

Validation of the One-Point Approach of the Elliptic Cone Model for the 13 December 2006 Frontside Full Halo Coronal Mass Ejection

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2 **Abstract.** To invert the radial propagation speed and acceleration from the
3 measured sky-plane speed and acceleration of frontside full-halo CMEs, an algorithm is
4 developed on the basis of the elliptic cone model parameters. The elliptic cone model
5 parameters for the 13 December 2006 frontside full halo CME are inverted using the
6 one-point approach, i.e., using the halo CME image and the position of associated flare.
7 In searching for the projection angle between the CME propagation direction and the
8 plane of the sky, it is assumed for fast halo CMEs that the candidate projection angle
9 should be located at the point on the α -curve [Zhao, 2008] that is the minimum distance
10 from the flare position to the α -curve. We show that the observed elliptic halo can be well
11 reproduced using the inverted model parameters; the inverted kinematic properties agree
12 well with those determined by Type II observations; and the solar wind disturbances
13 ahead of the ejection associated with the 13 December 2006 full-halo CME can also be
14 well reproduced. The agreement between calculations and observations suggests that
15 both the algorithm developed here for inverting the actual kinematic properties and
16 the minimum-distance criterion used for determining the projection angle of the fast
17 frontside full-halo CMEs are valid for fast frontside full-halo CMEs. It is also shown
18 that the condition of the minor axis of the halo passing through the solar disk center is a
19 necessary but not sufficient condition for using the circular cone model to invert actual
20 geometrical and kinematical properties for frontside full-halo CMEs.

21 1. Introduction

22 Coronal mass ejections (CMEs) with apparent (i.e., sky-plane) angular width of
23 360° and associated with near-surface activity are defined as frontside full-halo (FFH)
24 CMEs. FFH CMEs are mostly symmetric and ellipse-like. The geoeffectiveness rate of
25 FFH CMEs is greater than 70%, reaching the higher end of the range of geoeffectiveness
26 rate of all kinds of solar activities [Zhao and Webb, 2003; Gopalswamy, Yashiro, and
27 Akiyama, 2007]. The knowledge of actual geometric and kinematic properties of 3-D
28 CMEs which appear as 2-D FFH CMEs is essential for space weather forecasting. In
29 this study, we address the issue of best invert the actual geometric and kinematic
30 properties of 3-D CMEs from measured apparent geometric and kinematic properties of
31 2-D ellipse-like FFH CMEs on the plane of the sky (sky-plane).

32 Based on the observational fact that most limb CMEs propagate radially with
33 constant angular width, a geometrical model for the 3-D CMEs was developed for
34 inverting the actual geometric and kinematic properties of 3-D CMEs from observed
35 2-D FFH CMEs. This cone model, a hollow body which narrows to its apex located at
36 Sun's spherical center from a round, flat base [Zhao, Plunkett and Liu, 2002; Xie et al.,
37 2004] is topologically similar to the conical shell model suggested by Howard et al. [1982]
38 for understanding the formation of full-halo CMEs. The geometrical and kinematic
39 properties obtained using the cone model for the 12 May 1997 FFH CME have been
40 introduced at the boundary of a 3-D MHD solar wind model; and the arrival time at
41 the Earth's orbit and the sheath structure ahead of the ICME have been successfully
42 reproduced [Odstrcil, Riley and Zhao, 2004]. The success of the simulation indicates
43 that the use of a cone-like geometric model to invert model parameters of 3-D CMEs

44 from halo parameters of 2-D FFH CMEs is a valid means of estimating the actual
45 geometrical and kinematic properties for FFH CME, which then may be used to launch
46 CME structures at the inner boundary of MHD heliospherical models for numerically
47 forecasting the space weather.

48 It was found, however, that cone model inversion is applicable to less than 10% of
49 FFH CMEs because the semi-minor axis of the elliptic halos formed by the cone model
50 must pass through solar disk center (See Figure 2 of Zhao et al., 2002 for details), which
51 is not the case for the majority of events [Zhao, 2005; 2008].

52 With the aim of inverting the actual geometrical and kinematic properties of all
53 kinds of elliptic FFH CMEs, a new cone-like model is developed. This elliptic cone
54 model is defined as a hollow body which narrows to its apex located at Sun's center from
55 an elliptic, flat base [Zhao, 2005; Cremades and Bothmer, 2005]. CMEs are believed to
56 be driven by free magnetic energy stored in field-aligned electric currents, and before
57 eruption, the metastable structure with free magnetic energy is confined by overlying
58 arched field lines. The magnetic configuration of most, if not all, CMEs is thus expected
59 to be magnetic flux ropes with two ends anchored on the solar surface (Riley et al.,
60 2006). This kind of CME rope may be more correctly approximated by the elliptic cone
61 model than the circular cone model since the outer edge of the top (or leading) portion
62 of CME ropes appears more like an ellipse than a circle.

63 For the elliptic cone model, six model parameters are needed, three for the position
64 of the base center, and three for the size, shape and orientation of the elliptic base (for
65 the cone model, only four model parameters are necessary because only one parameter is
66 needed to describe the circular base [Xie et al., 2004]). Observed elliptic halos, however,
67 can provide only five halo parameters, two for the position of the halo center and three

68 for the size, shape and orientation. It is thus difficult to uniquely determine six model
69 parameters on the basis of five halo parameters [Cremades and Bothmer, 2005; Zhao,
70 2008].

71 We have established the equation system that relates model parameters with halo
72 parameters, and presented two approaches, i.e. two-point and one-point approach, to
73 uniquely find six model parameters [Zhao, 2008]. The present work will validate the
74 one-point approach of the elliptic cone model using the well recognized, fast 13 December
75 2006 FFH CME. In what follows we first develop an algorithm for inverting the radial
76 speed and acceleration on the basis of the measured sky-plane speed and acceleration at
77 a measurement position angle. We then calculate geometrical and kinematic properties
78 using five halo parameters and the position of the associated flare for the 2006 December
79 13 Disk FFH CME. To validate the one-point approach and newly established algorithm,
80 we reproduce the observed FFH CME using inverted model parameters, compare the
81 inverted speed with that from Type II observations, and compare the arrival time and the
82 sheath structure ahead of the simulated ICME at Earth's orbit with in situ observations.
83 Finally we summarize and discuss the results in the last section.

2. The Algorithm for Determining the Radial Kinematic

Property from the Sky-plane Kinematic Property

We work in the Heliocentric Ecliptic coordinate system $X_h Y_h Z_h$ with X_h axis pointing to the Earth, Y_h axis to the west, and Z_h axis to the north, the plane $Y_h Z_h$ denotes the plane of the sky. To express the orientation of an elliptic cone we introduce a coordinate system $X_c Y_c Z_c$, with its origin colocated with the origin of the $X_h Y_h Z_h$ system. Here X_c axis is aligned with the central axis of the elliptic cone (or the propagation direction of 3-D CMEs), and Y_c is the intersection between the plane $Y_h Z_h$ and the plane $Y_c Z_c$ normal to the X_c axis. Figure 1 shows the 13 December 2006 elliptic FFH CME on the $Y_h Z_h$ plane and the definition and measured values of five halo parameters (SA_{xh} , SA_{yh} , D_{se} , α and ψ) for the CME.

The white ellipse enveloping the halo CME in Figure 1 is obtained using the 5-point method (See Cremades, 2005 for details). The X'_c axis is in the direction from solar disk center to the center of the white ellipse. It is the projection of the CME propagation direction X_c on the $Y_h Z_h$ plane. Obviously, the Y'_c axis perpendicular to the X'_c axis must be aligned with the Y_c axis of the $X_c Y_c Z_c$, system, which may be used to relate the orientation of elliptic cone bases, χ , to the orientation of elliptic halos, ψ (see Zhao, 2008 for the details). The CME propagation direction X_c is often expressed in ecliptic latitude λ and ecliptic longitude ϕ . In Equations below, we use the sky-plane latitude β and sky-plane longitude α to express the CME propagation direction X_c . Here parameter β is the projection angle between X_c and X'_c , and α , the azimuthal of the X'_c axis from the Y_h axis (see Figure 1). By using the projection angle β , the unknown model parameters

106 are reduced to five from six, and the measured α may be helpful in determining the
 107 unknown β , as shown in next Section.

The white ellipse in Figure 1 can be reproduced by projecting the base of the elliptic cone first onto the $Y_c Z_c$ plane with the angle χ from $Y_e Z_e$ plane, then onto the $X'_c Y'_c$ plane with the angle β , and finally onto the $Y_h Z_h$ plane with the angle α (See Zhao, 2008 for the detailed derivation). Thus we have

$$y_h = R_c p_y, \quad z_h = R_c p_z \quad (1)$$

$$p_y = \cos \beta \cos \alpha + (\sin \beta \sin \chi \cos \alpha + \cos \chi \sin \alpha) \tan \omega_y \cos \delta_b -$$

$$-(\sin \beta \cos \chi \cos \alpha - \sin \chi \sin \alpha) \tan \omega_z \sin \delta_b \quad (2)$$

$$p_z = -\cos \beta \sin \alpha - (\sin \beta \sin \chi \sin \alpha - \cos \chi \cos \alpha) \tan \omega_y \cos \delta_b +$$

$$+(\sin \beta \cos \chi \sin \alpha + \sin \chi \cos \alpha) \tan \omega_z \sin \delta_b \quad (3)$$

108 Where R_c is the distance from Sun's spherical center to the center of the cone base;
 109 ω_y and ω_z are the half angular width corresponding to the semi-axes of the elliptic
 110 base, SA_{yb} and SA_{zb} respectively; and χ is the angle between the semi-axis SA_{yb} of
 111 the elliptic cone base and the Y_c axis. The variable δ_b is the angular distance of elliptic
 112 base radii from semi-axis SA_{yb} or Y_e axis, and increases clockwise along the edge of the
 113 elliptic base from 0° to 360° .

Except R_c , all parameters in Equations (1)–(3) are assumed to be time-independent. The time-variation of y_h and z_h depends on the time-variation of R_c alone. The sky-plane speed and acceleration of halo CMEs are determined using height-time plots of the CMEs, and the CME heights are measured with respect to the disk center. Setting $R_{es} = \sqrt{y_h^2 + z_h^2}$, the measured sky-plane speed and acceleration at edge, V_{es} and a_{es} , can

be calculated

$$V_{es} = \frac{dR_{es}}{dt}, \quad a_{es} = \frac{d^2R_{es}}{dt^2}$$

and the radial speed and acceleration at the center of the elliptic cone base are

$$V_{cr} = \frac{dR_c}{dt} = \frac{V_{es}}{\sqrt{p_y^2 + p_z^2}}, \quad (4)$$

$$a_{cr} = \frac{d^2R_c}{dt^2} = \frac{a_{es}}{\sqrt{p_y^2 + p_z^2}} \quad (5)$$

The radial speed and acceleration at the edge, V_{er} and a_{er} will be

$$V_{er} = V_{cr} / \cos \omega_e, \quad a_{er} = a_{cr} / \cos \omega_e \quad (6)$$

where the half angular width, ω_e , at δ_b can be calculated using model parameters ω_y

and ω_z ,

$$\cos \omega_e = \frac{1}{\sqrt{1 + \tan^2 \omega_y \cos^2 \delta_b + \tan^2 \omega_z \sin^2 \delta_b}} \quad (7)$$

114 Equation (7) shows that $\omega_e = \omega$ when $\omega_y = \omega_z = \omega$, indicating that the cone model is
115 just a specific case of the more general elliptic cone model.

116 **3. Determination of Actual Geometrical and Kinematical**

117 **Properties for the 13 December 2006 FFH CME**

118 Equations (1) – (7) show that given the sky-plane speed and acceleration of a FFH
119 CME, V_{es} and a_{es} , at an edge point of δ_b , the radial speed and acceleration at the center
120 and edge of the elliptic cone base, V_{cr} , a_{cr} and V_{er} , a_{er} can be inverted if the CME
121 propagation direction (β, α) , the elliptic cone base orientation, χ , and the half angular
122 widths (ω_y and ω_z) are given.

123 3.1. Inversion of Model Parameters from Halo Parameters

124 To invert the model parameters β , χ , ω_y and ω_z from the halo parameters, we use
 125 the inversion equation system of model parameters [Zhao, 2008]

$$\begin{aligned}
 R_c &= D_{se} / \cos \beta \\
 \tan \omega_y &= [-(a - c \sin \beta) + \sqrt{(a + c \sin \beta)^2 + 4 \sin \beta b^2}] / (2R_c \sin \beta) \\
 \tan \chi &= (R_c \tan \omega_y - c) / b \\
 \tan \omega_z &= -(a + b \tan \chi) / R_c \sin \beta
 \end{aligned} \tag{8}$$

126

where

$$\begin{aligned}
 a &= SA_{xh} \cos^2 \psi - SA_{yh} \sin^2 \psi \\
 b &= (SA_{xh} + SA_{yh}) \sin \psi \cos \psi \\
 c &= -SA_{xh} \sin^2 \psi + SA_{yh} \cos^2 \psi
 \end{aligned} \tag{9}$$

127 Equation systems (8) and (9) show that the four unknown model parameters, R_c ,
 128 ω_y , ω_z and χ , can be uniquely determined from the four given halo parameters, D_{se} ,
 129 SA_{xh} , SA_{yh} and ψ , if the projection angle β can be specified.

The candidate CME propagation direction may be, in general, at any λ and any ϕ .
 The relationship between the sky-plane latitude and longitude, β and α , and the ecliptic
 latitude and longitude, λ and ϕ , is

$$\sin \lambda = \cos \beta \sin \alpha, \quad \tan \phi = \cos \alpha / \tan \beta \tag{10}$$

$$\sin \beta = \cos \lambda \cos \phi, \quad \tan \alpha = \tan \lambda / \sin \phi \tag{11}$$

130 Equations (10) and (11) show that for a specific value of α , the candidate directions
 131 reduce to pairs of (λ, ϕ) that correspond to all possible β values. The curve in Figure
 132 2 corresponds to $\alpha = 55.9^\circ$, and is obtained by assuming that the possible values of β
 133 range from 45° to 90° for this disk FFH CME located within 45° of disk center.

Figure 2.

134 To select the candidate β from all possible points on the α -curve, we use the position
 135 of the flare associated with the FFH CME. The dark dot in Figure 2 denotes the flare
 136 position. It should be noted that the flare position is often specified using the latitude
 137 and longitude measured in the heliographic coordinate system, as shown by ϕ_{fs} , λ_{fs} in
 138 Figure 2. By correcting the effect of B0 angle (the heliographic latitude of the Earth)
 139 the flare position in the heliocentric ecliptic coordinate system, i.e., ϕ_{fe} , λ_{fe} in Figure 2,
 140 can be obtained.

141 CME-associated flares or active regions are often assumed to be located near the
 142 center of CME source region, and used to represent the CME propagation direction
 143 [e.g., Smith et al., 2008]. The dark dot would be located on the α -curve if it is the case.
 144 The deviation of the flare position from the α -curve, as shown in Figure 2, has been
 145 attributed to the effect of interaction between high-speed coronal hole streams and the
 146 propagating CMEs [e.g., Cremades et al., 2006]. For such fast CMEs as the 13 December
 147 2006 FFH CME with the linear sky-plane speed of 1774 km/s , the effect of stream-CME
 148 interaction on the CME propagation direction can be neglected. Observations have
 149 shown that associated flares are often located near one leg of limb CMEs [e.g., Plunkett
 150 et al., 2001]. Therefore, we select the candidate β among all possible β on the α -curve
 151 by assuming that the candidate β should be located at the point on the α -curve which
 152 minimizes the distance between the dot and the α -curve. The obtained β (or ϕ_{ce} , λ_{ce}) is
 153 shown in Figure 2.

154 Once the candidate projection angle β is determined, other model parameters can
 155 be inverted, as shown in Table 1.

Table 1.

156 As shown in Figure 1, the angle $\psi = 5.81^\circ$, and $SA_{xh} < SA_{yh}$, indicates that the

157 semi-minor axis of the 13 December 2006 FFH CME nearly passes through the solar disk
 158 center, suggesting that the FFH CME might be formed by the projection of a circular
 159 cone base on the sky-plane, and the geometrical properties of the 3-D CME rope may be
 160 uniquely inverted by the cone model. To compare the β values obtained from the two
 161 models and to see whether or not the condition of the halo's semi-minor axis passing
 162 through the solar disk center is a sufficient condition for using the cone model to invert
 163 model parameters, we calculate the four cone model parameters using the four halo
 164 parameters SA_{xh} , SA_{yh} , D_{sc} and α , as shown in Table 1. It is expected that the two sets
 165 of model parameters should be the same if the cone base is circular. Table 1 shows that
 166 the model parameters inverted by the cone model are significantly different from that
 167 by the elliptic cone model. To compare the results with observations, the kinematical
 168 properties of the event are needed.

169 3.2. Calculation of the Actual Kinematic Properties

As shown in http://cdaw.gsfc.nasa.gov/CME_list/, for the 13 December 2006 FFH
 CME, the linear sky-plane speed at the measurement position angle (MPA) of 193° is
 1774 km/s ; the acceleration is $-61.4 m/s^2$, and the second order speed at the time of
 2006/12/13_02:54:04 when the FFH CME shown in Figure 1 was observed is 1930 km/s .
 The MPA is defined counter clockwise from solar north in LASCO sky-plane, meaning

$$MPA = \tan^{-1}(-y_h/z_h) = \tan^{-1}(-p_y/p_z) \quad (12)$$

170 Thus the MPA depends on not only δ_b but also the five model parameters, ω_y , ω_z ,
 171 χ , β and α . Using the five model parameters listed in Table 1 and the given value of
 172 MPA, we can determine the variable δ_b (see Table 1). Using Equations (2) – (7) and
 173 the model parameters shown in Table 1 the linear radial CME propagation speed at

174 the edge of the elliptic cone base, V_{er1} , the second order radial propagation speed at
175 2006/12/13_02:54:04, V_{er2} , and the acceleration, a_{er} , can be calculated (See Table 1). To
176 compare such obtained radial speeds with the estimate from Type II observations and
177 with the MHD simulations, the second order radial propagation speed near the solar
178 surface, V_{er201} , and at the 30 solar radii (the helipspheric base), V_{er230} , are also calculated
179 (See Table 1).

180 4. Validation of the Inversion Solution

181 The calculations of the elliptic cone model by one-point approach made in Section 3
 182 for inverting actual geometrical and kinematic properties of the 13 December 2006 FFH
 183 CME are based on the assumption that the candidate β should be located at the point
 184 on the α -curve at the minimum distance between the flare position and the α -curve. The
 185 obtained β and other parameters are different from those of the cone model (See Table
 186 1). In the following sections we attempt to determine which set of parameters provides
 187 the most valid solution.

188 4.1. Test of the inverted geometrical property

189 By substituting the two sets of model parameters into Equations (1)–(3) we obtain
 190 the two modeled halos. The red dashed (green dotted) ellipse in Figure 3 is produced
 191 by the elliptic (circular) cone model parameters. Both ellipses agree well with the white
 192 ellipse, indicating that Equations (1) – (3) and (8) – (11) that are used in the calculations
 193 are valid. It also shows that this kind of agreement is not a solid argument for the
 194 validation of the candidate β , though the red ellipse matches the white one slightly
 195 better than the green one.

196 4.2. Test of the inverted kinematical property

197 By using Type II observations, the radial propagation speed near the solar surface
 198 for the 13 December 2006 event has been estimated to be 2212 *km/s* [Liu et al., 2008].
 199 The radial CME speed near the solar surface inverted by the elliptic cone model is 2323

200 km/s , agrees with the Type II estimate slightly better than the radial CME speed of
 201 $1997 km/s$ by the circular cone model. Furthermore, we have deduced the propagation
 202 speed of the CME's shock about half way its journey to Earth using Type II radio
 203 emissions in the kilometric domain. At those low frequencies it may be assumed that
 204 the CME's shock responsible for the emission has already undergone deceleration and
 205 that it is travelling at an approximately constant speed. The deduced speed near 120
 206 solar radii from the Sun yielded $V_{r120} = 1320 km/s$, which should be less than the radial
 207 CME speed at 30 solar radii for the fast CME. The radial CME speed at 30 solar radii
 208 inverted using the elliptic cone model is $1585 km/s$, greater than V_{r120} , but that inverted
 209 using the circular cone model is $1292 km/s$, slightly less than V_{r120} . Thus Type II radio
 210 emissions further support the elliptic cone model.

211 4.3. Test of inverted geometrical and kinematical properties

212 To reconstruct the near-Earth solar wind disturbance caused by the 13 December
 213 2006 FFH CME, we first use the ‘‘CORHEL’’ coupled 3-D magnetohydrodynamic (MHD)
 214 models of the corona-heliosphere system to simulate the ambient solar wind [Odstrcil et
 215 al., 2004a]. The corona in CORHEL is simulated by the SAIC ‘‘MHD Around a Sphere’’
 216 (MAS) model [Linker et al., 1999; Mikic et al., 1999], which solves the time-dependent
 217 3-D MHD equations using magnetograms as the inner boundary condition. At 30 solar
 218 radii, output from MAS is used as the inner boundary for the NOAA Space Weather
 219 Prediction Center (SWPC) ‘‘Enlil’’ model of the heliosphere [Odstrcil, 2003, and
 220 references therein], which solves the MHD equations on a Sun-centered spherical grid
 221 out to 2AU. This coupled scheme has been shown to match the bulk properties of the
 222 ambient solar wind very well [Owens et al., 2008].

223 Once the ambient solar wind conditions have been simulated, an overpressured
224 density cloud is inserted at 30 Rs as a proxy for the transient disturbance resulting
225 from the FFH CME. The cloud is assumed to be a spherical pulse, four times more
226 dense and at the same temperature as the ambient solar wind. It does not contain an
227 intrinsic magnetic field [Odstreil et al., 2004b]. The CME velocity, angular width and
228 the arrival time at 30 Rs can be specified by the circular or elliptic cone model fits to
229 the coronagraph observations of FFH CMEs [Zhao et al., 2002; Zhao, 2008]. In this
230 study, both the circular and elliptical cone model parameters are used to initialise the
231 simulations, with the second order fits to the reconstructed height-time profile used to
232 derive the time and speed at 30Rs: 1585 km/s at 05:28:13 UT (1217 km/s at 06:11:22)
233 for the elliptical (circular) cone model . For the elliptical model, the width is set to be
234 the average of the semi-major and semi-minor axes.

235 Figure 4 compares the ACE-observed near-Earth solar wind profile (black) with
236 the simulated solar wind, initialised with the elliptical (red) and circular (blue) cone
237 model parameters. In both cases, the disturbance arrives later than observed, which is
238 expected due to the ambient solar wind simulation underestimating the wind upstream
239 solar wind speed, and hence overestimating the drag force on the ejecta [Case et al.,
240 2008]. The disturbance initiated with the elliptic cone model, however, is only 0.5-days
241 late, compared to the 1-day error for the circular model. Both simulations produce
242 the correct field and flow deflections in the sheath region ahead of the actual ejecta,
243 suggesting the leading-edge orientation was correctly reproduced and thus supporting
244 the geometric and kinematic properties inverted by the elliptic cone model. The body of
245 the cloud is not expected to be reproduced, as the inner plasma and magnetic structure
246 of the overpressured cloud is not realistic.

247 5. Summary and Discussions

248 Based on the projection of the elliptic cone base on the plane of the sky, we have
 249 established the mathematical relationship between the apparent (sky-plane) speed and
 250 acceleration of frontside full-halo CMEs and the actual radial speed and acceleration of
 251 the 3-D CME ropes.

252 To invert the actual geometrical properties of the fast 13 December 2006 event we
 253 assume that the candidate projection angle β must be located at a specific point on the
 254 α -curve which minimizes the distance between the flare position and the α -curve.

255 To invert the actual kinematic properties from observations, it is necessary to
 256 determine the relationship between the position angle of the measurement of the apparent
 257 speed and the variable δ_b , the angular distance of radii of the elliptic cone base from its
 258 semi-axis near the Y'_c axis. We establish Equation (12) that can be used to accurately
 259 determine the δ_b value that corresponds to the measurement position angle.

260 By using the elliptic cone model, the obtained actual geometrical and kinematic
 261 properties for the 13 December 2006 frontside full-halo CME have been used to reproduce
 262 the observed halo CME, and to invert the radial propagation speed which agrees with
 263 the estimate of the radial propagation speed from Type II observations both near the
 264 Sun and near the Earth. The actual properties are also introduced at the inner boundary
 265 of the CORHEL model and the simulated solar wind disturbances near the Earth agree
 266 with the in situ observation of the solar wind disturbances associated with the 13
 267 December 2006 frontside full-halo CME.

268 Thus we suggest that both the mathematical relationship between the apparent

269 and actual kinematic properties established here and the minimum distance assumption
 270 used in the one-point approach for determining the projection angle are valid for fast
 271 frontside full-halo CMEs. We are examining more events to further validate the one-point
 272 approach.

273 Finally, the limitation of the circular cone model in inverting model parameters
 274 should be emphasized. As shown in Zhao [2008], the model parameters obtained using
 275 the elliptic cone model should be the same as those using the circular cone model if
 276 the elliptic halo is indeed formed by the projection of a circular cone base onto the
 277 sky-plane. The model parameters β and R_c (ω_y and ω_z) inverted by the elliptic
 278 cone model are, however, significantly greater (less) than those by the circular cone
 279 model (see Table 1), though the minor-axis of the 13 december 2006 halo CME passes
 280 close to the solar disk center. It indicates that the condition of passing through the
 281 solar disk center of the minor axis is only a necessary condition, but not a sufficient
 282 condition for using the circular cone model to invert model parameters. The significant
 283 difference is understandable. In the case of $\chi \sim 0$, we have $SA_{xh} = SA_{zb} \sin \beta$ and
 284 $SA_{yh} = SA_{yb}$. By using the circular cone model means assuming $SA_{zb} = SA_{yb} = SA_{yh}$
 285 so that we have $\sin \beta = SA_{xh} / SA_{yh}$. In fact, for the 13 december 2006 full-halo CME
 286 $SA_{zb} = R_c \tan \omega_z = 4.17$ Rs, and $SA_{yb} = R_c \tan \omega_y = 4.46$ Rs. Here $SA_{yh} = SA_{yb}$
 287 but $SA_{yh} > SA_{zb}$. Thus the parameter β obtained by $\sin \beta = SA_{xh} / SA_{yh}$ that is valid
 288 for the circular cone model is $\sim 10^\circ$ less than the real one, though the difference between
 289 ω_y and ω_z is only 2° . To avoid any misuse of the circular cone model, the elliptic cone
 290 model is strongly advised for inverting the model parameters for all kinds of frontside
 291 full-halo CMEs.

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358 **Figure Captions**

Halo parameters: $\psi=5.81$, $\alpha=55.99$, $SA_{xh}=3.96R_s$, $SA_{yh}=4.46R_s$, $D_{se}=1.39R_s$

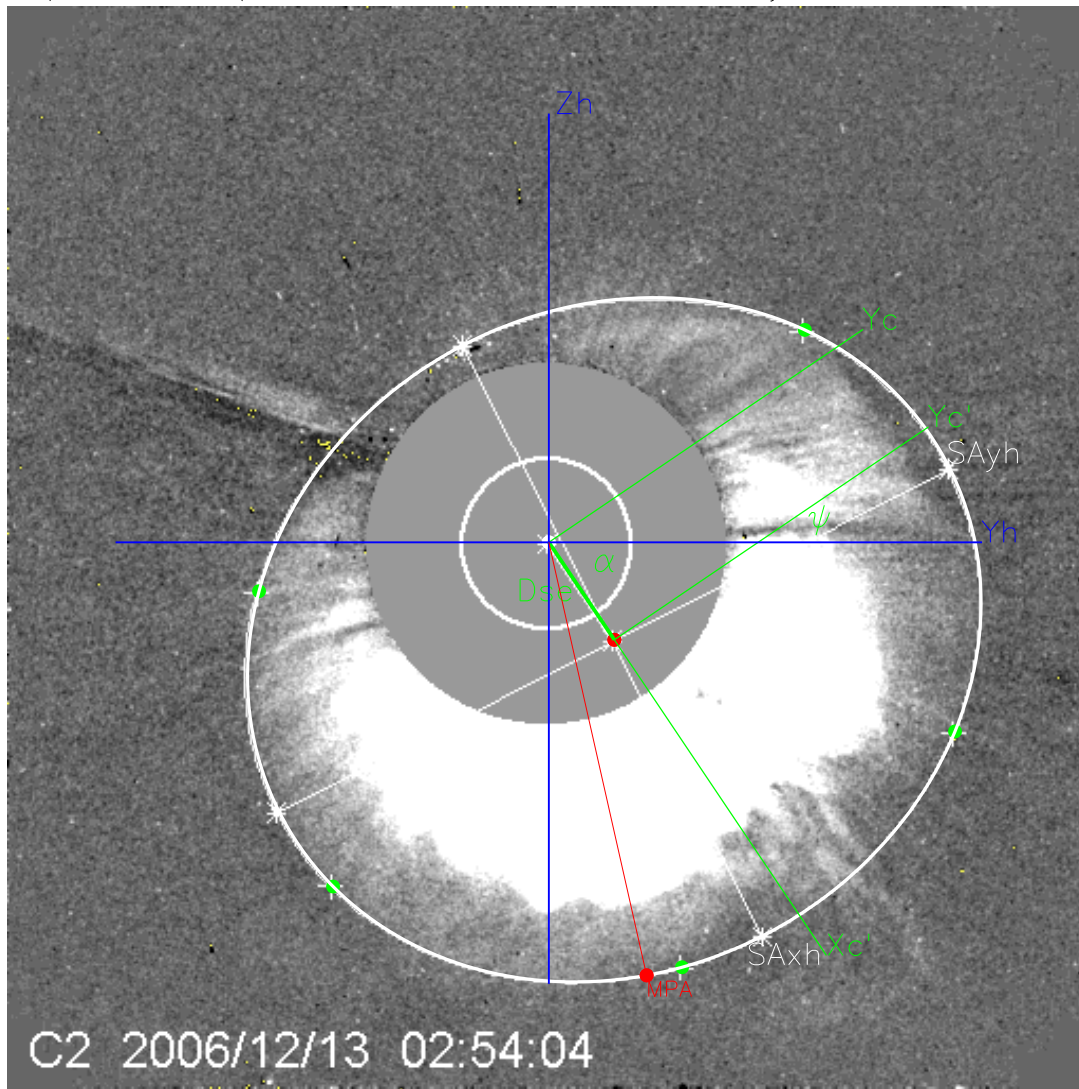


Figure 1. The definition and values of five halo parameters (SA_{xh} , SA_{yh} , ψ , D_{se} , α) for the 13 December 2006 frontside full-halo CME. Here X'_c and Y'_c are, respectively, aligned with and perpendicular to the direction from the solar disk center to the halo center, D_{se} (the short thick green line). Parameters ψ and α denote the angles between SA_{yh} and Y'_c and between X'_c and Y_h , respectively.

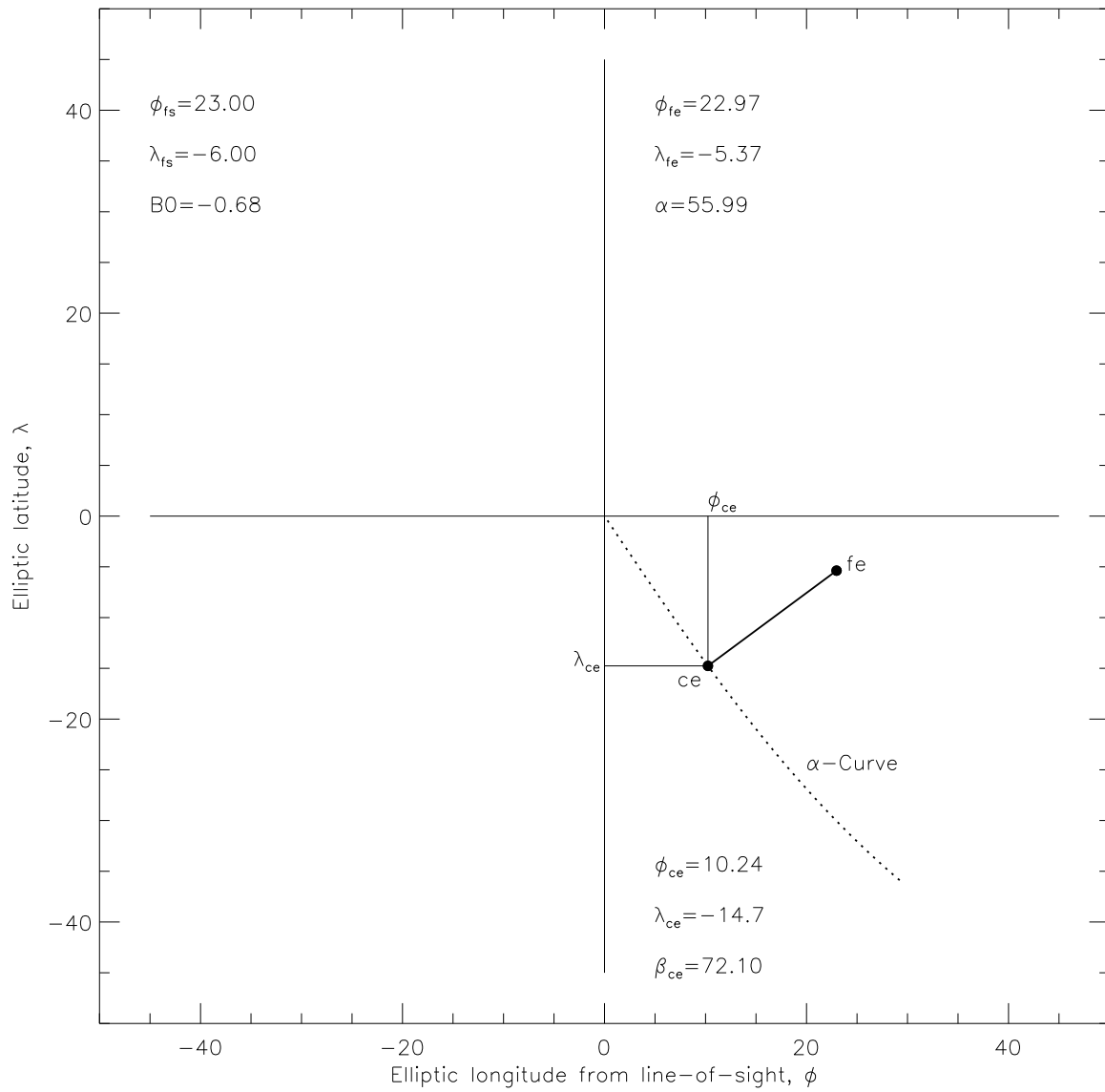


Figure 2. Description of the one-point approach for finding out the CME propagation direction $(\phi_{ce}, \lambda_{ce})$ or β on the basis of halo parameter α and the location of CME-associated flare $(\phi_{fs}, \lambda_{se})$. See text for details.

Model Params: $R_c=6.31R_s$, $\omega_y=59.63$, $\omega_z=63.58$, $\chi=-10.72$, $\beta=65.00$, $\alpha=-25.49$

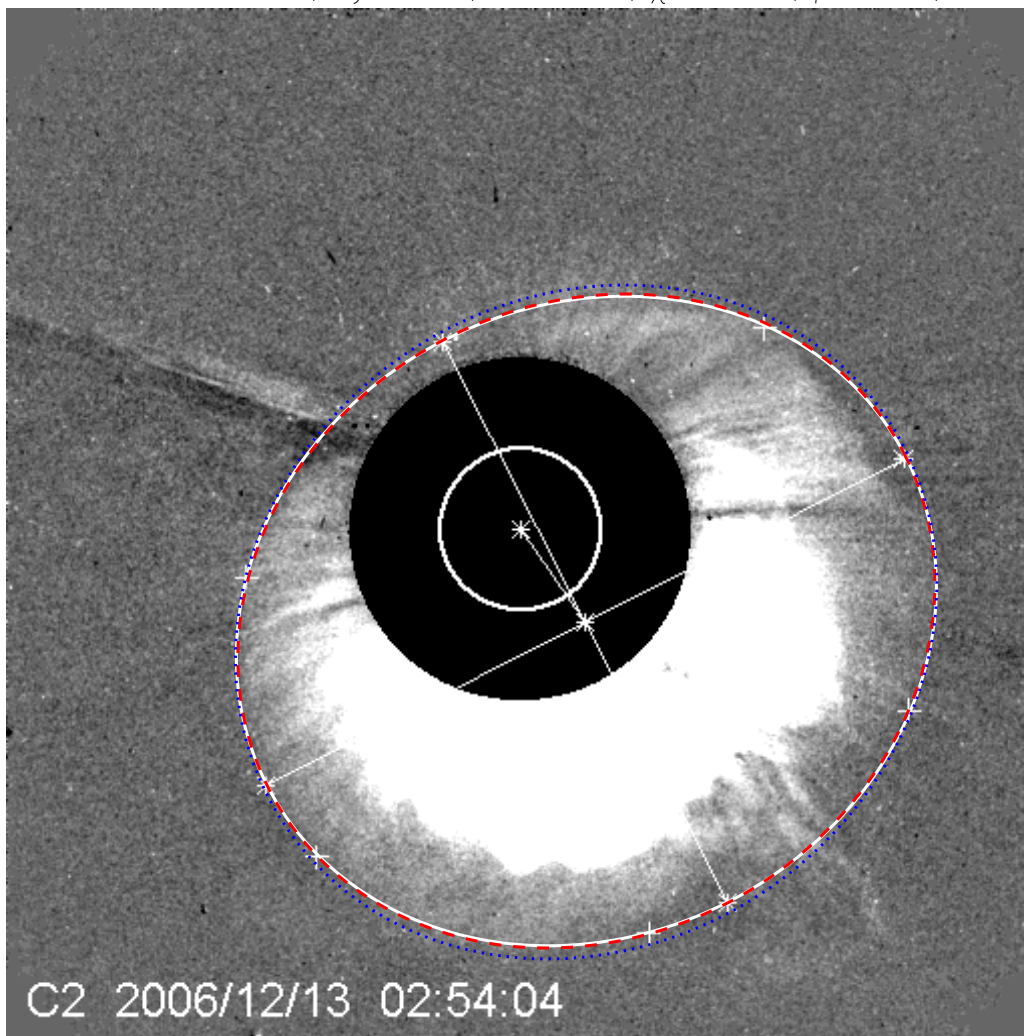


Figure 3. Comparison of the modeled halos by the elliptic cone model (Red dashed ellipse) and the cone model (Blue dotted ellipse) with the observed one (White solid ellipse). Both modeled halos agree with the observed one well.

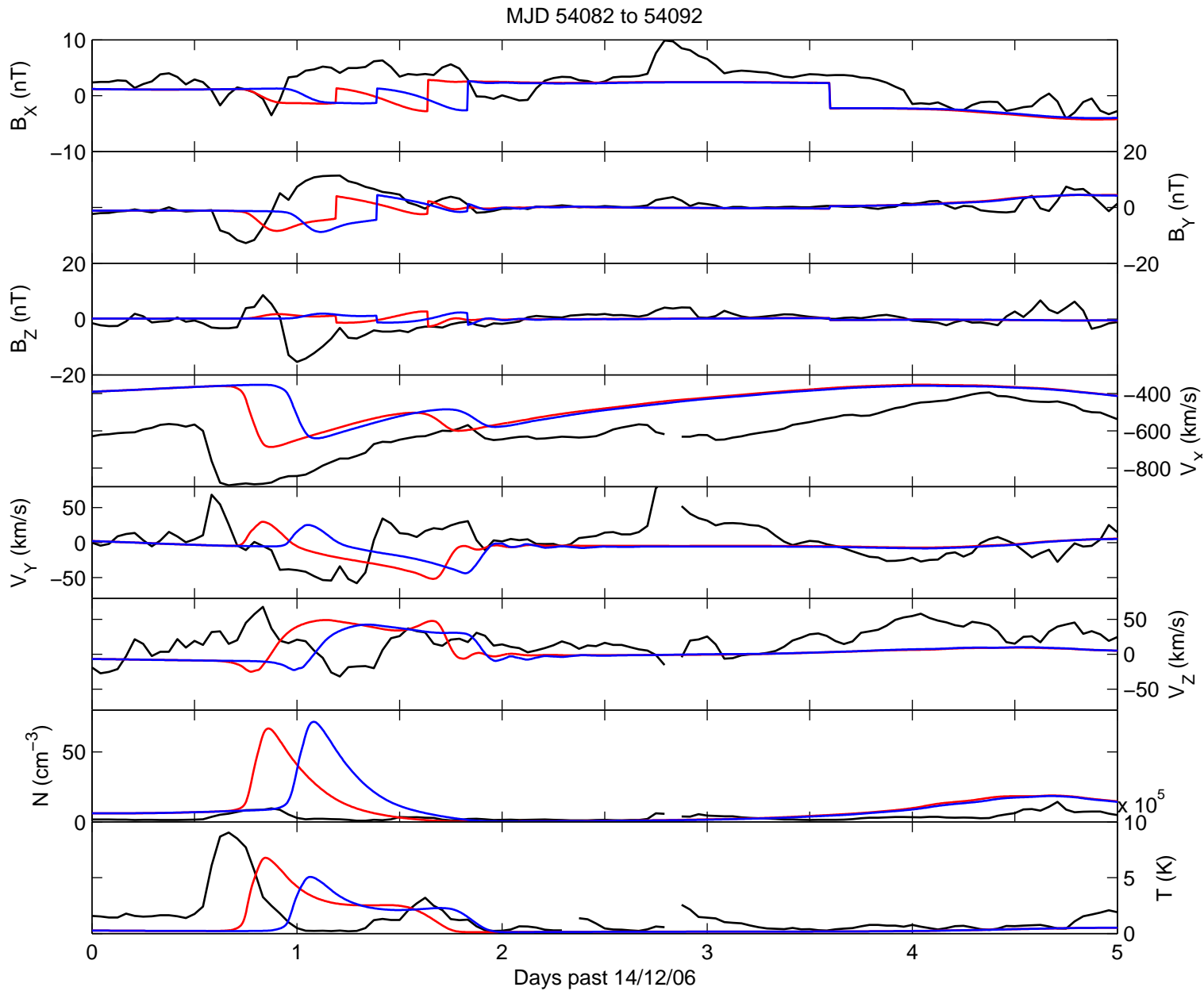


Figure 4. Comparison of simulated near-Earth solar wind disturbances initiated with elliptic (red) and circular (blue) cone model fits to the 13 December 2006 FFH CME observations to the ACE-observed near-Earth solar wind profile (black).

359 **Tables****Table 1.** Given Parameters and Model Parameters

Given parameters	Model parameters	Elliptic cone model	Cone model
Flare $\lambda=-6^\circ$	CME λ	-14.70°	-22.41°
Flare $\phi=23^\circ$	CME ϕ	10.24°	16.16°
$\alpha=55.99^\circ$	α	55.99°	55.99°
	β	72.10°	62.62°
$D_{se}=1.39R_s$	R_c	$4.53R_s$	$3.02R_s$
$SA_{xh}=3.96R_s$	ω_y	44.61°	55.88°
$SA_{yh}=4.46R_s$	ω_z	42.64°	55.88°
$\psi=5.81^\circ$	χ	5.97°	0.0°
MPA= 193°	δ_b	250°	245°
$V_{es1}=1774km/s$	V_{er1}	$2064km/s$	$1795km/s$
$V_{es2}=1930km/s$	V_{er2}	$2245km/s$	$1953km/s$
$a_{es}=-61.4m/s^2$	a_{er}	$-71.48m/s^2$	$-62.16m/s^2$
	V_{er201}	$2323km/s$	$1997km/s$
	V_{er230}	$1585km/s$	$1217km/s$