delta \simeq \chi - \psi$ here and $\Delta \delta \simeq \psi - \chi$ in Zhao08 reduce to $\Delta \delta \simeq 0$, and Equation system (3) here can be directly reduced to Equation system (6) here that is just the same as the equation system (10) in Zhao08.

Figures 3, 4, and 5 display non-disk Type C FFH CMEs (See white elliptic outline of halo CMEs) observed at 1999.05.03 08:42, 2000.02.09 20:30, and 2001.09.24 11:42 by SOHO/LASCO C2 and C3 coronagraph. The white ellipses are obtained based on the five points identified by Dr. Hebe Cremade around the edge of observed halos (See Cremade, 2005 for the details). The five halo parameters from the white ellipses are shown in the figures. Overplotted are reproduced halos obtained using new (green dashed ellipses) and old (red dashed ellipses) inversion equation system. The observed $\alpha$, estimated candidate $\beta$, and the other four inverted model parameters calculated using new (green) and old (red) inversion equation system are also shown in Figures 3, 4, and 5. The green ellipses agree with the white ellipse much better than the red ellipses, showing the improvement of the new inversion solutions over the old inversion solutions. All candidate $\beta$ for the three events are less than 50° that is away from the line-of-sight more than 40°, the "non-disk" halo CMEs.
5. Summary and Discussions

A successful determination of the geometrical properties from the elliptic cone model depends on the successful determination of (1) the outer edge of halo CMEs, (2) the sky-plane latitude $\beta$ by one-point or two-point approach, (3) the inversion solution from the inversion equation system.

Based on the projection of the elliptic cone base on the plane of the sky, we have established Equation system (3), the mathematical relationship between model parameters of the elliptic cone model and halo parameters of observed ellipse-like halo CMEs. By setting $\delta_h + \psi \simeq \delta_b + \chi' \simeq \delta_b + \chi$, we have $\Delta \delta = \delta_h - \delta_b \simeq \chi' - \psi \simeq \chi - \psi$ and derived the new inversion equation system (7).

Based on the same sky-plane latitudes of the radial CME propagation as obtained in Zhao08, the inversion solutions obtained from the new inversion equation system of the elliptic cone model are significantly improved over that from the old inversion equation system. Thus all three types of ellipse-like halo CMEs can be reproduced very well, not only disk halo CMEs, but also non-disk halo CMEs.

We have developed the algorithm for inverting 3-D kinematic properties on the basis of the inverted elliptic cone model parameters and the apparent kinematic properties of halo CMEs (Zhao, Cremades and Owen, 2010).

In addition to validating the determination of sky-plane latitude $\beta$ by one-point approach, we are trying to find a way to more objectively recognize the outer edge of halo CMEs so that we can reduce or avoid fitting errors occurred in the halo parameters.
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Figure 1. Relationship among coordinate systems $X_hY_hZ_h$, $X_cY_cZ_c$, and $X'_cY'_cZ'_c$. Axes $X'_c$, $Y'_c$, and $Y_c$ are all located in the sky-plane $Y_hZ_h$ normal to axis $X_c$. The axis $X_c$ denotes the direction of the central axis of the elliptic cone $(\beta, \alpha)$ and the distance of the cone base from the Sun’s center ($R_c$). The elliptic cone base at $R_c$ is parallel to the plane $Y_cZ_c$. The orientation of the elliptic cone base is expressed by parameter $\chi$, the angle between axis $Y_c$ and the semi-axis $SA_{yb}$ adjacent to $Y_c$. 

\[ Y_c' = Y_c = SAyb = Rc \cdot \tan(\omega_c) \text{ when } \chi = 0 \]

\[ z_c = SAyb = Rc \cdot \tan(\omega_c) \text{ when } \chi = 0 \]

$\beta$: the angle increases clockwise from green to blue line.
Figure 2. The four Type C halo CMEs studied in Zhao08, of which three cannot be reproduced using the inversion equation system in Zhao08. The measured values of the five halo parameters, $\psi$, $\alpha$, $SA_{xo}$, $SA_{yo}$, and $D_{se}$ are shown on the top of each panel. Shown on the bottom of each panel are the heliographic latitude and longitude of associated flares.
Figure 3. Comparison of reproduced halos inverted using old (red dashed) and new (green dashed) inversion equation systems with observed (white) FFH CME for 1999.05.03 08:42.
Figure 4. The same as Figure 3 but for 2000.02.09 20:30.
Figure 5. The same as Figure 3 but for 2001.09.24 11:42.