The Ice Cream Cone Model for Inversing Geometrical Properties of Halo Coronal Mass Ejections

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Abstract

It is suggested recently that most of halo coronal mass ejections (CMEs) may be formed by an ice cream cone-like shell of dense CME electrons. We improve our cone model (Zhao, Plunkett and Liu, 2002) to include the contribution of the top surface of the ice cream part as well as the cone part to the shape of the halo CMEs. It is found that for an ice cream cone with a half sphere sitting on the base of a cone, the projection of the top surface of the ice cream part on the plane of the sky is a half ellipse with its semi-minor axis aligned with the line connecting the center of the ellipse to the Sun’s disk center and located in the side of the major axis far away from the Sun’s disk center. This front half ellipse is greater than the front half ellipse produced by the cone part when the angle of the central axis of the ice cream cone from the plane of the sky is less than 45 degrees; when the angle is greater than 45 degrees, the whole elliptic halo is symmetric. In both cases, the such geometrical and kinematical properties as the angular width, the direction of the central axis, and the acceleration of the halo CME can be uniquely determined.
1. Introduction

Most of halo CMEs are believed to be formed due to Thompson scattering of the photospheric light in the line of sight by a broad shell of dense CME’s electrons. The shape of the shell-like plasma structures has been suggested to be conical, spherical (Howard et al., 1982) or ice cream cone-like (Fisher and Munro, 1984) (Figure 1).

For a conical shell of dense CME plasma we have developed a geometrical model, the circular cone model (see Figure 2), to determine the angular width, direction of central axis, and radial bulk speed for halo CMEs (Zhao, Plunkett and Liu, 2002). Since the four characteristic parameters for an elliptical halo CME (see Figure 3) depend on the four circular cone characteristic parameters, the four circular cone parameters can be uniquely inverted using the four ellipse’s parameters measured from halo CME images. By imputing the geometrical and knematical properties obtained using the cone model for the 12 May 1997 full halo CME into a MHD model of CME propagation, we have successfully predicted which part of and when the CME will hit the Earth (Odstrcil, Riley and Zhao, 2004).

Recent UV observations show that the shape of broad shell of dense plasma for most of halo CMEs may be ice cream cone-like. This work improves our cone model for reproducing the shape of halo CMEs formed by the ice cream cone-like shell of the dense CME plasma.
2. The ice cream cone model

As shown in the right bottom panel of Figure 1, the ice cream cone model contains two parts: the circular cone part and the ice cream part. Both the base of the cone (or ice cream) part and the top surface of the ice cream part contribute to the shape of the halo CMEs. Figure 2 shows the 3-D view of an circular conical shell of the CME plasma and its projection on the plane of the sky. It has been shown that the projection of the circular base of the cone part on the plane of the sky is an ellipse with its semi-minor axis aligned with the projection of the cone’s central axis on the plane of the sky (Xie et al., 2004). For the circular cone part there are four parameters that characterize the circular cone, i.e., the cone’s slant height, $s$, the half angular width, $\omega$, and the orientation of the cone’s central axis, $(\chi, \alpha)$ or $(\theta, \phi)$. Figure 1 and Figure 2 show the definition of the four circular cone parameters. For the ice cream part, in addition to the four parameters that characterize the base of both the cone and ice cream parts, other parameters are, in general, needed to describe the detailed shape of the ice cream part. However, for the small and large ice cream cone as shown in the right top and right bottom panels of Figure 1, the four parameters are enough to be used to characterize whole ice cream cone-like structure.

2.1 The cone part

As shown in Figure 4, the ellips parameters depend on the cone parameters as follows,

$$D_{dc} = s \cos \omega \cos \chi \quad (1)$$
$$S_{mj} = s \sin \omega \quad (2)$$
$$S_{mn} = s \sin \omega \sin \chi \quad (3)$$

and therefore, the three cone parameters can be uniquely inverted using the
Fig. 1.— Suggested shapes of the broad shell of dense CME plasma that form the halo CMEs through Thompson scattering (Schwenn et al., 2005). The panels from left top, left bottom, right top to right bottom denote, respectively, the spherical shell, the conical shell, the small ice cream cone-like shell (a sphere section with its center at apex), and the large ice cream cone-like shell (a half sphere sitting on the base of a cone).
Fig. 2.— The 3-D view of a circular cone in the XhYhZh system where Zh is in the line of sight (LOS) and XhYh denotes the plane of the sky. Zc, Dc and the black ellipse are, respectively, the projection of the cone axis Zc, the center and the rim (red) of the cone base on the plane of the sky.
Fig. 3.— The definition of the four parameters that characterize an ellipse on the plane of the sky, i.e., the distance of the ellipse from Sun’s disk center, $D_{dc}$, the semi-major axis, $S_{mj}$, the semi-minor axis, $S_{mn}$, and the angle of the projection of cone’s central axis, $Z’c$, from the east-west direction (Xh), $\alpha$. 
Fig. 4.— The dependence of the ellipse parameters, $S_{mj}$, $S_{mn}$, $S_{mni}$ and $D_{dc}$ on the parameters of the cone part, $s$, $\omega$, and $\chi$. 
three measured ellips parameters as follows

\[ \sin \chi = \frac{S_{mn}}{S_{mj}} \]  
(4)

\[ \tan \omega = \cos \chi \frac{S_{mj}}{D_{dc}} \]  
(5)

\[ s = \frac{S_{mj}}{\sin \omega} \]  
(6)

2.2 The ice cream part

The ice cream part may have various shapes and have, in general, more than four characteristic parameters. For simplicity we discuss two cases: large and small ice cream, as shown in Figure 1. The projection of the top surface of the ice cream part on the plane of the sky may form a half ellipse with its semi-minor axis, \( S_{mni} \), different from that of the circular cone, \( S_{mn} \), as shown in Figure 4. The half ellipse is located in the side of the major axis far away from the Sun’s disk center. The three parameters for the half ellipse depend on the cone parameters

\[ D_{dc} = s \cos \omega \cos \chi \]  
(7)

\[ S_{mj} = s \sin \omega \]  
(8)

\[ S_{mni} = s \sin \omega \cos \chi \]  
(9)

for the large ice cream part i.e., a half sphere sitting on the base of a cone, and

\[ D_{dc} = s \cos \omega \cos \chi \]  
(10)

\[ S_{mj} = s \sin \omega \]  
(11)

\[ S_{mni} = s(1 - \cos \omega) \cos \chi \]  
(12)

for the small ice cream part, i.e., a sphere section with its center at apex.

By compare the semi-minor axes corresponding to the cone part and ice cream part, the critical \( \chi \) at which \( S_{mn}/S_{mni} = 1 \) can be determined. The critical \( \chi \) is 45° for the large ice cream cone model, and the critical \( \chi \) depends on \( \omega \) as \( \tan \chi = (1 - \cos \omega)/\sin \omega \) for the small ice cream cone model.
3. Halo CMEs calculated using the ice cream cone model

Figures 5, 6, and 7 display the ellipses on the plane of the sky calculated using the ice cream cone model developed in the previous section. The ice cream cone parameters used in the calculation are $s = 12 \, R_\odot$, $\omega = 40^\circ$, and $\alpha = 0, 30, 90$ for Figures 5, 6, 7, respectively. The angle of the central axis from the plane of the sky, $\chi$, increases from 0, through 10, 17, 19, 21, 23, 25, 27, 30, 40, 45, 50, 60, 70, 80, to 90 degrees, as shown in the top of each panel. The red, blue and green half ellipses are obtained using the cone part, the large and small ice cream parts, respectively.

As $\chi$ increases the front green and blue half ellipse get smaller and smaller, and the red half ellipse gets bigger and bigger. The blue (green) and red half ellipses become the same when $\chi$ approaches a critical value of $45^\circ$ ($20^\circ$) for large (small) ice cream part.

The rear half ellipse produced using the ice cream cone model is exactly the same as what produced using the circular cone model.
4. Summary and discussion

• The ice cream cone model developed here consists of two parts, a cone part and a top “ice cream” part. The base of the cone and ice cream parts may be circular or elliptical. Because the legs of most limb CMEs are pointing in the radial direction, the apex of the cone part is assumed to be located at the center of the Sun. It suggests that CMEs do not origin in point sources at the solar surface.

• It has been shown that the projection of the base’s rim of a circular cone on the plane of the sky is an ellipse in general. The minor axis (major axis) of the ellipse is aligned with (perpendicular to) the projection of the central axis on the plane of the sky. The ellipse can be characterized using four parameters, and the four ellipse parameters depend on four circular cone parameters. Therefore, the measured four ellipse parameters can be used to uniquely inverse the four circular cone parameters. In other words, such geometrical properties of halo CMEs as the slat height $s$, the half angular width $\omega$, and the direction of the central axis ($\chi$, $\alpha$) or ($\theta$, $\phi$) can be uniquely determined on the basis of measured ellipse parameters, $S_{mj}$, $S_{mn}$, $D_{dc}$, and $\alpha$. The slat height $s$ determined using images observed at a series of time may be used to determine the acceleration of halo CMEs.

• If the top ice cream part is a half sphere sitting on the base of the cone part or a sphere section with its center at the apex of the
Fig. 5.— The dependence of the shape of ellipses calculated using the models for the cone part (red), small ice cream part (green) and large ice cream part (blue) on the angle of the central axis from the plane of the sky, $\chi$, as shown in the top of each panel.
Fig. 6.— The same as Figure 5 but $\alpha = 30$. As $\chi$ increases the front green and blue half ellipse decreases and the red half ellipse increases. The blue (green) and red half ellipses become the same when $\chi$ approaches a critical value of $45^\circ$ ($20^\circ$) for large (small) ice cream part.
Fig. 7.— The same as Figure 6 but $\alpha = 90$. 
cone part, the four cone parameters can be used to characterize the ice cream part either, and the projection of the top surface of the ice cream part on the plane of the sky is a half ellipse with its major axis same as that of the ellipse formed by the circular base’s rim and located in the side of the major axis far away from Sun’s disk center. The difference of the half ellipse formed by the ice cream part from the front half ellipse formed by the base’s rim of the circular cone is in the semi-minor axis and dependent on the angle of the central axis from the plane of the sky, $\chi$. As $\chi$ increases from 0 to 90 degrees, the former gets smaller and smaller and the latter gets bigger and bigger. The ellipse becomes symmetric about the major axis when $\chi$ greater than a critical angle.

- It is concluded that if the minor axis of an elliptical halo CME is aligned with the projection of the central axis on the plane of the sky, i.e., the line connecting the solar disk center to the ellipse center, the geometrical properties of full halo CMEs formed by Thompson scattering can be uniquely inversed using the cone model and the measurements of the rear half ellipse parameters; if the front half ellipse is greater than rear half ellipse, the plasma shell must be ice cream cone-like and the angle of the central axis from the plane of the sky is less than a critical angle, say 45 degrees; for a symmetric ellipse about its major axis, it is difficult to distinguish whether the plasma shell is conical or ice cream cone-like.
• If it is the major axis that is aligned with the projected central axis, the base of an ice cream cone-like shell of dense CME plasma must not be circular. The geometrical properties of the halo CME can not be uniquely determined using any geometrical model because the number of characteristic parameters for an non-circular cone is greater than the number of ellipse parameters.

• The success of the ice cream cone model in inversing the geometrical properties of the cone-like shell of the dense CME plasma depends on the accuracy of the measurement of the ellipse parameters. It is necessary to develop programs to automatically determine the ellipse parameters from observed halo CME images.