Problem 3.1. The equation of heat transport in plasma has the form
\[
\frac{\partial T}{\partial t} = \text{div} \chi \nabla T.
\]

(a) Assuming \(\chi=\text{constant}\) find a solution for the initial condition \(T(t = 0) = T_0 \delta(x)\), where \(\delta(x)\) is the delta-function.

(b) Calculate the FWHM (full width at half maximum) of the temperature profile.
Problem 3.2. (Sturrock, p.94) Radio signal from pulsars pass through the interstellar medium that contains free electrons.

(a) Show that the arrival time $t(\nu)$ of a signal will be a function of frequency of the form

$$t(\nu) = D\nu^{-2} + \text{constant},$$

where $\nu$ is the frequency in Hz, and the ‘dispersion coefficient’ $D$ is expressible as

$$D = C \int n_e ds,$$

where the integral represents the path integral of electron density along the propagation of the radio signal.

(b) Find the coefficient $C$.

(c) For a particular pulsar it is found that the signal at 100 MHz arrives 2 sec later than the signal at 200 MHz. What is the value of $D$ for that pulsar?

(d) If the mean electron density is $10^{-1.5} \text{ cm}^{-3}$, what is the distance to the pulsar in centimeters and in parsecs?

(e) What complicating factors are neglected in deriving the above simple expression for time delay as a function of frequency?
Problem 3.3. (Sturrock, p.94) Consider a plasma for which (neglecting the role of the ions) the normalized distribution of electron velocity $v_x$ has the form

$$f_0(v_x) = \begin{cases} 
0, & \text{if } |v_x| > V, \\
1/(2V), & \text{if } |v_x| < V 
\end{cases}$$

(a) Find the plasma dispersion relation.

(b) Define an one-dimensional temperature by the equation

$$\frac{1}{2} T = \frac{1}{2} m \langle v_x^2 \rangle$$

and express the dispersion relation in terms of $T$ rather than $V$. 
Problem 3.4. (Chen, Problem 7-2, p.263) An electron plasma wave with 1-cm wavelength is excited in a 10-eV plasma with \( n = 10^{15} \, \text{m}^{-3} \). The excitation is then removed, and the wave Landau damps away. How long does it take for the amplitude to fall by a factor of \( e \)?

Problem 3.5. (Chen, Problem 4-44, p.152) Intelligent life on a planet in the Crab nebula tries to communicate with us primitive creatures on the earth. We receive radio signals in the \( 10^8 - 10^9 \) Hz range, but the spectrum stops abruptly at 120 MHz. From optical measurements it is possible to place an upper limit of 36 G on the magnetic field in the vicinity of the parent star. If the star is located in the HII region (one which contains ionized hydrogen), and if the radio signals are affected by some sort of cutoff in the plasma there, what is a reasonable lower limit to the plasma density?