Models for Determining Geometrical Properties of Halo Coronal Mass Ejections

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1. Introduction

- Halo CMEs, the major cause of geomagnetic storms.
- Necessary conditions for forecasting the storm-effectiveness of halo CMEs.

  Real angular width of plasma clouds that form halo CMEs

  Propagation direction: latitude and longitude

  Propagation distance
- Approaches to infer the necessary conditions.

  Statistic methods: Schwenn et al., 2005 and the references therein
- A new approach: Inverse technique.

Circular cone	Zhao et al., 2002;	Michalek et al., 2003
	Xie et al., 2004;	Xue et al., 2005
Elliptic cone	Zhao, 2004;	Cremades el al., 2004
Ice Cream cone	Zhao, 2005	

- Issues to be answered:

  a. What kind of halo CMEs can be inverted?

  b. How to select the geometrical models given a specific halo CME?
  c. Whether or not the inverted solutions for halo CMEs are unique?

2. Models and Characteristic Geometric Parameters of CMEs

Coronagraphs measure the photospheric light scattered by coronal electrons along the line of sight. The rim of observed 2-D white-light images of CMEs may be approximated by the projection of the boundary surface of 3-D magnetized plasma clouds of CMEs on the plane of the sky.

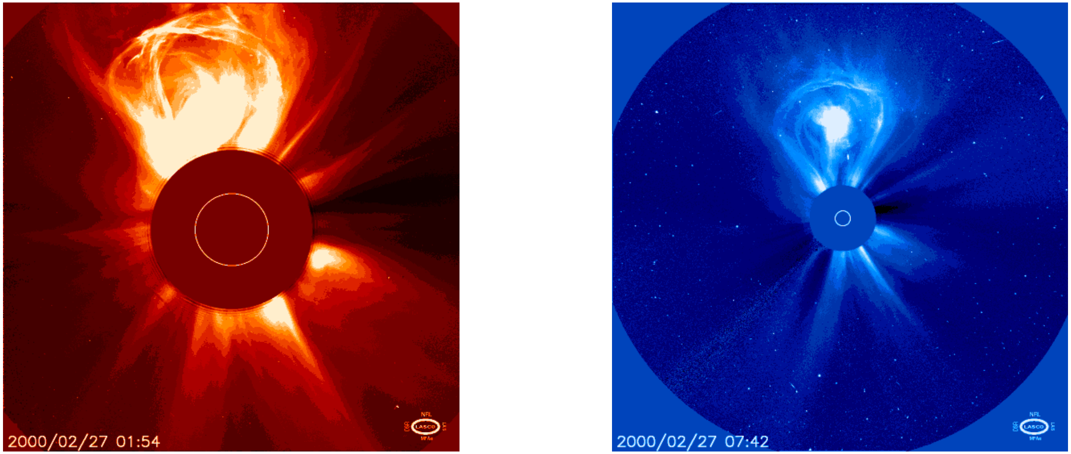

Figure 1. C2 and C3 images of a limb CME showing cone-like shape with the apex of the cone located at the center of the sun. Most of limb CMEs show radial propagation with constant angular width, implying cone-like magnetized plasma clouds with a shell-like outline . 2-D or 3-D structure?

2.1 Circular cone-like models suggested in literature

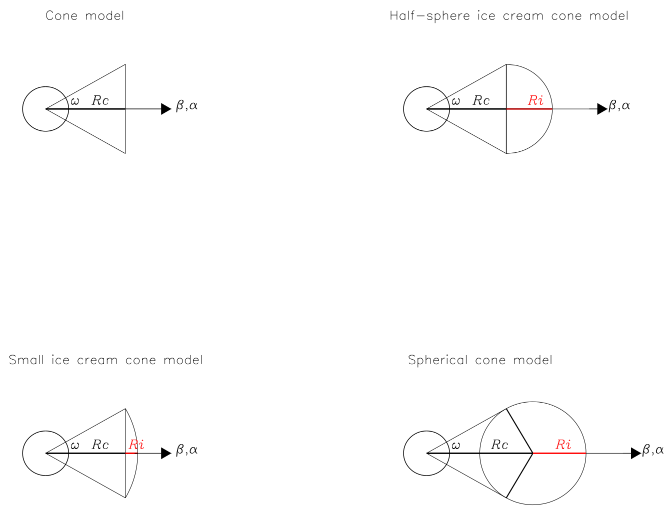

Figure 2. The detection of halo CMEs implies that the cone-like plasma clouds of CMEs are a 3-D structure. The conical, ice cream conical and spherical conical models have been suggested (e.g., Schwenn et al., 2005). All models here have a circular base of the cone. Both the rim of the base and the height of ice cream part, R_i, can be expressed by four characteristic parameters, i.e., the latitude and longitude of the central axis, β and α, the half angular width, ω, and the distance from apex to base of the cone (the apex-base distance), R_c. Recent studies show that the base of cone for many CMEs may be elliptic, and the shape of broad shell of dense plasma for most of halo CMEs may be ice cream cone-like.

2.2 Elliptic cone-like models

- Coordinate systems

$Coordinate\ System$	$Helio - Earth\ X_hY_hZ_h$	$Cone\ X_cY_cZ_c$	$Elliptic - Base\ X_eY_eZ_e$
$Xaxis$	$X_h{-}West$	$X_c \parallel Ct.axis$	$X_e \parallel X_c$
$Yaxis$	$Y_h{-}North$	$Y_c\ in\ X_hY_h$	$Y_e \parallel Semi-axis\ \sim Y_c$
$Zaxis$	$Z_h{-}Earth$	$Z_c\ in\ X_cZ_h$	$Z_e \parallel Semi-axis\ \sim Z_c$

The angle between Y_e and Y_c axes, $\underline{\chi}$, is used to characterize the tilt of the elliptic base of a cone. The direction of the central axis of the cone in $X_hY_hZ_h$ system, X_c, may be expressed using latitude $\underline{\beta\ (\lambda)}$ and longitude $\underline{\alpha\ (\phi)}$ with respect to the plane X_hY_h (Z_hX_h) and the axis X_h (Z_h). The relationship between (β, α) and (λ and ϕ) is

$$\left\{\begin{array}{lcl} \sin\lambda & = & \cos\beta\,\sin\alpha \\ \sin\phi & = & \frac{\cos\beta\,\cos\alpha}{\sqrt{1-\cos^2\beta\,\sin^2\alpha}} \end{array}\right\}\left\{\begin{array}{lcl} \sin\beta & = & \cos\lambda\,\cos\phi \\ \tan\alpha & = & \frac{\sin\lambda}{\cos\lambda\,\sin\phi} \end{array}\right\}$$

- The elliptic cone model

$$
\begin{aligned}
x_e &= R_c \\
y_e &= R_c \tan \omega_{ye} \cos \delta \\
z_e &= R_c \tan \omega_{ze} \sin \delta
\end{aligned} \tag{1}
$$

where $\underline{R_c}$ denote the apex-base distance, $\underline{\omega_{ye}}$ and $\underline{\omega_{ze}}$ the half angular widths corresponding to the semi-axes aligned with Y_e and Z_e axes. δ is the angle relative to the Y_e axis, varying between 0° and 360° along the rim of the base.

- The half-ellipsoid ice cream cone model

$$
\begin{aligned}
y_e &= R_c \tan \omega_{ye} \sin \delta \\
z_e &= -R_c \tan \omega_{ze} \sin \delta \nearrow R_c \tan \omega_{ze} \sin \delta \\
x_e &= R_c + R_i \left(1 - \cos^2 \delta - \frac{{z_e}^2}{(R_c \tan \omega_{ze})^2}\right)^{0.5}
\end{aligned} \tag{2}
$$

where δ runs from -90° to 90°. Additional parameter, $\underline{R_i}$ (the height of the ice cream part) is needed with respect to the elliptic cone model.

3. The shapes and measurable parameters of predicted halo CMEs

- Transformation from the elliptic base system ($X_eY_eZ_e$) to the cone system ($X_cY_cZ_c$)

$$
\begin{aligned}
x_c &= x_e \\
y_c &= y_e \cos\chi + z_e \sin\chi, \\
z_c &= -y_e \sin\chi + z_e \cos\chi
\end{aligned}
\tag{3}
$$

where x_e, y_e and z_e are given by Exp. (1) or (2), χ denotes the tilt angle of the elliptic base relative to Y_c axis.

- Transformation from the cone system ($X_cY_cZ_c$) to the Helio-Earth system ($X_hY_hZ_h$)

$$
\begin{bmatrix} x_h \\ y_h \\ z_h \end{bmatrix} = \begin{bmatrix} \cos\alpha\cos\beta & -\sin\alpha & -\cos\alpha\sin\beta \\ \sin\alpha\cos\beta & \cos\alpha & -\sin\alpha\sin\beta \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} \tag{4}
$$

where β and α denote the direction of central axis of the cone in $X_hY_hZ_h$ system.

3.1 Circular cone and half spheroid ice cream cone models

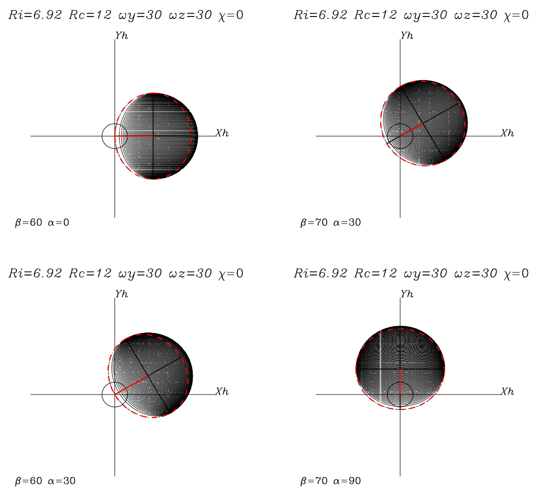

Figure 3. Four predicted halo CMEs. The red dashed ellipses and the grey elliptic areas are obtained using the circular cone and half spheroid ice cream cone models ($\omega_{ye} = \omega_{ze}$ and $\chi = 0$ in Exps (1) and (2)), respectively. The red solid line, h, is the distance between centers of the solar disk and ellipse. It is the projection of the apex-base distance (R_c) on the plane of the sky and aligned with the semi-minor axis. The front and rear half parts are symmetric (asymmetric) for circular (half spheroid ice cream) cone models. There are four (five) measurable parameters for the red ellipse (grey elliptic area): the semi-minor axis, semi-major axis, the length and direction of the red between-centers line, i.e., S_{mn}, S_{mj}, h, and α (S_i).

3.2 Elliptic cone and half ellipsoid ice cream cone models ($\chi = 0$)

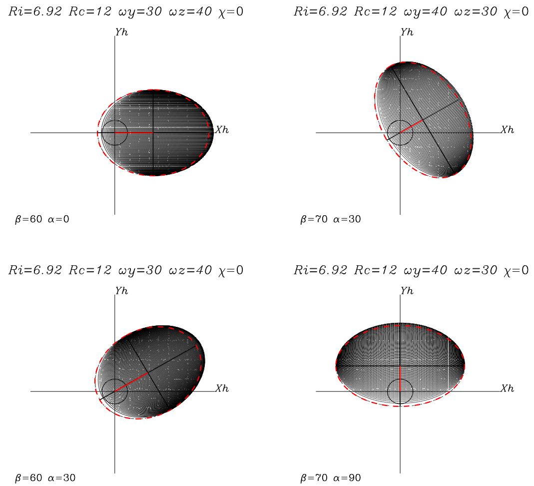

Figure 4. Four predicted halo CMEs. The red dashed ellipses and the grey elliptic areas are obtained using the elliptic cone and half ellipsoid ice cream cone models with $\chi = 0$, respectively. The red solid line, $\underline{h}$, is the distance between centers of the solar disk and ellipse. It is the projection of the apex-base distance (R_c) on the plane of the sky and aligned with ANY semi-axis, minor or major. The front and rear half parts are symmetric (asymmetric) for elliptic (half ellipsoid ice cream) cone models. There are four (five) measurable parameters for the red ellipse (grey elliptic area): the semi-minor axis, semi-major axis, the length and direction of the projection of the apex-base distance, i.e., S_{mn}, S_{mj}, h, and α (S_i).

3.3 Elliptic cone and half ellipsoid ice cream cone models ($\chi \neq 0$)

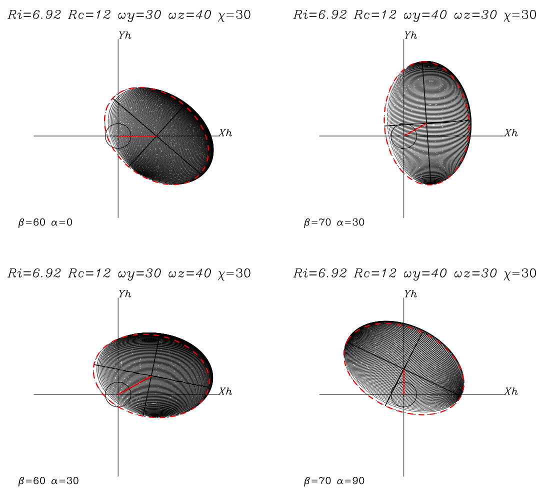

Figure 5. Four predicted halo CMEs. The red dashed ellipses and the grey elliptic areas are obtained using the elliptic cone and half ellipsoid ice cream cone models with $\chi \neq 0$, respectively. The red solid line, h, is the distance between centers of the solar disk and ellipse. It is the projection of the apex-base distance (R_c) on the plane of the sky and NOT aligned with any semi-axis, minor or major. The front and rear half parts are symmetric (asymmetric) for elliptic (half ellipsoid ice cream) cone models. There are five (six) measurable parameters for the red ellipse (grey elliptic area): In addition to S_{mn}, S_{mj}, h, and α (S_i) for the case of $\chi = 0$, there is a new parameter, the angle between the semi-minor axis and the local west (X_h), ψ.

4. Inverted Solutions

• To determine the necessary conditions for forecasting the geoeffectiveness of halo CMEs, i.e., to determine the real angular width, the propagation direction and speed, only the rear half ellipse of halo CMEs is necessary to use. In other words, only circular or elliptic cone models are enough to invert the necessary condition.

• In the coordinate system $X'_cY'_cZ'_c$ where Y'_c and Z'_c are parallel to Y_c and Z_h, respectively, and X'_c is the projection of the central axis of the cone in the plane of the sky (X_hY_h), the relationship between the measurable parameters of observed 2-D halo CMEs and the characteristic geometric parameters of the 3-D CME plasma clouds can be expressed clearly.

• Circular cone model

$$\left(\frac{x'_c - R_c\cos\beta}{R_c\tan\omega\sin\beta}\right)^2 + \left(\frac{y'_c}{R_c\tan\omega}\right)^2 = 1 \tag{5}$$

$$\left\{\begin{array}{lcl} h & = & R_c\cos\beta \\ S_{x'_c} & = & R_c\tan\omega\sin\beta \\ S_{y'_c} & = & R_c\tan\omega \end{array}\right\} \left\{\begin{array}{lcl} \sin\beta & = & S_{x'_c}/S_{y'_c} \\ R_c & = & h/\cos\beta \\ \tan\omega & = & S_{y'_c}/R_c \end{array}\right\} \left\{\begin{array}{l} S_{x'_c} < S_{y'_c} \\ Unique \\ \end{array}\right\}$$

Here $S_{x'_c}$ and $S_{y'_c}$ denote the semi-minor and semi-major axes.

• Elliptic cone model with $\chi = 0$

$$\left(\frac{x'_c - R_c \cos\beta}{R_c \tan\omega_{ze} \sin\beta}\right)^2 + \left(\frac{y'_c}{R_c \tan\omega_{ye}}\right)^2 = 1 \tag{6}$$

the halo CME becomes circular if $\sin\beta = \tan\omega_{ye}/\tan\omega_{ze} < 1$.

$$\left\{\begin{array}{lcl} h & = & R_c \cos\beta \\ S_{x'_c} & = & R_c \tan\omega_{ze} \sin\beta \\ S_{y'_c} & = & R_c \tan\omega_{ye} \end{array}\right\} \left\{\begin{array}{l} S_{x'_c} >=< S_{y'_c} \\ not - unique \\ \end{array}\right\}$$

• Elliptic cone model with $\chi \neq 0$

$$\left\{\begin{array}{lcl} h & = & R_c \cos\beta \\ S_{x'_c} \cos\psi & = & R_c \tan\omega_{ze} (\sin\alpha \sin\chi + \sin\beta \cos\alpha \cos\chi) \\ S_{y'_c} \sin\psi & = & R_c \tan\omega_{ye} (\sin\alpha \cos\chi - \sin\beta \cos\alpha \sin\chi) \\ S_{x'_c} \sin\psi & = & R_c \tan\omega_{ze} (-\cos\alpha \sin\chi + \sin\beta \sin\alpha \cos\chi) \\ S_{y'_c} \cos\psi & = & R_c \tan\omega_{ye} (-\cos\alpha \cos\chi - \sin\beta \sin\alpha \sin\chi) \end{array}\right\} \left\{\begin{array}{l} S_{x'_c} >=< S_{y'_c} \\ not - unique \\ \\ \\ \end{array}\right\}$$

The case of elliptic cone model with $\chi \neq 0$ is more complicated than the case of $\chi = 0$, and the inverted solution is not unique!

5. Summary and Discussions

5.1 What kind of halo CMEs can be inverted?

• Only halo CMEs that are formed fully by Thompson scattering, the characteristic geometrical parameters of cone-like CME plasma clouds can be inverted using the measurable parameters of observed halo CMEs.

5.2 How to select models given a specific halo CME?

• The halo CMEs consist of a rear half ellipse corresponding to the rear half rim of the cone base and a front half part corresponding to the front half rim of the cone base or the top surface of the ice-cream part. If the front half part and the rear half part are symmetric, the CME plasma clouds may be expressed by the circular cone or elliptic cone models. Otherwise, the CME plasma clouds must be expressed by ice-cream cone models, such as the half spheroid and half ellipsoid ice cream cone models.

• The tilt angle of observed elliptic halo CMEs, ψ, i.e. the angle between the semi-minor axis of halo CMEs and the local west direction, X_h, is determined by the orientation of the central axis of the cone-like CMEs and the tilt of the elliptic base of the cone. By comparing ψ with α the question of whether the base of the cone is circular or elliptic may be determined. Therefore, by comparing front with rear half ellipses and comparing ψ with α, the model used to invert the characteristic parameters can be selected (See Table for details).

5.3 The uniqueness of the inverted solution

• For Thompson scattering-formed halo CMEs, the shapes of halo CMEs predicted using the circular cone model and half spheroid cone model provide four parameters that are equal to the number of the chracteristic parameters of these circular cone-like models, and the inverted solution is unique.

• The shapes of halo CMEs predicted using the elliptic cone model provide four or five parameters depending on the tilt of the elliptic base of the cone. Since it is less than the number of characteristic parameters of the elliptic cone, the inverted solution is not unique. For the case of half ellipsoid cone ice cream model, the solution is also not unique. The up-coming multi-point observations from STEREO may finally solve the problem of uniqueness of the inverted solution (See Table for details).

	Parameters for Cone-like M.	Parameters for Predicted Ellipse
Circular C.	$\omega,\ R_c,\ \beta,\ \alpha$	$h,\ \alpha,\ S_{mn},\ S_{mj}$
H. Sphe. C.	$\omega,\ R_c,\ \beta,\ \alpha,\ R_i$	$h,\ \alpha,\ S_{mn},\ S_{mj},\ S_i$
Elliptic C.	$\omega_{ye},\ \omega_{ze},\ R_c,\ \beta,\ \alpha,\ \chi = 0$	$h,\ \alpha,\ S_{mn},\ S_{mj}$
	$\omega_{ye},\ \omega_{ze},\ R_c,\ \beta,\ \alpha,\ \chi$	$h,\ \alpha,\ S_{mn},\ S_{mj},\ \psi$
H. Elli. C.	$\omega_{ye},\ \omega_{ze},\ R_c,\ \beta,\ \alpha,\ \chi = 0,\ R_i$	$h,\ \alpha,\ S_{mn},\ S_{mj},\ S_i$
	$\omega_{ye},\ \omega_{ze},\ R_c,\ \beta,\ \alpha,\ \chi,\ R_i$	$h,\ \alpha,\ S_{mn},\ S_{mj},\ S_i,\ \psi$

$\lvert\psi - \alpha\rvert$	0	90	*Other*
Circular Cone	Unique		
Half Spheroid Cone	Unique		
Elliptic Cone ($\chi = 0$)	Not Unique	Not Unique	
Elliptic Cone			Not Unique
Half Ellipsoid Cone ($\chi = 0$)	Not Unique	Not Unique	
Half Ellipsoid Cone			Not Unique